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Stabilization of a Class of Linearly Unobservable Nonlinear Systems by Dynamic Output Feedback *

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Abstract: This paper deals with the problem of stabilization of a class of nonlinear systems with linearly unobservable modes by dynamic output feedback. Based on the concept of the stability in approximation, sufficient conditions for dynamic output feedback stabilizability are obtained. An immediate consequence of our result is that if a nonlinear system has relative degree and its zero dynamics is asymptotically stable in approximation, then it can be stabilized by dynamic output feedback. Our main result may handle some non-minimum phase nonlinear systems.

Key words: stabilization; nonlinear systems; dynamic output feedback

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一类线性不可观非线性系统的动态输出反馈镇定

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摘要: 本文研究一类不可观非线性系统的动态输出反馈镇定.基于逼近渐近稳定性的概念,给出了动态输出反馈可镇定的充分条件.本文主要结果的直接推论是零动态逼近渐近稳定的最小相位系统能用动态输出反馈镇定.本文的方法也能处理非最小相位系统.

关键词: 镇定; 非线性系统; 动态输出反馈

1 Introduction

Over the past decade, the problem of stabilization of nonlinear systems by output feedback has been extensively studied. Dynamic output feedback stabilization is divided into local stabilization, semi-global stabilization and global stabilization. For the results of semi-global stabilization and global stabilization, the interested reader may refer to [1 ~ 7] and others cited in those papers. In the paper, we will study the problem of local stabilization by dynamic output feedback of the affine nonlinear system

$$\begin{cases} \dot{x} = f(x) + g(x)u, \\ y = h(x), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$; $f: u \to \mathbb{R}^n$, $g: U \to \mathbb{R}^{n\times m}$, and $h: U \to \mathbb{R}^p$ are smooth functions, $f(0) = 0$, $h(0) = 0$, and U is a neighborhood of $x = 0$.

Although the problem of local stabilization of (1) by dynamic output feedback has been studied by many authors, the achievements are rather limited. In fact, the systems considered in the previous studies are mainly linearizable nonlinear systems[8,9] and locally uniformly completely observable nonlinear systems[9,10]. Local state feedback stabilization of minimum phase nonlinear systems is studied in [11]. But the technique used in [11] is not applicable to dynamic output feedback stabilization. Our result may handle some unlinearizable and nonminimum phase nonlinear systems, which can not be handled by the methods in proposed [8 ~ 10]. The method for the design of dynamic compensators in the paper is based on the concept of asymptotic stability in

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approximation[12] and the design proves to be constructive.

The paper is organized as follows. Section 2 deals with dynamic output feedback stabilization. In Section 3, two examples including a nonminimum phase system with unobservable linear approximation are given to illustrate the applications of our major result. Section 4 draws some conclusions from the study we have made.

2 Stabilization

In this section, we will study the problem of local stabilization of (1) by dynamic output feedback. We first introduce some operations and concepts.

Consider the dynamically extended system of (1)

$$\begin{cases} \dot{x} = f(x) + g(x)\lambda_1, \\ \dot{\lambda}_1 = \lambda_2, \\ \quad \vdots \\ \dot{\lambda}_{l-1} = \lambda_l, \\ \dot{\lambda}_l = v, y = h(x), \end{cases} \tag{2}$$

where $\lambda_i \in \mathbb{R}^m$, $i = 1,2,\cdots,l$, and $v \in \mathbb{R}^m$.

Define the following differential operations:

$$y = h(x),$$

$$\dot{y} = \dot{y}(x,\lambda_1) = \frac{\partial h}{\partial x}(f(x) + g(x)\lambda_1),$$

$$y^{(i+1)} = y^{(i+1)}(x,\lambda_1,\lambda_2,\cdots,\lambda_i) = \frac{\partial y^{(i)}}{\partial x}(f(x) + g(x)\lambda_1) + \sum_{j=1}^{i} \frac{\partial y^{(i)}}{\partial \lambda_j} u^{(j+1)},$$

$$i = 1,2,3,\cdots,k,$$

where $\lambda_{l+1} = v$.

Write

$$Y_k = Y_k(x,\lambda) = (y^{\mathrm{T}}, \dot{y}^{\mathrm{T}}, \cdots, (y^{(k)})^{\mathrm{T}})^{\mathrm{T}},$$

$$\Lambda_l = (\lambda_1^{\mathrm{T}}, \lambda_2^{\mathrm{T}}, \cdots, \lambda_l^{\mathrm{T}})^{\mathrm{T}}.$$

In what follows, we always assume that

$$\frac{\partial Y_k}{\partial v} = 0.$$

Definition 2.1 The equilibrium of system (2) at the origin is said to be asymptotically stabilizable according to the Nth order approximation by feedback $u = \alpha(Y_k,\Lambda_l)$ if the closed loop system of (2) with $u = \alpha(Y_k,\Lambda_l)$ is asymptotically stable according to the Nth order approximation.

Definition 2.2 The equilibrium of system (2) is asymptotically stabilizable in approximation by feedback $u = \alpha(Y_k,\Lambda_l)$ if there exists a positive integer N such that the closed loop system of (2) with $u = \alpha(Y_k,\Lambda_l)$ is asymptotically stable according to the Nth order approximation.

For the concept of asymptotic stability according to the Nth order approximation, see, for instance, [12].

The following Theorem 2.1 is our main result.

Theorem 2.1 If (2) is asymptotically stabilizable in approximation by a feedback law of the form $u = \alpha(Y_k,\Lambda_l)$, then (1) can be stabilized by dynamic output feedback.

Proof According to the assumption of Theorem 2.1, the following system is asymptotically stable in approximation:

$$\begin{cases} \dot{x} = f(x) + g(x)\lambda_1, \\ \dot{\lambda}_1 = \lambda_2, \\ \quad \vdots \\ \dot{\lambda}_{l-1} = \lambda_l, \\ \dot{\lambda}_l = \alpha(Y_k,\Lambda_l). \end{cases} \tag{3}$$

Note that the feedback law $u = \alpha(Y_k,\Lambda_l)$ depends on the derivatives of the output y. Thus we introduce the dynamic system

$$\begin{cases} \dot{\theta}_1 = \theta_2, \\ \quad \vdots \\ \dot{\theta}_k = \theta_{k+1}, \\ \dot{\theta}_{k+1} = \vartheta, \end{cases} \tag{4}$$

where $\theta_i \in \mathbb{R}^p$, $i = 1,2,\cdots,k+1$.

Let $\theta = (\theta_1^{\mathrm{T}}, \theta_2^{\mathrm{T}}, \cdots, \theta_{k+1}^{\mathrm{T}})^{\mathrm{T}}$. For system (2), we now use the new feedback law

$$v = \alpha(\theta,\Lambda_l), \tag{5}$$

which is an output feedback law for system (1).

By the definition of $y^{(k+1)}$, $y^{(k+1)}$ can be expressed as

$$y^{(k+1)} = F(x,\Lambda_l). \tag{6}$$

Write $e_i = y^{(i-1)} - \theta_i$, $i = 1,2,\cdots,k+1$ and $e = (e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, \cdots, e_{k+1}^{\mathrm{T}})^{\mathrm{T}}$. Then $Y_k = \theta + e$. By (6), the closed loop system consisting of (2), (4) and (5) can be written as

$$\begin{cases} \dot{x} = f(x) + g(x)\lambda_1, \\ \dot{\lambda}_1 = \lambda_2, \cdots, \dot{\lambda}_{l-1} = \lambda_l,\ \dot{\lambda}_l = \alpha(Y_k + e,\Lambda_l), \\ \dot{e}_1 = e_2, \cdots, \dot{e}_k = e_{k+1},\ \dot{e}_{k+1} = F(x,\Lambda_l) - \vartheta. \end{cases} \tag{7}$$

Take e_1 as an output of (7). Then it is easy to see that (7) with the output e_1 is minimum phase and the equation of its zero dynamics is (3), which is asymptotically stable in approximation at $z = 0, \Lambda_l = 0$. By the result in [13], system (7) can be stabilized by dynamic output feedback, that is, there exists a dynamic compensator

$$\begin{cases} \dot{w} = \eta(w, e_1), \\ v = \psi(w, e_1), \end{cases} \tag{8}$$

such that the closed loop system consisting of (7) and (8) is asymptotically stable. Since $e_1 = y - \theta_1$, the feedback law $v = \psi(w, e_1)$ in (8) is a dynamic output feedback. This means that (1) is stabilizable by dynamic output feedback.

Corollary 2.2 Let $m = p$. If (1) has relative degree $\{r_1, r_2, \cdots, r_m\}$ at $x = 0$ and the equation of its zero dynamics is asymptotically stable in approximation at $x = 0$, then it can be stabilized by dynamic output feedback.

Proof We can assume without loss of generality that $r_1 = r_2 = \cdots = r_m = r$, since this can always be achieved by performing dynamic extension on (1). Let (1) be asymptotically stable according to the Nth order approximation. It is known from (13) that there exist a positive integer l, a linear feedback $v = \sum_{i=1}^{l} \alpha_i \lambda_i + \hat{v}$, α_i constant, and a local nonlinear coordinate transformation such that (2) can be transformed into the following form:

$$\begin{cases} \dot{z} = f_0(z) + g_0(z, \xi_1, \cdots, \xi_{r+1})\xi, \\ \dot{\xi}_1 = \xi_2, \cdots, \dot{\xi}_r = \xi_{r+1}, \cdots, \dot{\xi}_{r+l-1} = \xi_{r+l}, \\ \dot{\xi}_{r+l} = f_1(z, \xi_1, \cdots, \xi_{r+l}) + g_1(z, \xi_1, \cdots, \xi_r)\hat{v}, \\ y = \xi_1, \end{cases} \tag{9}$$

where $\xi = (\xi_1, \xi_2, \cdots, \xi_{r+1})^{\mathrm{T}}$ and f_0, g_0, f_1 and g_1 have the properties:

a) $g_0(0, 0, \cdots, 0) = 0$;

b) $f_1(z, 0, \cdots, 0) = O(\| z \|^N)$;

c) $g_1(0, 0, \cdots, 0)$ is invertible;

d) The zero dynamics

$$\dot{z} = f_0(z)$$

is asymptotically stable according to the Nth order approximation.

It is easy to see that there exists a linear feedback

$$\hat{v} = \sum_{i=1}^{r+l} \beta_i \xi_i, \tag{10}$$

such that the closed loop system (9) and (10) is asymptotically stable according to the Nth order approximation. Since $y = \xi_1, y^{(i)} = \xi_{i+1}, i = 1, 2, \cdots, r+1$. (10) implies that (2) can be asymptotically stabilizable in approximation by a feedback of the form $v = \alpha(Y_k, \Lambda_l)$. It follows from Theorem 2.1 that Corollary 2.2 holds.

3 Examples

Example 3.1 Consider the system

$$\begin{cases} \dot{z} = z^4 + \xi_1 + \xi_2^2, \\ \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = -\xi_1 - z^3 + u, \\ y = \xi_1. \end{cases} \tag{11}$$

The system (11) is linearly unobservable and nonminmum phase, so the methods presented in [3,8 ~ 10] are not applicable to (11). If the feedback $u = -y^{(1)} = \xi_2$ is used, then system (11) is transformed into

$$\begin{cases} \dot{z} = z^4 + \xi_1 + \xi_2^2, \\ \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = -\xi_1 - \xi_2 - z^3, \\ y = \xi_1. \end{cases} \tag{12}$$

We now prove that (12) is asymptotically stable according to the fourth order approximation. It is clear that (12) has an invariant manifold

$$\begin{cases} \xi_1 = \pi_1(z), \\ \xi_2 = \pi_2(z), \end{cases} \tag{13}$$

which satisfy equation[14]

$$\begin{pmatrix} \dfrac{\partial \pi_1(z)}{\partial z} \\ \dfrac{\partial \pi_2(z)}{\partial z} \end{pmatrix} (z^4 + \pi_1(z) + \pi_2^2(z)) = \begin{pmatrix} \pi_2(z) \\ -\pi_1 - \pi_2(z) - z^3 \end{pmatrix}. \tag{14}$$

By solving (14) approximately, we obtain

$$\pi_1(z) = -z^3 + O(|z|^4), \ \pi_2(z) = O(|z|^4). \tag{15}$$

The reduction system of (12) is

$$\dot{z} = -z^3 + O(|z|^4),$$

which is asymptotically stable according to the fourth order approximation[13]. This implies that (12) is asymptotically stable according to the fourth order approximation. Thus (11) satisfies the condition of Theorem 2.1, so it can be stabilized by dynamic output feedback. The order of a dynamic compensator for (11) may be larger if we imitate the method presented in the proof of Theorem 2.1 to design the dynamic compensator. We can use some better methods to design a dynamic compensator so long as the condition of Theorem 2.1 is satisfied. By using a revised method, a dynamic compensator of order 3 for (11) can be given as follows:

$$\begin{cases} u = \theta_2, \\ \dot{\theta}_1 = \theta_2 + 6(y - \theta_1), \\ \dot{\theta}_2 = \theta_3 + 12(y - \theta_1), \\ \dot{\theta}_3 = \theta_2 - \theta_3 + 20(y - \theta_1). \end{cases} \tag{16}$$

Details of the design are omitted.

Example 3.2 Consider the system

$$\begin{cases} \dot{z} = -z^3 + \frac{1}{2}\xi_2^3, \\ \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = -\xi_1 + z^4 + \xi_2 u, \\ y = \xi. \end{cases} \tag{17}$$

It is easy to see that the system (17) has neither the relative degree at $(z, \xi_1, \xi_2) = 0$ nor the zero dynamics. By taking the feedback $u = -2(y^{(1)})^2 = -2\xi_2^2$, we obtain

$$\begin{cases} \dot{z} = -z^3 + \frac{1}{2}\xi_2^3, \\ \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = -\xi_1 + z^4 - 2\xi_2^3, \\ y = \xi. \end{cases} \tag{18}$$

We now show that (18) is asymptotically stable in approximation. Let

$$V = \frac{1}{2}z^2 + \frac{1}{2}\xi_1^2 + \frac{1}{2}\xi_2^2 + \frac{1}{3}\xi_1^3\xi_2.$$

Then V is positive definite on some neighborhood of $z = 0, \xi_1 = 0, \xi_2 = 0$. By a direct computation, we have, on some neighborhood of $z = 0, \xi_1 = 0, \xi_2 = 0$,

$$\begin{aligned} \dot{V} = & -z^4 + \frac{1}{2}z\xi_2^3 + \xi_1\xi_2 - \xi_1\xi_2 - 2\xi_2^4 + z^4\xi_2 + \\ & \xi_1^2\xi_2^2 - \frac{1}{3}\xi_1^4 + \frac{1}{3}z^4\xi_1^3 - \frac{1}{3}\xi_1^3\xi_2^3 \leqslant \\ & -(1 - \xi_1 - \frac{1}{3}\xi_1^3)z^4 + \frac{1}{4}z^2\xi_2^2 + \frac{1}{4}\xi_2^4 - \\ & 2\xi_2^4 + \frac{1}{4}\xi_1^4 + \xi_2^4 - \frac{1}{3}\xi_1^4 + \frac{1}{6}\xi_1^6 + \frac{1}{6}\xi_2^6 \leqslant \\ & -\frac{1}{2}z^4 + \frac{1}{8}z^4 + \frac{1}{8}\xi_2^4 - \frac{3}{4}\xi_2^4 - \frac{1}{12}\xi_1^4 + \frac{1}{6}\xi_1^6 + \frac{1}{6}\xi_2^6 \leqslant \\ & -\frac{1}{4}z^4 - \frac{1}{15}\xi_1^4 - \frac{1}{2}\xi_2^4. \end{aligned}$$

This implies that (18) is asymptotically stable according to the fourth order approximation. By Theorem 2.1, (17) can be stabilized by dynamic output feedback. The design of a dynamic compensator for (17) is omitted.

4 Conclusions

Based on the concept of asymptotic stability in approximation, we have presented a constructive method to stabilize a class of nonlinear systems by means of dynamic output feedback. Our result does not assume local uniform complete observability, and may handle some nonminimum phase nonlinear systems that cannot be stabilized by existing methods. Two examples have been used to illustrate the effectiveness of our approach. How to find the minimum order dynamic output feedback controller remains to be further studied.

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