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The Comparison Principle of Multidelay Hyperneutral Type Continuous System

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Abstract: We prove the comparison principles of the multidelay hyperneutral constant linear continuous system, the multidelay hyperneutral time-varying linear continuous system, the multidelay hyperneutral non-linear continouous system by inducing step by step, respectively, and obtain the precise sufficient conditions.

Key words: multidelay; hyperneutral type; comparison principle
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多滞后超中立型连续系统的比较原理

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摘要: 采用归纳法, 逐次分段证明了多组多滞后定常线性连续系统, 多组多滞后时变线性连续系统, 多组多滞后非线性连续系统的比较原理, 获得了若干简洁的充分条件.

关键词: 多滞后; 超中立型; 比较原理

1 Introduction

There have been perfect results on the comparison principle of the general linear continuous system without delay and the multidelay neurtral type continuous system[1~3]. But the extensive application of the comparison principle in the study of the stability of the time-varying large scale system, the robust stability of the interval coefficient (large) system, the stability of some uncertain (large) system and the robust stability of its different control system, and the study of the unconditional stability of multi-group delay time-varying continuous system with the frequency domain method, make it very important and necessary to study the comparison principle of the multidelay hyperneutral type continuous system. In this paper, we provided the comparison principle of the multidelay hypemeutral type continuous system and obtained the precise sufficient conditions.

2 Main results

Consider the multidelay hyepemeurtral type constant linear differential inequality

$$\dot{X}(t) \leqslant A^{(0)}X(t) + \sum_{r=1}^{N_1} L_1^{(r)}(X,t,\tau_1^{(r)}) + \sum_{l=1}^{N_2} L_2^{(l)}(X,t,\tau_2^{(l)}) + \sum_{p=1}^{N_3} L_3^{(p)}(X,t,\tau_3^{(p)}), \tag{1}$$

here $A^{(0)} = (a_{ij}^{(0)})_{n\times n}$ are constant matrixes, $G(t,\theta)$ is the function matrix about t,θ,

$X(t) = (x_1(t), x_2(t), \cdots, x_n(t))^{\mathrm{T}}$,

$\tau_1^{(r)} = \tau_{1ij}^{(r)}$, $\tau_2^{(l)} = \tau_{2ij}^{(l)}$,

$\tau_{1ij}^{(r)} > 0$, $\tau_{2ij}^{(l)} > 0$, $0 < S < \tau^{(p)}$,

$i,j = 1,2,\cdots,n$, $r = 1,2,\cdots,N_1$,

$l = 1,2,\cdots,N_2$, $p = 1,2,\cdots,N_3$,

$G(t,\theta) = (g_{ij}(t,\theta))_{n\times n}$,

$L_1^{(r)}(X,t,\tau_1^{(r)}) =$
$(L_{11}^{(r)}(X,t,\tau_{11}^{(r)}), L_{12}^{(r)}(X,t,\tau_{12}^{(r)}), \cdots, L_{1n}^{(r)}(X,t,\pi_{1n}^{(r)}))^{\mathrm{T}}$,

$L_{1i}^{(r)}(X,t,\tau_{1i}^{(r)}) =$
$(a_{1i1}^{(r)}, a_{1i2}^{(r)}, \cdots, a_{1in}^{(r)})(x_1(t-\tau_{1i1}^{(r)}), \cdots, x_n(t-\tau_{1in}^{(r)}))^{\mathrm{T}}$,

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$L_2^{(l)}(X,t,\tau_2^{(l)}) =$
$(L_{21}^{(l)}(X,t,\tau_{21}^{(l)}), L_{22}^{(l)}(X,t,\tau_{22}^{(l)}),\cdots,L_{2n}^{(l)}(X,t,\pi_{2n}^{(l)}))^{\mathrm{T}}$,
$L_{2i}^{(l)}(X,t,\tau_{2i}^{(l)}) =$
$(a_{2i1}^{(l)}, a_{2i2}^{(l)},\cdots,a_{2in}^{(l)})(x_2(t-\tau_{2i1}^{(l)}),\cdots,x_n(t-\tau_{2in}^{(l)}))^{\mathrm{T}}$,
$L_3^{(p)}(X,t,\tau_3^{(p)}) =$
$(L_{31}^{(p)}(X,t,\tau_{31}^{(p)}), L_{32}^{(p)}(x,t,\tau_{32}^{(p)})\cdots,L_{3n}^{(p)}(X,t,\pi_{3n}^{(p)}))^{\mathrm{T}}$,
$L_{3i}^{(p)}(X,t,\tau_{3i}^{(p)}) =$
$(a_{3i1}^{(p)}, a_{3i2}^{(p)},\cdots,a_{3in}^{(p)})(\int_{-\tau_3^{(p)}}^{-s}\sum_{q=1}^{n} g_{1q}(t,\theta)\dot{x}_q(t+\theta)\mathrm{d}\theta,\cdots,\int_{-\tau_3^{(p)}}^{-s}\sum_{q=1}^{n} g_{nq}(t,\theta)\dot{x}_q(t+\theta)\mathrm{d}\theta)^{\mathrm{T}}$.

Suppose that $G(t,\theta)$ is bounded, and $\| G(t,\theta)\| \leqslant M$, then we can rewrite (1) as follows:

$$\dot{x}_i(t) \leqslant \sum_{j=1}^{n} a_{ij}^{(0)}x_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)}x_j(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)}\dot{x}_j(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)}\int_{-\tau^{(p)}}^{-s}\dot{x}_j(t+\theta)\mathrm{d}\theta = \sum_{j=1}^{n} a_{ij}^{(0)}x_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)}x_j(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)}\dot{x}_j(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)}\int_{0}^{\tau^{(p)}-s}\dot{x}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta,\ (i=1,2,\cdots,n), \tag{2}$$

moreover, we denote

$$\begin{cases} h = \max\{\tau_{1ij}^{(r)},\tau_{2ij}^{(l)},\tau^{(p)}: i,j=1,2,\cdots,n, \\ \quad r=1,2,\cdots,N_1, l=1,2,\cdots,N_2,\ p=1,2,\cdots,N_3\}, \\ \tau = \min\{\tau_{1ij}^{(r)},\tau_{2ij}^{(l)},\tau^{(p)}: i,j=1,2,\cdots,n, \\ \quad r=1,2,\cdots,N_1, l=1,2,\cdots,N_2,\ p=1,2,\cdots,N_3\}. \end{cases} \tag{3}$$

Suppose that we had given continuous differential initial function $x_{0j}(t)$ for $t_0 - h \leqslant t \leqslant t_0$, and

$$\begin{cases} x_j(t) = x_{0j}(t),\ \dot{x}_j(t) = \dot{x}_{0j}(t), \\ \int_0^{\tau^{(p)}-s}\dot{x}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s}\dot{x}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \\ (j=1,2,\cdots,n,\ 1\leqslant p\leqslant N_3,\ t_0-h\leqslant t\leqslant t_0), \end{cases} \tag{4}$$

we can describe the multidelay hyperneutral constant linear comparison system of (2) as follows

$$\dot{y}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)}y_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)}y_j(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)}\dot{y}_j(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)}\int_0^{\tau^{(p)}-s}\dot{y}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta,\ (i=1,2,\cdots,n), \tag{5}$$

with initial function $y_{0j}(t)$, and

$$\begin{cases} y_j(t) = y_{0j}(t),\ \dot{y}_j(t) = \dot{y}_{0j}(t), \\ \int_0^{\tau^{(p)}-s}\dot{y}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s}\dot{y}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \\ (j=1,2,\cdots,n,\ 1\leqslant p\leqslant N_3,\ t_0-h\leqslant t\leqslant t_0). \end{cases} \tag{6}$$

Suppose that the coefficients of (2) and (5) satisfy the condition

$$\begin{cases} a_{ii}^{(0)} < 0,\ a_{ij}^{(0)} \geqslant 0\ (j\neq i), \\ a_{1ij}^{(r)} \geqslant 0,\ a_{2ij}^{(l)} \geqslant 0,\ a_{3ij}^{(p)} \geqslant 0, \\ (i,j=1,2,\cdots,n,\ r=1,\cdots,N_1, \\ l=1,\cdots,N_2,\ p=1,\cdots,N_3). \end{cases} \tag{7}$$

And $x_j(t)$ and $y_j(t)(j=1,2,\cdots,n)$ are separately the solutions of inequality (2) and the comparison system (5), we have

Theorem 1 Suppose that the coefficients of the multidelay hyperneural linear constant differential inequality (2) and the comparison (5) satisfy conditions (7), namely, the right function of (5) is quasi-monotonic increasing function about its variable, and $\| G(t,\theta)\| \leqslant M$, then the initial functions of (2) and (5) satisfying

$$x_{0j}(t) \leqslant y_{0j}(t),\ \dot{x}_{0j}(t)\leqslant \dot{y}_{0j}(t),$$
$$\int_0^{\tau^{(p)}-s}\dot{x}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta \leqslant \int_0^{\tau^{(p)}-s}\dot{y}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta,$$
$$(j=1,2,\cdots,n,\ 1\leqslant p\leqslant N_3,\ t_0-h\leqslant t\leqslant t_0),$$

imply

$$x_j(t) \leqslant y_j(t),\ (j=1,2,\cdots n,\ t_0-h\leqslant t<\infty).$$

When we consider the multidelay hyperneutral type linear time-varying differential inequality

$$\dot{X}(t) \leqslant A^{(0)}(t)X(t) + \sum_{r=1}^{N_1}A_1^{(r)}(t)X(t-\tau_1^{(r)}) + \sum_{l=1}^{N^2}A_2^{(l)}(t)\dot{X}(t-\tau_2^{(l)}) + \sum_{p=1}^{N^3}A_3^{(p)}(t)\int_{-\tau^{(p)}}^{-s}G(t,\theta)\dot{X}(t+\theta)\mathrm{d}\theta,$$

or

$$\dot{x}_i(t) \leqslant \sum_{j=1}^{n} a_{ij}^{(0)}(t)x_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)}(t)x_j(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)}(t)\dot{x}_j(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)}(t)\int_0^{\tau^{(p)}-s}\dot{x}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta,$$

$$(\| G(t,\theta) \| \leqslant M,\ i = 1,2,\cdots,n), \tag{8}$$

and the corresponding comparison system

$$\dot{y}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)}(t)y_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)}(t)y_j(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)}(t)\dot{y}_j(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)}(t)\int_0^{\tau^{(p)}-s}\dot{y}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta,\ (i = 1,2,\cdots,n). \tag{9}$$

Suppose that for $t_0 - h \leqslant t < \infty$, the coefficients of (8) and (9) satisfy

$$\begin{cases} a_{ii}^{(0)}(t) < 0,\ a_{ij}^{(0)}(t) \geqslant 0 (j \neq i), \\ a_{1ij}^{(r)}(t) \geqslant 0,\ a_{2ij}^{(l)}(t) \geqslant 0, a_{3ij}^{(p)}(t) \geqslant 0, \\ (i,j = 1,2,\cdots,n,\ r = 1,\cdots,N_1, \\ l = 1,\cdots,N_2,\ p = 1,\cdots,N_3), \end{cases} \tag{10}$$

then, we have

Theorem 2 Suppose that the coefficients of the multidelay hyperneutral linear time-varying differential inequality (8) and the comparison system (9) satisfy conditions (10), namely, the right function of (9) is quasi-monotonic increasing function about its variable, and $\| G(t,\theta \| \leqslant M$, then the initial function of (8) and (9) satisfying

$$x_{0j}(t) \leqslant y_{0j}(t),\ \dot{x}_{0j}(t) \leqslant \dot{y}_{0j}(t),$$

$$\int_0^{\tau^{(p)}-s}\dot{x}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta \leqslant \int_0^{\tau^{(p)}-s}\dot{y}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta,$$

$$(j = 1,2,\cdots,n,\ 1 \leqslant p \leqslant N_3,\ t_0-h \leqslant t \leqslant t_0),$$

imply $x_j(t) \leqslant y_j(t), (j = 1,2,\cdots,n, t_0 - h \leqslant t < \infty)$, where $x_j(t)$ and $y_j(t)(j = 1,2,\cdots,n)$ are respectively the solutions of inequality (8) and the comparison system (9).

Furtherly, for the multidelay hyperneutral type nonlinear differential inequality

$$\dot{x}_i(t) \leqslant f_i(t, x_1(t),\cdots,x_n(t), x_1(t-\tau_{1i1}^{(1)}),\cdots,x_n(t-\tau_{1in}^{(1)}), \dot{x}_1(t-\tau_{2i1}^{(1)}),\cdots,\dot{x}_n(t-\tau_{2in}^{(1)}),\cdots,x_n(t-\tau_{2in}^{(N_2)}), M\int_{-\tau^{(1)}}^{-s}\dot{x}_1(t+\theta)\mathrm{d}\theta,\cdots,M\int_{-\tau^{(1)}}^{-s}\dot{x}_n(t+\theta)\mathrm{d}\theta,\cdots, M\int_{-\tau^{(N_3)}}^{-s}\dot{x}_n(t+\theta)\mathrm{d}\theta),\ (i = 1,2,\cdots,n), \tag{11}$$

and its comparison system

$$\dot{y}_i(t) \leqslant f_i(t, y_1(t),\cdots,y_n(t), y_1(t-\tau_{1i1}^{(1)}),\cdots,y_n(t-\tau_{1in}^{(1)}),\cdots,y_n(t-\tau_{1in}^{(N_1)}), \dot{y}_1(t-\tau_{2i1}^{(1)}),\cdots,\dot{y}_{11}(t-\tau_{2in}^{(1)}),\cdots,\dot{y}_{in}(t-\tau_{2in}^{(N_2)}), M\int_{-\tau^{(1)}}^{-s}\dot{y}_1(t+\theta)\mathrm{d}\theta,\cdots,M\int_{-\tau^{(1)}}^{-s}\dot{y}_n(t+\theta)\mathrm{d}\theta\cdots, M\int_{-\tau^{(N_3)}}^{-s}\dot{y}_n(t+\theta)\mathrm{d}\theta),\ (i = 1,2,\cdots,n), \tag{12}$$

we also have

Theorem 3 Suppose $f_i(i = 1,2,\cdots,n)$ are quasi-monotonically increasing about all variable, and the initial functions $x_{0j}(t)$ and $y_{0j}(t)$ satisfy

$$x_{0j}(t) \leqslant y_{0j}(t), \dot{x}_{0j}(t) \leqslant \dot{y}_{0y}(t),$$

$$\int_0^{\tau^{(p)}-s}\dot{x}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta \leqslant \int_0^{\tau^{(p)}-s}\dot{y}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta,$$

$$(j = 1,2,\cdots,n,\ 1 \leqslant p \leqslant N_3,\ t_0-h \leqslant t \leqslant t_0),$$

then $x_j(t) \leqslant y_j(t), (j = 1,2,\cdots,n, t_0 - h \leqslant t < \infty)$, where $x_j(t)$ and $y_j(t)(j = 1,2,\cdots,n)$ are respectively the solutions of inequality (11) and comparison system (12).

3 The proof of the theorems

The proof methods of the three theorems in Section 1 are similar, so we only give the proof of Theorem 1.

The proof of Theorem 1. For $t_0 \leqslant t \leqslant t_0 + \tau$, and each $i: 1 \leqslant i \leqslant n$, we have

$$\begin{cases} x_j(t-\tau_{1ij}^{(r)}) = x_{0j}(t-\tau_{1ij}^{(r)}) \leqslant y_{0j}(t-\tau_{1ij}^{(r)}) = y_j(t-\tau_{1ij}^{(r)}), \\ \dot{x}_j(t-\tau_{1ij}^{(r)}) = \dot{x}_{0j}(t-\tau_{1ij}^{(r)}) \leqslant \dot{y}_{0j}(t-\tau_{1ij}^{(r)}) = \dot{y}_j(t-\tau_{1ij}^{(r)}), \\ \int_0^{\tau^{(p)}-s}\dot{x}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s}\dot{x}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta \leqslant \\ \int_0^{\tau^{(p)}-s}\dot{y}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s}\dot{y}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \\ (j = 1,2,\cdots,n), \end{cases} \tag{13}$$

because $x_{0j}(t)$ and $y_{0j}(t)$ are initial continuous functions, in $t_0 \leqslant t \leqslant t_0+\tau$, putting (13) into (2) and (5), we can get

$$\dot{x}_i(t) \leqslant \sum_{j=1}^{n} a_{ij}^{(0)} x_j(t) + \sum_{r=1}^{1}\sum_{j=1}^{n} a_{1ij}^{(r)} x_{0j}(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{x}_{0j}(t-\tau_{2ij}^{(r)}) + M\sum_{l=1}^{3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau-s} \dot{x}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \quad (i=1,\cdots,n) \tag{14}$$

$$\dot{y}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)} y_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{0j}(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{y}_{0j}(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{0j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \quad (i=1,\cdots,n) \tag{15}$$

thus, (14) and (15) are separately non-delay linear constant differential inequality and its linear constant comparison system.

In $t_0 \leqslant t \leqslant t_0+\tau$, for each $i: 1 \leqslant i \leqslant n$, if

$$\begin{cases} y_j' \leqslant y_j'',\ y_{0j}' \leqslant y_{0j}'',\ \dot{y}_{0j}' \leqslant \dot{y}_{0j}'', \\ \int_0^{\tau^{(p)}-s} y_{0j}'\mathrm{d}\theta \leqslant \int_0^{\tau^{(p)}-s} y_{0j}''\mathrm{d}\theta, (j \neq i), \\ y_i' = y_i'',\ y_{0i}' = y_{0i}'',\ \dot{y}_{0i}' = \dot{y}_{0i}'', \\ \int_0^{\tau^{(p)}-s} \dot{y}_{0i}'\mathrm{d}\theta = \int_0^{\tau^{(p)}-s} \dot{y}_{0i}''\mathrm{d}\theta, \\ (j=i,\ j=1,2,\cdots,n,\ 1 \leqslant p \leqslant N_3), \end{cases} \tag{16}$$

from the coefficient conditions (7) and (16), and for $i, j = 1,2,\cdots,n, 1 \leqslant p \leqslant N_3$, then we obtain

$$\sum_{j=1}^{n} a_{ij}^{(0)} y_j' + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{0j}' + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{y}_{0j}' + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{0j}'\mathrm{d}\theta \leqslant \sum_{j=1}^{n} a_{ij}^{(0)} y_j'' + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{0j}'' + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{y}_{0j}'' + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{0j}''\mathrm{d}\theta. \tag{17}$$

For $t_0 \leqslant t \leqslant t_0+\tau$, from (17) and the comparison principle of the general linear constant system without delay[1], we can get $x_j(t) \leqslant y_j(t), (j=1,2,\cdots,n)$.

Furthermore, in $t_0+(m-1)\tau \leqslant t \leqslant t_0+m\tau$, we may assume that $x_{mj}(t)$ and $y_{mj}(t)$ are respectively solutions of non-delay linear constant differential inequality

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)} x_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} x_{m-1,j}(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{x}_{m-1,j}(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{m-1,j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \quad (i=1,2,\cdots,n), \tag{18}$$

and its linear constant comparison system

$$\dot{y}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)} y_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{m-1,j}(t-\tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{y}_{m-1,j}(t-\tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(r)} \int_0^{\tau^{(p)}-s} \dot{y}_{m-1,j}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \quad (i=1,2,\cdots,n) \tag{19}$$

and

$$\begin{cases} x_j(t) \leqslant y_j(t), \\ z_j(t) = z_{mj}(t),\ \dot{z}_j(t) = \dot{z}_{mj}(t), \\ \int_0^{\tau^{(p)}-s} \dot{z}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s} \dot{z}_{mj}(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \\ z_{m-1,j}(t_0+(m-1)\tau) = z_{mj}(t_0+(m-1)\tau), \\ \dot{z}_{m-1,j}(t_0+(n-1)\tau) = z_{mj}(t_0+(m-1)\tau), \\ \int_0^{\tau^{(p)}-s} \dot{z}_{m-1j},(t_0+(m-1)\tau-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s} \dot{z}_{mj}(t_0+(m-1)\tau-\tau^{(p)}+\theta)\mathrm{d}\theta, \\ z=x,y,\ j=1,2,\cdots,n,\ 1 \leqslant p \leqslant N_3. \end{cases} \tag{20}$$

Now we will prove that there is Theorem's result in $t_0+m\tau \leqslant t \leqslant t_0+(m+1)\tau$, by (20), we have

$$\begin{cases} x_j(t-\tau_{1ij}^{(r)}) = x_{mj}(t-\tau_{1ij}^{(r)}) \leqslant y_{mj}(t-\tau_{1ij}^{(r)}) = y_j(t-\tau_{1ij}^{(r)}), \\ \dot{x}_j(t-\tau_{1ij}^{(r)}) = \dot{x}_{mj}(t-\tau_{1ij}^{(r)}) \leqslant \dot{y}_{mj}(t-\tau_{1ij}^{(r)}) = \dot{y}_j(t-\tau_{1ij}^{(r)}), \\ \int_0^{\tau^{(p)}-s} \dot{x}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s} \dot{x}_{mj}(t-\tau^{(p)}+\theta)\mathrm{d}\theta \leqslant \\ \int_0^{\tau^{(p)}-s} \dot{y}_{mj}(t-\tau^{(p)}+\theta)\mathrm{d}\theta = \int_0^{\tau^{(p)}-s} \dot{y}_j(t-\tau^{(p)}+\theta)\mathrm{d}\theta, \end{cases} \tag{21}$$

and putting (21) into (2) and (5), we obtain for $1 \leqslant p \leqslant N_3$,

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)} x_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} x_{mj}(t - \tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{x}_{mj}(t - \tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{x}_{mj}(t - \tau^{(p)} + \theta)\mathrm{d}\theta,$$
$$(i = 1,\cdots,n) \quad (22)$$

and

$$\dot{y}_i(t) = \sum_{j=1}^{n} a_{ij}^{(0)} y_j(t) + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{mj}(t - \tau_{1ij}^{(r)}) + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{y}_{mj}(t - \tau_{2ij}^{(l)}) + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{mj}(t - \tau^{(p)} + \theta)\mathrm{d}\theta.$$
$$(i = 1,\cdots,n) \quad (23)$$

In $t_0 + \tau m \leqslant t \leqslant t_0 + (m+1)\tau$, for each $i: 1 \leqslant i \leqslant n$, for $j = 1,2,\cdots,n$, $1 \leqslant p \leqslant N_3$, if

$$\begin{cases} y_j' \leqslant y_j'',\ y_{mj}' \leqslant y_{mj}'',\ \dot{y}_{mj}' \leqslant \dot{y}_{mj}'', \\ \int_0^{\tau^{(p)}-s} \dot{y}_{mj}' \mathrm{d}\theta \leqslant \int_0^{\tau^{(p)}-s} \dot{y}_{mj}'' \mathrm{d}\theta, (j \neq i), \\ y_i' = y_i'',\ y_{mi}' = y_{mi}'',\ \dot{y}_{mi}' = \dot{y}_{mi}'', \\ \int_0^{\tau^{(p)}-s} \dot{y}_{mi}' \mathrm{d}\theta = \int_0^{\tau^{(p)}-s} \dot{y}_{mi}'' \mathrm{d}\theta, (j = i), \end{cases} \quad (24)$$

from coefficient conditions (7) and (24), for $i, j = 1, 2, \cdots, n, 1 \leqslant p \leqslant N_3$, we have

$$\sum_{j=1}^{n} a_{ij}^{(0)} y_j' + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{0j}' + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(l)} \dot{y}_{0j}' + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{0j}' \mathrm{d}\theta \leqslant$$
$$\sum_{j=1}^{n} a_{ij}^{(0)} y_j'' + \sum_{r=1}^{N_1}\sum_{j=1}^{n} a_{1ij}^{(r)} y_{0j}'' + \sum_{l=1}^{N_2}\sum_{j=1}^{n} a_{2ij}^{(r)} \dot{y}_{0j}'' + M\sum_{p=1}^{N_3}\sum_{j=1}^{n} a_{3ij}^{(p)} \int_0^{\tau^{(p)}-s} \dot{y}_{0j}'' \mathrm{d}\theta. \quad (25)$$

By (25) and the comparison principle of nodelay linear constant system, we can obtain

$$x_j(t) \leqslant y_j(t),$$
$$(j = 1,2,\cdots,n, t_0 + m\tau \leqslant t \leqslant t_0 + (m+1)\tau).$$

As this going on, we can prove that the solutions of the multidelay hyperneutral type constant inequality (2) and its comparison system (5) satisfy

$$x_j(t) \leqslant y_j(t),\ (j = 1,2,\cdots,n,\ t_0 - h \leqslant t \leqslant \infty).$$

Remark With respect to one dimensional multidelay hyperneutral type linear continuous system

$$\dot{x}(t) \leqslant ax(t) + bx(t - \tau_1) + c\dot{x}(t - \tau_2) + d\int_{-\tau_3}^{0} g(t,\theta)\dot{x}(t + \theta)\mathrm{d}\theta,$$

where a, b, c, d are proper constant number, $g(t,\theta)$ is continuous about t, θ. If $|g(t,\theta)| \leqslant M$, then the above inequality can be reduced to one dimensional multidelay neutral type linear continuous system

$$\dot{x}(t) \leqslant (a + dM)x(t) + bx(t - \tau_1) + dMx(t - \tau_3) + c\dot{x}(t - \tau_2),$$

if $c = d = 0$, one dimensional hyperneutral type multidelay linear continuous system can be reduced to one dimensional multidelay system $\dot{x}(t) \leqslant ax(t) + bx(t - \tau_1)$.

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