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Robust H_∞ Output Feedback Control for a Class of Uncertain Linear Systems with Time-Delay *

ZHENG Lianwei and HUANG Shoudong
(School of Science, Northeastern University · Shenyang, 110006, P.R.China)
LIU Xiaoping
(School of Information Science and Engineering, Northeastern University · Shenyang, 110006, P.R.China)

Abstract: This paper considers the problem of robust H_∞ output feedback controller design for linear time-delay systems with polytopic uncertainty. A sufficient condition in terms of quadratic matrix inequalities is obtained, which guarantees the system is quadratically stabilizable with disturbance attenuation in the sense of H_∞-norm via a strictly proper dynamic output feedback controller of full order. Such a controller can be constructed from the solutions of quadratic matrix inequalities. It is shown that in some cases the quadratic matrix inequalities can be solved by converting them into linear matrix inequalities with the same variables.

Key words: time-delay systems; H_∞ control; output feedback; uncertainty; linear matrix inequalities
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一类不确定线性时滞系统的输出反馈鲁棒 H_∞ 控制

郑连伟　黄守东　　刘晓平
(东北大学理学院·沈阳,110006)　(东北大学信息科学与工程学院·沈阳,110006)

摘要: 研究具有仿射参数不确定性的线性时滞系统的鲁棒 H_∞ 输出反馈控制器设计问题.得到了一些二次矩阵不等式,证明了如果它们有解,则从它们的解可以构造全阶严格真动态输出反馈控制器,它能保证闭环系统二次稳定且具有 H_∞ 范数意义下的干扰抑制能力.进一步证明了在某些情况下,这些二次矩阵不等式可以化为线性矩阵不等式来求解.

关键词: 时滞系统; H_∞ 控制; 输出反馈; 不确定性; 线性矩阵不等式

1 Introduction

Since time-delays are frequently encountered in physical processes and very often are the causes of instability and poor performance of control systems, time-delay systems constitute an important class of system of both theoretical and practical significance. In recent years, considerable attention has been paid to the H_∞ control problems for linear time-delay systems with uncertainty, e. g., for norm-bounded uncertainty, state feedback H_∞ controller design methods have been proposed in [1 ~ 3], and an observer-based H_∞ controller design method has been given in [4]. In the research of uncertain linear systems, two typical uncertainties are norm-bounded uncertainty and polytopic uncertainty. So far, the control problems for linear time-delay systems with polytopic uncertainty have not been reported.

In this paper, we consider the problem of robust H_∞ output feedback controller design for uncertain linear time-delay systems. The uncertainty is assumed to be polytopic, i. e., the uncertain parameters appear affinely in the matrices of the state-space model. As a stability criterion, the notion of quadratic stability is adopted. A sufficient condition, under which a robust H_∞ strictly proper dynamic output feedback controller of full order exists, is derived in terms of quadratic matrix inequalities. By means of their solutions, the controller is cons-

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tructed, which quadratically stabilizes the system and achieves a prescribed level of H_∞-norm bound of the closed-loop system. It is shown that in some cases such quadratic matrix inequalities can be solved by converting them into linear matrix inequalities with the same variables, which can be easily solved with efficient numerical method, see, e.g., [5].

2 Problem statement

Consider the following uncertain linear system with delayed state

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x(t-d) + \\ \qquad (B_1 + \Delta B_1)w(t) + (B_2 + \Delta B_2)u(t), \\ z(t) = (C_1+\Delta C_1)x(t) + (C_{11}+\Delta C_{11})x(t-d) + \\ \qquad (D_{11} + \Delta D_{11})w(t) + (D_{12} + \Delta D_{12})u(t), \\ y(t) = (C_2+\Delta C_2)x(t) + (C_{21}+\Delta C_{21})x(t-d) + \\ \qquad (D_{21} + \Delta D_{21})w(t), \\ x(t) = \varphi(t), t \in [-d, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^n$ is the control input, $w(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2[0,\infty)$, $z(t) \in \mathbb{R}^q$ is the controlled output, $y(t) \in \mathbb{R}^r$ is the measurement output, and $d > 0$ is the unknown time delay, $\varphi(t) \in \mathbb{R}^n$ is the continuous initial function, $A, A_1, B_1, B_2, C_1, C_{11}, C_2, C_{21}, D_{11}, D_{12}$ and D_{21} are known constant matrices with appropriate dimensions. It is assumed that (A, B_2, C_2) is stabilizable and detectable. $\Delta A, \Delta A_1, \Delta B_1, \Delta B_2, \Delta C_1, \Delta C_{11}, \Delta C_2, \Delta C_{21}$, $\Delta D_{11}, \Delta D_{12}$ and ΔD_{21} are polytopic uncertainty matrices, which are linear matrix functions of possibly time-varying vector $\delta(t) = (\delta_1(t), \cdots \delta_s(t)) \in \mathbb{R}^s$, e.g., $\Delta A = \delta_1 A_1 + \cdots + \delta_s A_s$ where $A_1, \cdots, A_s$ are known constant matrices. The uncertainty vector $\delta(t)$ is assumed to range in the polytope

$$\Omega = \{(\delta_1, \cdots, \delta_s) \in \mathbb{R}^s : \delta_i \in [\underline{\delta}_i, \bar{\delta}_i]\}.$$

Let

$$\Omega_{\text{vex}} = \{(\delta_1, \cdots, \delta_s) \in \mathbb{R}^s : \delta_i = \underline{\delta}_i \text{ or } \delta_i = \bar{\delta}_i\}$$

denote the vertex set of Ω, which contains 2^s elements.

The aim of this paper is to design a full order output feedback controller

$$\begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c y(t), \\ u(t) = C_c x_c(t), \end{cases} \tag{2}$$

such that the resulting closed-loop system

$$\dot{\xi}(t) = \bar{A}(\delta)\xi(t) + \bar{A}_1(\delta)\xi(t-d) + \bar{B}(\delta)w(t),$$
$$z(t) = \bar{C}(\delta)\xi(t) + \bar{C}_1(\delta)\xi(t-d) + (D_{11} + \Delta D_{11})w(t)$$

is quadratically stable and has the H_∞-norm bound γ, i.e., subject to the assumption of the zero initial condition, the constraint $\| z(t) \|_2 \leq \gamma \| w(t) \|_2$ is satisfied.

3 Main results

Our design procedure is based on the following lemma, which can be easily proved by Theorem 1 in [6].

Lemma 1 If there exist symmetric positive definite matrices Q and S such that

$$\begin{bmatrix} \bar{A}(\delta)Q + Q\bar{A}^{\mathrm{T}}(\delta) + S & Q\bar{C}^{\mathrm{T}}(\delta) & \bar{B}(\delta) & \bar{A}_1(\delta)Q \\ \bar{C}(\delta)Q & -I & D_{11}+\Delta D_{11} & \bar{C}_1(\delta)Q \\ \bar{B}^{\mathrm{T}}(\delta) & D_{11}^{\mathrm{T}}+\Delta D_{11}^{\mathrm{T}} & -\gamma^2 I & 0 \\ Q\bar{A}_1^{\mathrm{T}}(\delta) & Q\bar{C}_1^{\mathrm{T}}(\delta) & 0 & -S \end{bmatrix} < 0 \tag{3}$$

for all $\delta \in \Omega_{\text{vex}}$, then system (3) is quadratically stable and has the H_∞-norm bound γ.

In the sequel, denote

$$A_{1\Delta} = A_1 + \Delta A_1,\ B_{1\Delta} = B_1 + \Delta B_1,$$
$$C_{1\Delta} = C_1 + \Delta C_1,\ C_{11\Delta} = C_{11} + \Delta C_{11},$$
$$D_{11\Delta} = D_{11} + \Delta D_{11},\ D_{12\Delta} = D_{12} + \Delta D_{12},$$
$$C_{21\Delta} = C_{21} + \Delta C_{21},\ D_{21\Delta} = D_{21} + \Delta D_{21},$$

for saving space.

Theorem 1 There exist a controller (2) and symmetric positive definite matrices Q and S such that (3) holds if and only if there exist matrices $X > 0, Y > 0$, $S_{11} > 0, S_{22} > 0, U, V, Z$, and S_{12} such that

$$\begin{bmatrix} \Pi_{11} + S_{11} & \Pi_{12} + S_{12} & C_{1\Delta}^{\mathrm{T}} & YB_{1\Delta} + VD_{21\Delta} & YA_{1\Delta} + VC_{21\Delta} & YA_{1\Delta}X + VC_{21\Delta}X \\ \Pi_{12}^{\mathrm{T}} + S_{12}^{\mathrm{T}} & \Pi_{22} + S_{22} & XC_{1\Delta}^{\mathrm{T}} + U^{\mathrm{T}}D_{21\Delta}^{\mathrm{T}} & B_{1\Delta} & A_{1\Delta} & A_{1\Delta}X \\ C_{1\Delta} & C_{1\Delta}X + D_{12\Delta}U & -I & D_{11\Delta} & C_{11\Delta} & C_{11\Delta}X \\ B_{1\Delta}^{\mathrm{T}}Y + D_{21\Delta}^{\mathrm{T}}V^{\mathrm{T}} & B_{1\Delta}^{\mathrm{T}} & D_{11\Delta}^{\mathrm{T}} & -\gamma^2 I & 0 & 0 \\ A_{1\Delta}^{\mathrm{T}}Y + C_{21\Delta}^{\mathrm{T}}V^{\mathrm{T}} & A_{1\Delta}^{\mathrm{T}} & C_{11\Delta}^{\mathrm{T}} & 0 & -S_{11} & -S_{12} \\ XA_{1\Delta}^{\mathrm{T}}Y + XC_{21\Delta}^{\mathrm{T}}V^{\mathrm{T}} & XA_{1\Delta}^{\mathrm{T}} & XC_{11\Delta}^{\mathrm{T}} & 0 & -S_{12}^{\mathrm{T}} & -S_{22} \end{bmatrix} < 0 \tag{4}$$

for all $\delta \in \Omega_{vex}$, where

$$\Pi_{11} = Y(A + \Delta A) + (A + \Delta A)^{T}Y + V(C_2 + \Delta C_2) + (C_2 + \Delta C_2)^{T}V^{T}, \quad (5)$$

$$\Pi_{12} = Z + Y\Delta AX + Y\Delta B_2 U + V\Delta C_2 X + \Delta A^{T},$$

$$\Pi_{22} = (A + \Delta A)X + X(A + \Delta A)^{T} + (B_2 + \Delta B_2)U + U^{T}(B_2 + \Delta B_2)^{T}, \quad (6)$$

and

$$\begin{bmatrix} Y & I \\ Y & X \end{bmatrix} > 0.$$

Moreover, the controller matrices can be taken as

$$A_c = X(YX-I)^{-1}(YAX+YB_2U+VC_2X+A^{T}-Z)X^{-1},$$

$$C_c = UX^{-1},\ B_c = X(I - YX)^{-1}V.$$

Proof Partition Q in (3) as

$$Q = \begin{bmatrix} X & Q_{12} \\ Q_{12}^{T} & Q_{22} \end{bmatrix}$$

with $Q_{12} \in \mathbb{R}^{n \times n}$. Without loss of generality, Q_{12} can be assumed to be invertible. Let

$$T_1 = \mathrm{diag}\{I, Q_{12}^{-1}X\},\ T_2 = \mathrm{diag}\{T_1, I, I, T_1\},$$

$$T_3 = \begin{bmatrix} Y & -R^{-1} \\ I & 0 \end{bmatrix},$$

and $T_4 = \mathrm{diag}\{T_3, I, I, T_3\}$. Left and right multiply (3) by $T_4 T_2^{T}$ and $T_2 T_4^{T}$. Define

$$R = XQ_{12}^{-T}Q_{22}Q_{12}^{-1}X - X,\ Y = R^{-1} + X^{-1}.$$

Performing the state transformation $x_c = Q_{12}^{T}X^{-1}\hat{x}_c$ to the controller (2), and defining

$$U = C_c X,\ V = -R^{-1}B_c,$$

$$Z = YAX + YB_2U - R^{-1}A_cX + VC_2X + A^{T},$$

the theorem can be proved.

Since Ω_{vex} contains only 2^s elements, (4) are 2^s quadratic matrix inequalities. We now give a sufficient condition for their solvability in terms of linear matrix inequalities. The condition needs the decompositions

$$\Delta A = D_a E_a,\ \Delta B_2 = D_b E_b,\ \Delta C_2 = D_c E_c \quad (7)$$

where the factors are linear matrix functions of δ, or constant matrices with appropriate dimensions. Note that such decompositions always exist since some factors can be chosen as identity matrices if necessary.

Theorem 2 Decompose $\Delta A, \Delta B_2, \Delta C_2$ as in (7). If for some positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$, the matrices $X > 0, Y > 0, S_{11} > 0, U, V$, and Z are solutions of the following LMIs for all $\delta \in \Omega_{vex}$, then they together with $S_{22} = \varepsilon_4 X^2$ and $S_{12} = 0$ are solutions of (4) for all $\delta \in \Omega_{vex}$.

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^{T} & -\Phi_{22} \end{bmatrix} < 0,$$

where

$$\Phi_{11} = \begin{bmatrix} \Pi_{11} + S_{11} & Z + \Delta A^{T} & C_{1\Delta}^{T} & YB_{1\Delta} + VD_{21\Delta} & YA_{1\Delta} + VC_{21\Delta} & YA_{1\Delta} + VC_{21\Delta} \\ Z^{T} + \Delta A & \Pi_{22} & XC_{1\Delta}^{T} + U^{T}D_{21\Delta}^{T} & B_{1\Delta} & A_{1\Delta} & A_{1\Delta} \\ C_{1\Delta} & C_{1\Delta}X + D_{12\Delta}U & -I & D_{11\Delta} & C_{11\Delta} & C_{11\Delta} \\ B_{1\Delta}^{T}Y + D_{21\Delta}^{T}V^{T} & B_{1\Delta}^{T} & D_{11\Delta}^{T} & -\gamma^2 I & 0 & 0 \\ A_{1\Delta}^{T}Y + C_{21\Delta}^{T}V^{T} & A_{1\Delta}^{T} & C_{11\Delta}^{T} & 0 & -S_{11} & 0 \\ A_{1\Delta}^{T}Y + C_{21\Delta}^{T}V^{T} & A_{1\Delta}^{T} & C_{11\Delta}^{T} & 0 & 0 & -\varepsilon_4 I \end{bmatrix}$$

with Π_{11} and Π_{22} given by (5) and (6), respectively, and

$$\Phi_{12} = \begin{bmatrix} YD_a & YD_b & VD_c & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & XE_a^{T} & U^{T}E_b^{T} & XE_c^{T} & X \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{22} = \mathrm{diag}\{\varepsilon_1^{-1}I, \varepsilon_2^{-1}I, \varepsilon_3^{-1}I, \varepsilon_1 I, \varepsilon_2 I, \varepsilon_3 I, \varepsilon_4^{-1}I\}.$$

Proof Setting $S_{12} = 0, S_{22} = \varepsilon_4 X^2$ in (4), and left and right multiplying (4) by diag $\{I, I, I, I, I, X^{-1}\}$, the theorem can be proved by the Schur complement formula and the inequality

$$A^{T}B + B^{T}A \leqslant A^{T}GA + B^{T}G^{-1}B$$

for

$$G = \mathrm{diag}\{\varepsilon_1 I, \varepsilon_2 I, \varepsilon_3 I\}.$$

Remark 1 In the Riccati equation approach for norm-bounded uncertainty developed in [4], some matrices must be specified at first, while in our LMI approach every matrices are determined according to the inequality (3). This is one advantage of our approach. One more advantage of our approach is that it applies to more general systems when the uncertainty is polytopic. In [4], the measured output equation and the controlled output equation contain only the present state and do not allow the uncertainty. Even if the polytopic uncertainty is converted into norm-bounded uncertainty, the approach given in [4] does not apply to the system (1).

4 Conclusion

We have proposed a robust H_∞ output feedback controller design method for linear time-delay systems with polytopic uncertainty. Based on the notion of quadratic stability, a sufficient condition, under which a robust H_∞ strictly proper dynamic output feedback controller of full order exists, has been derived in terms of quadratic matrix inequalities. By means of their solutions, the controller has been constructed, which stabilizes the system and achieves a prescribed level of H_∞-norm bound of the closed-loop system. It has been shown that in some cases such quadratic matrix inequalities can be solved by converting them into linear matrix inequalities with the same variables, which can be numerically solved very efficiently, see, e.g., [5].

References

[1] Ge J H, Frank P M and Lin C F. Robust H_∞ state feedback control for linear systems with state delay and parameter uncertainty [J]. Automatica, 1996, 32(8): 1183 – 1185

[2] Yu L, Chu J and Su H Y. Robust memoryless H_∞ controller design for linear time-delay systems with norm-bounded time-varying uncertainty [J]. Automatica, 1996, 32(12), 1759 – 1762

[3] Kim J K and Park H B. State feedback control for generalized continuous/discrete time-delay system [J]. Automatica, 1999, 35(8): 1443 – 1452

[4] Wang J H, Su H Y, Chu J, et al. Robust H_∞ feedback controller design for linear time-varying uncertain systems with delayed state [J]. Control Theory and Applications, 1999, 16(3): 334 – 338, 344

[5] Boyd S, Ei Ghaoui L, Feron E, et al. Linear Matrix Inequalities in System and Control Theory [M]. Philadelphia: SIAM, 1994

[6] Kokame H, Kobayashi H and Mor T. Robust H_∞ performance for linear delay-differential systems with time-varying uncertainties [J]. IEEE Trans. Automat. Contr., 1998, 43(2): 223 – 226

本文作者简介

郑连伟 1963 年生.分别于 1989 年和 1992 年在东北大学数学系获学士和硕士学位.1997 年起在东北大学自动控制系攻读博士学位.目前主要从事时滞系统的鲁棒控制,H_∞控制等方面的研究.

黄守东 1969 年生.分别于 1987 年和 1990 年在东北大学数学系获学士和硕士学位.于 1998 年在东北大学自动控制系获博士学位.目前主要从事复杂系统的分析与控制研究.

刘晓平 1962 年生.分别于 1984 年、1987 年和 1989 年在东北大学自动控制系获学士、硕士和博士学位.现任东北大学自动控制系统教授,博士生导师.目前主要从事非线性系统的鲁棒控制研究.