第 18 卷第 6 期
2001 年 12 月

控制理论与应用
CONTROL THEORY AND APPLICATIONS

Vol.18 No.6
Dec. 2001

Article ID: 1000 - 8152(2001)06 - 0817 - 06

On Leader-Follower Model of Traffic Rate Control for Networks *

JING Yuanwei and CHEN Bing
(Faculty of Information Science and Engineering, Northeastern University · Shenyang, 110006, P.R.China)
Georgi.M.Dimirovski
(Faculty of Electrical Engineering. Saint Cyril and Saint Methodius University · Skopje. MK-91000, R.Macedonia)
Khosrow Sohraby
(Faculty of Computer Science Telecommunications, University of Missouri-Kansas City · Kansas City. MO 64110, U.S.A.)

Abstract: This paper deals with traffic rate control problems of networks. The incentive Stackelberg strategy concept was introduced to the networking model that comprises subsidiary systems of users and network. A linear strategy and a nonlinear strategy were proposed to the elastic traffic problem. The results were then extended to the non-elastic traffic problem. Numerical examples and simulations were given to illustrate the proposed method.

Key words: rate control; pricing; elastic traffic; non-elastic traffic; incentive Stackelberg strategy
Document code: A

网络通信量控制的主从模型

井元伟 陈 兵 乔治·迪米罗夫斯基
(东北大学信息科学与工程学院·沈阳,110006) (圣西里尔和麦色迪大学电子工程学院·基科普里,MK-91000,马其顿)
科斯罗·索拉比
(密苏里大学堪萨斯城分校计算机科学远程通讯学院·堪萨斯,MO 64110,美国)

摘要: 讨论了网络的通信量控制问题.将激励 Stackelberg 策略的概念引入到具有用户和网络两层子系统的网络模型中.针对弹性通信量问题.提出了一个线性策略和一个非线性策略.并将此方法扩展到非弹性通信量情形.数值例子及仿真结果说明了此方法的适用性.

关键词: 比率控制; 价控; 弹性通信量; 非弹性通信量; 激励 Stackelberg 策略

1 Introduction

We focus on the system model of charging, routing and flow control, where the system comprises both users with utility functions and a network with capacity constraints. Kelly[1] showed that the optimization of the system may be decomposed into subsidiary optimization problems, one for each user and one for the network, by using price per unit flow as a Lagrange multiplier that mediates between the subsidiary problems. Low and Varaiya[2] and Murphy et al[3] described how such results may be used as the basis for distributed pricing algorithms, and MacKie-Mason and Varian[4] described a "smart market" based on a per-packet charge when the network is congested.

As mentioned in Kelly's work[1], price per unit flow is the mediating variable. The system optimum can be achieved when users' choice of charges and the network's choice of allocated rates are in equilibrium. The equilibrium exists for elastic traffic systems. But for most nonelastic traffic, the equilibrium does not exist and the system optimum can not be achieved.

By using the incentive Stackelberg strategy concept, we try to find a new way to deal with such a kind of routing control problems. In a game theoretic model[5,6], there are at least two players who control their own inputs to make the state of the system to reach their own outcomes from the system, respectively. Therefore, the game theory[7] provides a systematic framework

* Foundation item: supported by Foundation for University Key Teacher by Ministry of Education and Foundation for Returned Scholar Abroad by Ministry of Education.
Received date: 2000 - 07 - 10; Revised date: 2001 - 07 - 06.

to treat the dynamic behavior of noucooperative networks. Two major concepts in game theory, Nash and Stackelberg equilibria, have been applied to the study of noncooperative networks[8-11]. In these references, the game theoretic models are all based on the classical noncooperate strategy concepts.

In this paper, we consider the traffic rate control problem by means of the incentive Stackelberg strategy, which was introduced into the game theory by Ho et al[12]. The incentive strategy proposed here consists of two parts, one of them being the regular price, and the other being punishment price which varies on the variance of traffic rate linearly or functionally.

2 System model

Consider a network with a set J of resources, and let C_j be the finite capacity of resource j, for $j \in J$. A set S of users use the network with rates $x = (x_1, x_2, \cdots, x_s)$. For each user s, the utility maximization is as follows.

$\mathrm{USER}_s(U_s;\lambda_s)$:

$$\text{maximize} \quad U_s(x_s) - \lambda_s x_s,$$
$$\text{over} \quad x_s \geqslant 0, \tag{1}$$

where $U_s(x_s)$ is the utility function of user s, and it is an increasing, strictly concave and continuously differentiable function of x_s over the range $x_s \geqslant 0$. λ_s is a price charged to user s per unit flow, and also is the component of the vectors of Lagrange multipliers for the following problem of overall system.

$\mathrm{SYSTEM}(U, H, A, C)$:

$$\text{maximize} \quad \sum_{s=1}^{S} U_s(x_s),$$
$$\text{subject to} \quad Hy = x,\ Ay \leqslant C,$$
$$\text{over} \quad x, y \geqslant 0, \tag{2}$$

where H and A are the 0 – 1 matrixes, y is the flow pattern. And the Lagrangian form of the problem is

$$L(x, y, z; \lambda, \mu) = \sum_{s=1}^{S} U_s(x_s) - \lambda^{\mathrm{T}}(x - Hy) + \mu^{\mathrm{T}}(C - Ay - z), \tag{3}$$

where z is a vector of slack variables.

If the network receives a revenue λ_s per unit flow from user s, then the revenue optimization problem for the network is as follows.

$\mathrm{NETWORK}(H, A, C; \lambda)$:

$$\text{maximize} \quad \sum_{s=1}^{S} \lambda_s x_s,$$
$$\text{subject to} \quad Hy = x, Ay \leqslant C,$$
$$\text{over} \quad x, y \geqslant 0. \tag{4}$$

From Kelly's work[1], there exists a price vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_s)$ such that the vector $x = (x_1, x_2, \cdots, x_S)$, formed from the unique solution x_s to $\mathrm{USER}_s(U_s; \lambda_s)$ for each $s \in S$, solves $\mathrm{NETWORK}(H, A, C; \lambda)$. The vector x then also solves $\mathrm{SYSTEM}(U, H, A, C)$.

The problem $\mathrm{NETWORK}(H, A, C; \lambda)$ is just opposite to the problem $\mathrm{USER}_s(U_s; \lambda_s)$, because there are the same parts $\lambda_s x_s$ in their formulations with the opposite symbols. So, they are non-cooperate in general.

We introduce the Stackelberg strategy in game theory to the model[5]. The network should be the leader in the game, and the users should be the followers who act at the Nash equilibrium among them. Note that both of the users and network are allowed to freely vary the flow x_s, $s = 1, 2, \cdots, S$. So if the leader wants users to be at the rates which are arranged by the network, the leader must have the leadership in the game which is indicated in the following Stackelberg strategy.

$$\xi_s(x_s) = \lambda_s + p_s(x_s) - p_s(x_s^a), \tag{5}$$

where $p_s(x_s)$ is any function of x_s to be determined. x_s^a is a desired point arranged by the network. Generally, we can take $p_s(x_s)$ as a linear function

$$p_s(x_s) = q_s x_s, \tag{6}$$

where q_s is some kind of punishment price. It will be determined by the leader.

3 Incentive strategy for elastic traffic problem

3.1 Linear incentive strategy

In this section, we consider the linear function (6) as the Stackelberg incentive strategy to force users to act at the point x_s^a. Replacing λ_s in (1) by ξ_s with linear structure, the problem $\mathrm{USER}_s(U_s; \lambda_s)$ becomes

$\mathrm{USER}_s(U_s; q_s)$:

$$\text{maximize} \quad U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^a)x_s,$$
$$\text{over} \quad x_s \geqslant 0. \tag{7}$$

To get q_s, calculate the derivative of (7) with respect to x_s, and let it be zero. Then let x_s take the value at x_s^a. We get

$$q_s = \frac{U_s'(x_s^a) - \lambda_s}{x_s^a}. \tag{8}$$

So, the strategy should be

$$\xi_s(x_s) = \lambda_s + \frac{U_s'(x_s^a) - \lambda_s}{x_s^a}(x_s - x_s^a). \tag{9}$$

The following work is intended to prove that (9) is an incentive Stackelberg strategy, i.e. (see [5])

$$\xi_s(x_s^a) = \lambda_s \tag{10}$$

and

$$\arg\max[U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^a)x_s] = x_s^a. \tag{11}$$

It is obvious that (10) is held from the structure of $\xi_s(x_s)$ in (9), Eq. (11) means that the following inequality should hold

$$U_s(x_s^a) - \lambda_s x_s^a \geqslant U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^a)x_s, \quad \forall x_s \neq x_s^a. \tag{12}$$

Now, denote by x_s^u the optimal rate of user s which maximizes the problem USER$_s(U_s;\lambda_s)$. So we have

$$U_s'(x_s^u) - \lambda_s = 0 \tag{13}$$

and

$$U_s'(x_s) - \lambda_s > 0, \text{ if } x_s < x_s^u, \tag{14}$$

$$U_s'(x_s) - \lambda_s < 0, \text{ if } x_s > x_s^u. \tag{15}$$

If the optimal rates of users coincide with the arranged rates of the network, i.e. $x_s^u = x_s^a$, then preferred rate of user s is just x_s^a. Therefore,

$$U_s(x_s^a) - \lambda_s x_s^a = U_s(x_s^u) - \lambda_s x_s^u > U_s(x_s) - \lambda_s x_s. \tag{16}$$

So (12) is satisfied in the case of $x_s^u = x_s^a$.

If $x_s^u \neq x_s^a$, we have two cases to discuss.

i) $x_s^u > x_s^a$. Denote by $V_s(x_s)$ the entire utility function of user s in problem USER$_s(x_s;q_s)$. Substituting (8) into $V_s(x_s)$, and calculating the first and second derivatives of (7) with respect to x_s, we can see $V_s'(x_s^a) = 0$ and $V_s''(x_s) < 0$. So we can come to the conclusion that $V_s(x_s^a) > V_s(x_s)$. i.e. Eq.(12) holds.

ii) $x_s^u < x_s^a$. Actually, it could not occur in this case. It is evident that $x_s^u = C$. At most, $x_s^a = C$. So there cannot be $x_s^u < x_s^a$.

3.2 Non-linear incentive strategy

In this section, we deal with such a non-linear function as

$$p_s(x_s) = \begin{cases} \lambda_s(x_s^a - x_s)/x_s, & \text{if } x_s < x_s^a, \\ 0, & \text{if } x_s = x_s^a, \\ (U_s(x_s) - U_s(x_s^a))/x_s, & \text{if } x_s > x_s^a. \end{cases} \tag{17}$$

It is easy to see that (5) becomes $\xi_s(x_s) = \lambda_s$, if $x_s = x_s^a$. It is just the first condition (10). To meet the second condition, substitute (5) into (1) with the structure described in (17). The problem USER$_s(U_s;\lambda_s)$ becomes

$$\text{USER}_s(U_s;p_s(\cdot)): \quad \text{maximize } W_s(x_s), \quad \text{over } x_s \geqslant 0, \tag{18}$$

where $W_s(x_s) = U_s(x_s) - \lambda_s x_s - p_s(x_s)x_s$.

We choose $p_s(x_s) = \lambda_s(x_s^a - x_s)/x_s$, when $x_s < x_s^a$. We have $W_s(x_s) = U_s(x_s) - \lambda_s x_s^a$. So, we can see $W_s(x_s) < W_s(x_s^a)$ for $U_s(x_s) < U_s(x_s^a)$.

We choose $p_s(x_s) = (U_s(x_s) - U_s(x_s^a))/x_s$, when $x_s > x_s^a$. We have $W_s(x_s) = U_s(x_s^a) - \lambda_s x_s$. So, we can see $W_s(x_s) < W_s(x_s^a)$ for $\lambda_s x_s > \lambda_s x_s^a$.

In both cases, it is shown that $W_s(x_s^a) > W_s(x_s)$ which indicates the satisfaction of the second condition for incentive strategy.

4 Incentive strategy for non-elastic traffic problems

In a practical network, however, the delay occurs very often. The more the traffic rate is closed to the capacity of resources, the higher the delay will be. Therefore, the utility function can not be always increasing. In this section, we are to deal with the problem that the utility function $U_s(x_s)$ is concave but no longer increasing. In such conditions, the Lagrange multipliers of such problem should be zero and can not be taken as the price for per unit flow.

So we must consider now $\lambda_s > 0$ as a regular price determined by the network. Assume that x_s^* is the optimal rate for problem SYSTEM (U, H, A, C), i. e. $U_s'(x_s^*) = 0$. And assume also that x_s^u is the optimal rate for problem USER$_s(U_s;\lambda_s)$, i.e. $U_s'(x_s^u) - \lambda_s = 0$. It is obvious that $x_s^* \neq x_s^u$ if $\lambda_s \neq 0$. The problem here is to find an incentive strategy to force users to act at the point x_s^* rather than x_s^u. We use the linear function here again, under which problem USER$_s(U_s;\lambda_s)$ becomes

$$\text{USER}_s(U_s; p_s):$$
$$\text{maximize} \quad U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^*)x_s,$$
$$\text{over} \quad x_s \geqslant 0. \tag{19}$$

By taking the following steps, we can determine what q_s should be.

i) Calculate the derivative of (19) with respect to x_s, and let it be zero;

ii) Let x_s take the value at x_s^*. By rearrangement, we get $q_s = -\lambda_s / x_s^*$.

It can be easily shown that there is an incentive Stackelberg strategy under the condition

$$\lambda_s < -\frac{U_s''(x_s^*)x_s^*}{2}. \tag{20}$$

Eq. (20) means that the regular price should be determined in a reasonable range.

5 Numerical example and simulation

5.1 Elastic case

We take the example from Kelly's work[1]. Take $U_s(x_s) = m_s \log x_s$ and let finite capacity is $C = 10$. The optimal points for problems SYSTEM (U, H, A, C), $\text{USER}_s(U_s; \lambda_s)$ and NETWORK $(H, A, C; \lambda_s)$ are the right end of the interval $(0, 10]$, respectively, i.e. $x_s^u = C = 10$.

Figure 1 gives out the result of $\text{USER}_s(U_s; q_s)$ with $m_s = 5$ and $\lambda_s = 0.5$ for two different points $x_s^a = 8$ and $x_s^a = 6$, respectively.

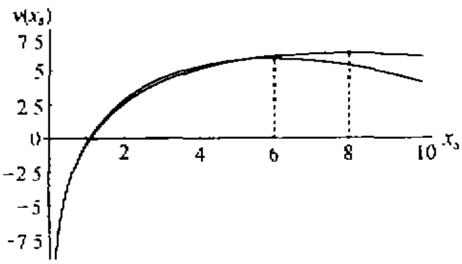

Fig. 1 Curves of the functions for problem USER_s

In each case, the maximum is really at x_s^a. For $x_s^a = 6$, a contour illustration is given in Fig. 2 from which one can see the optimal point clearly. The set of curves is the contour of the function for user s in problem $\text{USER}_s(U_s; \lambda_s)$ and the line with tangent $q_s = 1/18$ is the incentive strategy. We can see that the maximal value of the function of the user along this line is got at $x_s = x_s^a = 6$, the tangent point of the line and the contour curves.

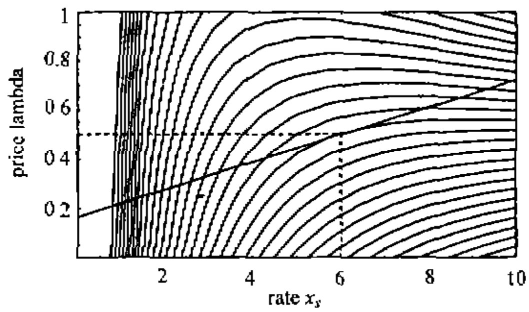

Fig 2 Contour illustration of the optimal problem

From Fig. 3, we can see how a non-linear incentive strategy forces users to act at x_s^a. According to (17), for $x_s^a = 6$, we have

$$\xi_s(x_s) = \begin{cases} \dfrac{1}{2} + \dfrac{(6 - x_s)}{2x_s}, & \text{if} \quad x_s < 6, \\ \dfrac{1}{2}, & \text{if} \quad x_s = 6, \\ \dfrac{1}{2} + \dfrac{5(\log x_s - \log 6)}{x_s}, & \text{if} \quad x_s > 6. \end{cases}$$

Fig. 3 gives out the result in the contour curves, where $m_s = 5, \lambda_s = 0.5$. The folding curve is the non-linear incentive strategy $\xi_s(x_s)$. We can see that, along the curve, the maximal point of $m_s \log x_s - \lambda_s x_s$ is at $x_s = 6$ and $\lambda_s = 0.5$.

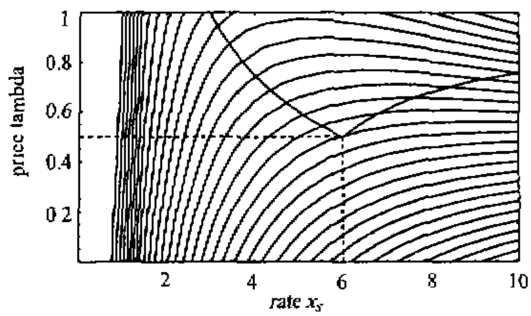

Fig. 3 Contour illustration of the optimal problem with non-linear incentive strategy

5.2 Non-elastic case

To illustrate the result of a non-elastic problem, we introduce $U_s(x_s) = m_{s1} \log x_s + m_{s2} \log(C - x_s)$. The example is just the extension of that in the previous subsection. It is easy to see that $x_s^* = \arg\max U_s(x_s)$ is taken in the open set $(0, C)$. For instance, $x_s^* = 3$ when $(m_{s1}, m_{s2}) = (3, 7)$, $x_s^* = 5$ when $(m_{s1}, m_{s2}) = (5, 5)$, $x_s^* = 7$ when $(m_{s1}, m_{s2}) = (7, 3)$.

From Fig. 4, however, we can see that $x_s^u = \arg\max[U_s(x_s) - \lambda_s x_s]$ are a) 2.15477, b) 3.81966 and (c) 5.78046, respectively. It indicates that users prefer x_s^u (for instance, 5.78046 in the case $m_{s1} = 7$) to $x_s^* (= 7)$.

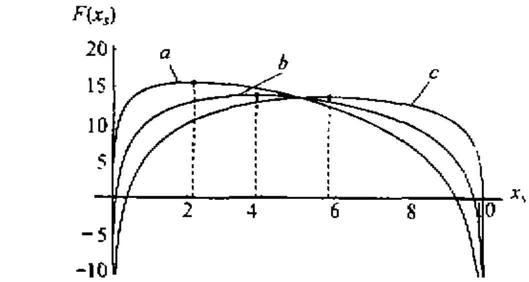

Fig. 4 Non-increasing concave functions of users and their optimum

It can also be seen in Fig.5 that the line $\xi_s(x_s) = 0.5$ is just tangent to contour of the function $U_s(x_s) - \lambda_s x_s$ at $x_s = 5.78046$. Note that $\xi_s(x_s) = 0.5$ means that the regular price is $\lambda_s = 0.5$ and the punishment price is null. Users would rather take $x_s^u = 5.78046$ than $x_s^* = 7$, if there were no punishment price in the strategy.

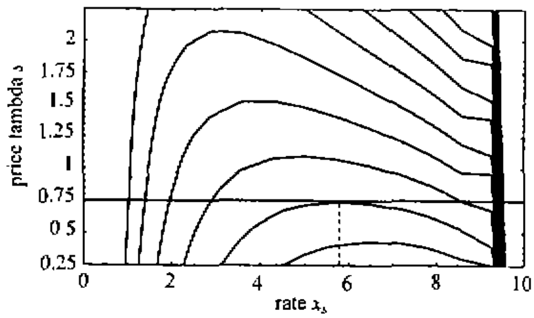

Fig. 5 Contours for problem (1) with non-increasing concave utility function and the linear incentive strategy

Figure 6 gives out the geometric illustration of the incentive strategy. The line of the incentive function is just tangent to the contour of the utility function $U_s(x_s) - \lambda_s x_s$ at $x_s = 7$. Along the line, the maximum point is just $x_s = 7$. So, under the incentive strategy, users have to choose the rate $x_s^* = 7$.

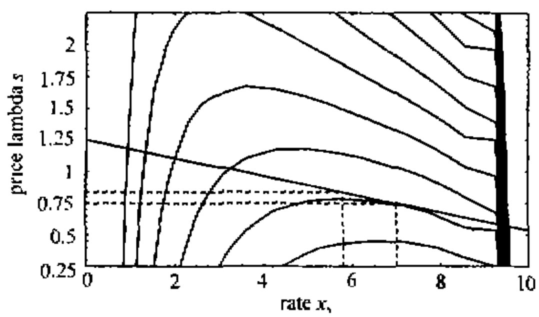

Fig. 6 Contours for problem (1) with non-increasing concave utility function and the linear incentive strategy

6 Conclusions

In this paper, we discussed the traffic rate control problem for a kind of network systems, by introducing the concepts of non-cooperative game theory. The networking models based on elastic and non-elastic traffic are considered and the valid incentive Stackelberg strategies are proposed and illustrated. It is quite a new way that the networking traffic control problem is dealt with by using the game theory. Still, much challenging work is waiting for us to cope with, such as the studies on practicality and technicality.

References

[1] Kelly F. Charging and rate control for elastic traffic [J]. European Transactions on Telecommunications, 1997, 8(1): 33 – 37

[2] Low S H and Varaiya P P. A new approach to service provisioning in ATM networks [J]. IEEE Transactions on Networking, 1993, 1(4): 547 – 553

[3] Murphy J, Murphy L and Posner E C. Distributed pricing for embedded ATM network [A]. In: Labetoulle J, Roberts J W, ed. The Fundamental Role of Teletraffic in the Evolution of Telecommunications Networks [C], Amsterdam: Elsevier, 1994, 1053 – 1063

[4] MacKie-Mason J K and Varian H R. Pricing the Internet [A]. In: Keller J, Kahin B, ed. Public Access to the Internet [M], Englewood Cliffs, New Jersey: Prenticee Hall, 1994

[5] Basar T and Olsder G J. Dynamic Noncooperative Game Theory [M]. 2rd ed. Philadelphia: SIAM, 1998

[6] Fudenberg D and Tirole J. Game Theory [M]. Cambridge, Massachusetts: MIT Press, 1992

[7] Myerson R B. Game Theory: Analysis of Conflict [M]. Cambridge, Massachusetts: Harvard University Press, 1991

[8] Bertsekas D and Gallager R. Data Networks [M]. 2rd ed. Englewood Cliffs, New Jersey: Prentice-Hall, 1992

[9] Economides A A and Silvester J A. Priority load sharing: An approach using Stackelberg games [A]. In: Proceedings of the 28th Annual Allerton Conference on Communications, Control, and Computing [C], Baltimore, MD, 1990

[10] Korilis Y A, Lazar A A and Orda A. The role of the manager in a noncooperative network [A]. In Proceeedings of the IEEE INFOCOM'96 [C], San Francisco, CA, 1996

[11] Orda A, Rom R and Shimkin N. Competitive routing in multiuser communication networks [J]. IEEE/ACM Transactions on Networking, 1993, 1(10): 510 – 521

[12] Ho Y C, Luh P B and Olsder G J. A control-theoretic view on incentive [J]. Automatica, 1982, 18(2): 167 – 179

本文作者简介

井元伟 1956年生.教授.1988年于东北大学获博士学位.1996年在马其顿圣西里尔和麦色迪大学电子工程学院做博士后研究工作.1998～1999年在美国密苏里大学堪萨斯城分校计算机科学远程通讯学院做高级访问学者.研究方向为复杂控制系统结构分析与控

制,对策论,远程通讯网络通信控制等.

陈 兵 1958年生.副教授.1998年于东北大学获博士学位.现在东北大学计算机应用博士后流动站做研究工作.主要研究方向为复杂非线性系统结构分析及鲁棒控制,计算机网络拓扑结构分析等.

乔治·迪米罗夫斯基 1941年生.教授.马其顿工程与科学协会主席,圣西里尔和麦色迪大学电子工程学院首席教授.主要研究方向为非线性复杂系统控制理论及应用,对策论,柔性制造系统的建模与控制,网络通信协议等.

科斯罗·索拉比 1954年生.教授.美国密苏里大学堪萨斯城分校计算机科学远程通讯学院通信网络系主任.研究方向为高速计算机及远程通信网络的设计和分析,通信量控制与分析,非合作对策理论及其在远程通信网络控制中的应用等.

会 议 征 文

第一届控制与自动化国际年会(ICCA'02)将于2002年6月16日至6月19日在中国厦门举行.本次会议由厦门大学主办、香港中文大学、弗吉尼亚大学、华盛顿大学、IEEE新加坡自动控制学会和IEEE香港机器人自动控制系统联合会共同协办.会议地点厦门市位于中国东南沿海的台湾海峡,是著名的"海上花园",与之相邻的武夷山在1999年被联合国授予"世界自然和文化遗产".该会议的宗旨是为系统和自动控制研究领域的专家学者和工程技术人员提供一个该领域最新研究成果的学术交流的机会.会议关注理论及应用的发展,设立了许多特邀报告、专题报告和会议报告.优秀论文将被推荐到亚洲控制杂志发表.

征文范围(不限于以下领域)

·线性系统 ·非线性系统 ·随机系统 ·综合管理系统 ·复杂系统建模 ·自适应自学习系统 ·实时系统 ·预测与辨识 ·故障检测 ·优化控制 ·多变量控制 ·离散事件系统 ·鲁棒控制 ·非线性控制 ·智能控制 ·模糊神经网络 ·过程控制 ·运动控制 ·机器人控制系统 ·控制应用 ·系统工程理论与方法 ·复杂系统分析 ·信息管理系统 ·系统建模和仿真 ·决策支持系统 ·ERP和BRP应用 ·算法分析与设计 ·人工智能及应用 ·专家系统及应用 ·数据挖掘 ·柔性制造系统 ·过程自动化 ·计算机集成制造系统 ·制造系统的建模和自动化 ·Petri网及应用 ·人机交互

投稿说明:

1. 论文及摘要必须用英文撰写,只有作者参加会议的论文才会被采纳.
2. 论文摘要字数1000字左右,论文中必须提供详细的论文要点,如主要思想、方法和主要特点等,同时必须包括论文名称、作者、联系地址、邮政编码、E-mail、电话、传真等.
3. 稿件要求一式三份,可通过邮局投寄,或通过EMAIL投稿.
4. 中国大陆的作者请将稿件投寄到:

   福建省厦门大学自动化系(361005) 李茂青教授

   Tel:(0592)2185649 Fax:(0592)2182407 Email:mqli@xmu.edu.cn

   其他地区和国家的作者请将稿件投寄到:

   Professor Lihua Xie

   School of EEE, BLK S2

   Nanyang Technological University

   Singapore 639798

   Fax:(65)7920415 Email:elhxie@ntu.edu.sg
5. 特邀组织委员会和相关领域的专家报告,请与秦化淑教授联系:

   中国科学院系统科学研究所(100080) 秦化淑教授 Email:qin@iss03.iss.ac.cn

重要日期:

论文摘要和特邀专题提交的截止日期:2002年3月15日

论文录取通知时间:2002年4月15日

论文全文交稿时间:2002年5月15日

会议网址:

http://cies.xmu.edu.cn/icca(中文网址)

http://hdd.ece.nus.edu.sg/~icca(英文网址)