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一类非线性系统的 H_∞ 鲁棒控制

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摘要：考虑了一类带有扰动的仿射非线性系统的 H_∞ 控制问题.包括状态反馈与动态输出反馈两种情形,我们基于 HJI 不等式的转换,直接给出了相应解的一种构造以及一种构造性判据,从而避免了通常的从数值求解 HJI 不等式的困难.

关键词：非线性系统；鲁棒 H_∞ 控制；HJI 不等式；状态反馈；动态输出反馈
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Robust H_∞ Control for Nonlinear Systems

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Abstract: This paper is concerned with robust H_∞ control for nonlinear systems with disturbance. State feedback and dynamic output feedback are considered. Our results are given through HJI inequality's transformation. A constructional solution and a constructional criterion are obtained without giving a solution to an HJI inequality.

Key words: nonlinear system; robust H_∞ control; HJI inequality; state feedback; dynamic output feedback

1 引言(Introduction)

非线性系统控制理论中一个长期困扰人们的问题是所得的理论上的结果往往形式复杂,而导致实用价值的大打折扣.例如,精确线性化的结论涉及过于复杂的偏微分方程组的求解,仿射非线性系统的关于 H_∞ 控制的结论涉及一个困难的 HJI 不等式,等等.有鉴于此,近年来,针对仿射非线性系统发展了一些构造性的理论,以回避这些数学上的困难(参见文献[1~7]).这些结果包括著名的 Backstepping 方法.它依赖一系列的迭代过程来实现一个稳定控制器的设计.

所有这些结果都是从某种意义上来说基于无源性理论的,而且基本上可以分成两类;考虑相对阶与不考虑相对阶.当然,并不是所有的系统都具有相对阶,因此,后一种情形更具一般性.

本文力图强调 HJI 不等式的联系作用.事实上,鲁棒 KYP 引理[1]与 H_∞ 控制给出了形式上相同的 HJI 不等式.这启发我们:既然这些不同的问题具有相同的不等式形式要求求解,我们就可以在一定意义上等效这些不同的问题,从而在适当的时候把问题转化为较易解决的形式.

基于这种思想,本文考虑了一类具有不确定性的仿射非线性系统的 H_∞ 控制问题,分别在状态反馈与动态输出反馈两种情形下给出了有关结果.我们的主要方法是基于 HJI 不等式的转换,直接给出了解的一种构造以及一种构造性判据,从而避免了通常的从数值求解 HJI 不等式的困难.

2 系统描述与基本假设(System description and basic assumption)

我们首先考虑如下的非线性系统:

$$\begin{cases} \dot{x}(t) = f(x) + \Delta f(x) + (g_1(x) + \Delta g_1(x))\omega + \\ \qquad (g_2(x) + \Delta g_2(x))u, \\ z = h_1(x) + k_{12}(x)u. \end{cases} \tag{1}$$

其中, $x(t)$ 是在原点某领域的状态向量 $x(0)=0$, z 是评价输出向量, u 为控制输入, ω 为外界扰动. $f(x)$、$g_1(x)$, $g_2(x)$, $h_1(x)$, $h_2(x)$、$k_{12}(x)$、$k_{21}(x)$ 为具有适当维数的已知函数向量. Δf, Δg_1, Δg_2 作为系统的不确定性,满足如下假设:

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假设 1

$$\Delta f(x) = e_f\delta_f,\ \|\delta_f\| \leqslant \|m_f(x)\|,$$
$$\Delta g_1 = e_\omega\delta_\omega,\ \|\delta_\omega\| \leqslant \|m_\omega(x)\|,$$
$$\Delta g_2 = e_u\delta_u,\ \|\delta_u\| \leqslant \|m_u(x)\|.$$

上式中，e_f, e_ω, e_u，是常矩阵，m_f, m_ω, m_u 是适当维数的向量函数.假设 1 是仿射不确定非线性系统的通常假设.本节与下一节的结论均不考虑 H_∞ 控制问题的所谓的标准假设，但在第 4 节我们为简便起见，仍将启用这一假设，详见后文.

3 状态反馈(State feedback)

作为预备性结果，我们先考虑标称系统：

$$\begin{cases}\dot{x}(t) = f(x) + g_1(x)\omega + g_2(x)u,\\ z = h_1(x) + k_{12}(x)u.\end{cases} \tag{2}$$

定理 1 设系统(2)是零状态可检测的，若存在适当的常数 $\lambda > 0$ 如下的 HJI 不等式

$$\frac{\partial V}{\partial x}f + \frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_1g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x} - (\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})^{\mathrm T} + h_1^{\mathrm T}h_1 \leqslant 0 \tag{3}$$

有光滑正定解，且 $I - k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T} > 0$，则状态反馈律

$$u = -(k_{12}^{\mathrm T}k_{12})^{-1}[\frac{1}{2}g_2^{\mathrm T}(x)\frac{\partial^{\mathrm T}V}{\partial x} + k_{12}^{\mathrm T}h_1] \tag{4}$$

使得系统(2)满足 H_∞ 指标

$$\|z\|^2 \leqslant \gamma^2\|\omega\|^2,\ \forall\omega \in L_2[0,\infty). \tag{5}$$

且 $\omega = 0$ 时，平衡点 $x = 0$ 渐近稳定.

证 考虑指标

$$H(x,u,\omega) =$$
$$\frac{\partial V}{\partial x}(f + g_1\omega + g_2u) + \|z\|^2 - \gamma^2\|\omega\|^2 =$$
$$\frac{\partial V}{\partial x}(f + g_1\omega + g_2u) + h_1^{\mathrm T}h_1 + 2h_1^{\mathrm T}k_{12}u + u^{\mathrm T}k_{12}^{\mathrm T}k_{12}u - \gamma^2\omega^{\mathrm T}\omega.$$

取鞍点：

$$\frac{\partial H}{\partial u} = 0,\ \frac{\partial H}{\partial \omega} = 0,$$

有 $u = -(k_{12}^{\mathrm T}k_{12})^{-1}[\frac{1}{2}g_2^{\mathrm T}(x)\frac{\partial^{\mathrm T}V}{\partial x} + k_{12}^{\mathrm T}h_1]$，

$$\omega = \frac{1}{2\gamma^2}g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x}.$$

代入 $H(x,u,\omega)$，进而可得

$$H(x,u,\omega) \leqslant$$
$$\frac{\partial V}{\partial x}f + \frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_1g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x} - (\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})^{\mathrm T} + h_1^{\mathrm T}h_1.$$

从而系统满足 H_∞ 指标(5).下面证自由系统的稳定性，$\omega = 0$ 时，

$$\dot V =$$
$$\frac{\partial V}{\partial x}f - \frac{\partial V}{\partial x}g_2(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})^{\mathrm T} \leqslant$$
$$-\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_1g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x} - (\frac{1}{2}\frac{\partial V}{\partial x}g_2 - h_1^{\mathrm T}k_{12})(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})^{\mathrm T} - h_1^{\mathrm T}h_1 \leqslant$$
$$-\frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_1g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x} - \frac{1}{4}\frac{\partial V}{\partial x}g_2(k_{12}^{\mathrm T}k_{12})^{-1}g_2^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x} - h_1^{\mathrm T}(I - k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1 \leqslant 0.$$

由系统零状态可检测性可知，$\dot V = 0$ 仅当 $x = 0$. 根据 LaSalle 不变集原理，自由系统是渐近稳定的.

证毕.

定理 1 给出了相应 H_∞ 控制的一个较为直接的结论——HJI 不等式，下面的定理进一步给出了该不等式的一个构造性解，它的构造思想来自文献[6]，但我们证明它确系(3)的一个解.

定理 2 设 HJI 不等式(3)中：$g_1 = g_2p + q$，且 k_{12} 行满秩，则如下构造的正定函数是(3)的一个解：

i) $\frac{\partial W}{\partial x}f(x) \leqslant -\alpha(x),\ W(x) > 0, \alpha(x) > 0$;

ii) $\frac{h_1^{\mathrm T}h_1}{\alpha} < \infty$，当 $x \to 0$;

iii) $\beta < \gamma^2,\ \|k_{12}p\| < \gamma^2, 0 < \eta < \frac{\gamma^2}{\|k_{12}p\|^2} - 1$;

iv) $\frac{1}{\alpha}h_1^{\mathrm T}(I - \frac{(1+\eta)}{\gamma^2}k_{12}pp^{\mathrm T}k_{12}^{\mathrm T})h_1 \leqslant K(W(x))$;

v) $K(W)[\frac{\partial W}{\partial x}qq^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}] \leqslant \beta\alpha(1 + \frac{1}{\eta})^{-1}$;

vi) $\frac{1}{2}(1 - \sqrt{1 - \frac{\beta}{\gamma^2}}) < \varepsilon < \frac{1}{2}(1 + \sqrt{1 - \frac{\beta}{\gamma^2}})$;

vii) $V(W) = \frac{1}{\varepsilon}\int_0^W K(t)\mathrm{d}t$.

证 直接将式(7)代入式(3)，并注意到 $\frac{\partial V}{\partial x} = \frac{1}{\varepsilon}K\frac{\partial W}{\partial x}$，我们有

$$\frac{\partial V}{\partial x}f + \frac{1}{4\gamma^2}\frac{\partial V}{\partial x}g_1g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x} - (\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}g_2 + h_1^{\mathrm T}k_{12})^{\mathrm T} + h_1^{\mathrm T}h_1 =$$
$$\frac{1}{\varepsilon}K\frac{\partial W}{\partial x}f + \frac{1}{4\gamma^2\varepsilon^2}K^2\frac{\partial W}{\partial x}(g_2p +$$

$$q)(g_2p+q)^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}-(\frac{1}{2\epsilon}K\frac{\partial W}{\partial x}g_2+h_1^{\mathrm T}k_{12})\cdot$$

$$(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2\epsilon}K\frac{\partial W}{\partial x}g_2+h_1^{\mathrm T}k_{12})^{\mathrm T}+h_1^{\mathrm T}h_1\leqslant$$

$$-\frac{1}{\epsilon}\alpha K+\frac{1}{4\gamma^2\epsilon^2}K^2\frac{\partial W}{\partial x}(g_2p+q)(g_2p+q)^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}-$$

$$(\frac{1}{2\epsilon}K\frac{\partial W}{\partial x}g_2+h_1^{\mathrm T}k_{12})(k_{12}^{\mathrm T}k_{12})^{-1}(\frac{1}{2\epsilon}K\frac{\partial W}{\partial x}g_2+$$

$$h_1^{\mathrm T}k_{12})^{\mathrm T}+h_1^{\mathrm T}h_1=$$

$$-\frac{1}{\epsilon}\alpha K-\frac{1}{4\epsilon^2}K^2\frac{\partial W}{\partial x}g_2((k_{12}^{\mathrm T}k_{12})^{-1}-\frac{1}{\gamma^2}pp^{\mathrm T})g_2^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}-$$

$$\frac{1}{\epsilon}K\frac{\partial W}{\partial x}g_2((k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1+\frac{1}{2\gamma^2\epsilon^2}K^2\frac{\partial W}{\partial x}\cdot$$

$$g_2pq^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}h_1^{\mathrm T}(I-k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1+$$

$$\frac{1}{4\gamma^2\epsilon^2}K^2\frac{\partial W}{\partial x}qq^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}\leqslant$$

$$-\frac{1}{\epsilon}\alpha K-\frac{1}{4\epsilon^2}K^2\frac{\partial W}{\partial x}g_2((k_{12}^{\mathrm T}k_{12})^{-1}-$$

$$\frac{1+\eta}{\gamma^2}pp^{\mathrm T})g_2^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}-\frac{1}{\epsilon}K\frac{\partial W}{\partial x}g_2((k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1+$$

$$\frac{1}{4\gamma^2\epsilon^2}K^2(1+\frac{1}{\eta})\frac{\partial W}{\partial x}qq^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}h_1^{\mathrm T}(I-$$

$$k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1=$$

$$-\frac{1}{\epsilon}\alpha K-\frac{1}{4\epsilon^2}K^2\frac{\partial W}{\partial x}g_2(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}(I-$$

$$\frac{1+\eta}{\gamma^2}k_{12}pp^{\mathrm T}k_{12}^{\mathrm T})k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}g_2^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}-$$

$$\frac{1}{\epsilon}K\frac{\partial W}{\partial x}g_2(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T}h_1+\frac{1}{4\gamma^2\epsilon^2}(1+$$

$$\frac{1}{\eta})K^2\frac{\partial W}{\partial x}qq^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}+h_1^{\mathrm T}(I-k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1\leqslant$$

$$-\frac{1}{\epsilon}\alpha K+\frac{1}{4\gamma^2\epsilon^2}(1+\frac{1}{\eta})K^2\frac{\partial W}{\partial x}qq^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}+$$

$$h_1^{\mathrm T}(I-(1+\eta)k_{12}(k_{12}^{\mathrm T}k_{12})^{-1}k_{12}^{\mathrm T})h_1\leqslant$$

$$-\frac{1}{\epsilon}\alpha K+\frac{\beta}{4\gamma^2\epsilon^2}\alpha K+\alpha K.\tag{6}$$

在上面的式(6)以下的证明中,我们使用了矩阵反演公式.根据定理条件 vi),我们易得:

$$\epsilon^2-\epsilon+\frac{\beta}{4\gamma^2}<0.$$

从而,式(3)成立.　　证毕.

推论 1　设 HJI 不等式(3)满足匹配条件: $g_1=g_2p$,且 k_{12} 行满秩.则如下构造的正定函数是式(3)的一个解:

i) $\frac{\partial W}{\partial x}f(x)\leqslant-\alpha(x),W(x)>0,\alpha(x)>0$;

ii) $\frac{h_1^{\mathrm T}h_1}{\alpha}<\infty$,当 $x\to0$;

iii) $\|k_{12}p\|<\gamma^2$;

iv) $\frac{1}{\alpha}h_1^{\mathrm T}(I-\frac{1}{\gamma^2}k_{12}pp^{\mathrm T}k_{12}^{\mathrm T})h_1\leqslant K(W(x))$;

v) $V(W)=\frac{1}{\epsilon}\int_0^W K(t)\mathrm dt$.

证　注意到在匹配条件成立时,定理 2 的条件 v)成为多余,经过简单的推导,即可得到我们所要的结论.

考虑系统(1),则我们可得以下结论.

定理 3　设系统(1)是对于所有 Δf 是零状态可检测的,若存在适当的常数 $\lambda_f,\lambda_\omega,\lambda_u>0$ 使得如下的 HJI 不等式

$$\frac{\partial V}{\partial x}f+\frac{1}{4}\frac{\partial V}{\partial x}(g_1MM^{\mathrm T}g_1^{\mathrm T}+\frac{1}{\lambda_f^2}e_fe_f^{\mathrm T}+$$

$$\frac{1}{\lambda_\omega^2}e_\omega e_\omega^{\mathrm T}+\frac{1}{\lambda_u^2}e_ue_u^{\mathrm T})\frac{\partial^{\mathrm T}V}{\partial x}-(\frac{1}{2}\frac{\partial V}{\partial x}g_2+$$

$$h_1^{\mathrm T}k_{12})(k_{12}^{\mathrm T}k_{12}+\lambda_u^2m_u^{\mathrm T}m_u)^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}g_2+h_1^{\mathrm T}k_{12})^{\mathrm T}+$$

$$h_1^{\mathrm T}h_1+\lambda_f^2m_f^{\mathrm T}m_f\leqslant0\tag{7}$$

有光滑正定解,则状态反馈律

$$u=-(k_{12}^{\mathrm T}k_{12}+\lambda_u^2m_u^{\mathrm T}m_u)^{-1}[\frac{1}{2}g_2^{\mathrm T}(x)\frac{\partial^{\mathrm T}V}{\partial x}+k_{12}^{\mathrm T}h_1]\tag{8}$$

使得系统(1)满足 H_∞ 指标(5),且 $\omega=0$ 时,平衡点 $x=0$ 渐近稳定.式(7)中:

$$MM^{\mathrm T}=(\gamma^2I-\lambda_\omega^2m_\omega^{\mathrm T}m_\omega)^{-1}.\tag{9}$$

证　考虑指标

$$H(x,u,\omega)=$$

$$\frac{\partial V}{\partial x}(f(x)+\Delta f(x)+(g_1(x)+\Delta g_1(x))\omega+$$

$$(g_2(x)+\Delta g_2(x))u)+\|z\|^2-\gamma^2\|\omega\|^2.$$

注意到:

$$\frac{\partial V}{\partial x}\Delta f(x)=\frac{\partial V}{\partial x}e_f\delta_f\leqslant\frac{1}{4\lambda_f^2}\frac{\partial V}{\partial x}e_fe_f^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x}+\lambda_f^2m_f^{\mathrm T}m_f,$$

$$\frac{\partial V}{\partial x}\Delta g_1\omega=\frac{\partial V}{\partial x}e_\omega\delta_\omega\leqslant\frac{1}{4\lambda_\omega^2}\frac{\partial V}{\partial x}e_\omega e_\omega^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x}+\lambda_\omega^2\omega^{\mathrm T}m_\omega^{\mathrm T}m_\omega\omega,$$

$$\frac{\partial V}{\partial x}\Delta g_2u=\frac{\partial V}{\partial x}e_u\delta_uu\leqslant\frac{1}{4\lambda_u^2}\frac{\partial V}{\partial x}e_ue_u^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x}+\lambda_u^2u^{\mathrm T}m_u^{\mathrm T}m_uu,$$

代入指标并取鞍点:

$$\frac{\partial H}{\partial u}=0,\ \frac{\partial H}{\partial\omega}=0,$$

有

$$u=-(k_{12}^{\mathrm T}k_{12}+\lambda_u m_u^{\mathrm T}m_u)^{-1}\Big[\frac{1}{2}g_2^{\mathrm T}(x)\frac{\partial^{\mathrm T}V}{\partial x}+k_{12}^{\mathrm T}h_1\Big],$$

$$\omega=\frac{1}{2}M^{\mathrm T}g_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x}.$$

余下的证明与定理1类似,略.

下面是本节的主要结论.

定理 4 在系统(2)中,设

$$G_1=\Big[g_1M\frac{1}{\lambda_f}e_f\quad\frac{1}{\lambda_u}e_u\quad\frac{1}{\lambda_\omega}e_\omega\Big],$$

$$G_1=g_2P_e+Q,\ K_1=\begin{bmatrix}k_{12}\\0\\\lambda_u m_u'\end{bmatrix},\ m_u'=[m_u\ 0],$$

且 K_1 列满秩. P_e,Q 是适当维数的矩阵,则如下构造的正定函数是HJI不等式(7)的一个解:

i) $\frac{\partial W}{\partial x}f(x)\leqslant-\alpha(x),W(x)>0,\alpha(x)>0$;

ii) $\frac{h_1^{\mathrm T}h_1+\lambda_f^2m_f^{\mathrm T}m_f}{\alpha}<\infty$,当 $x\to0$;

iii) $\beta<1,\|K_1P_e\|<1,0<\eta<\frac{1}{\|K_1P_e\|^2}-1$;

iv) $\frac{1}{\alpha}[h_1^{\mathrm T}(I-(1+\eta)k_{12}P_eP_e^{\mathrm T}K_{12}^{\mathrm T})h_1+\lambda_f^2m_f^{\mathrm T}m_f]\leqslant K(W(x))$;

v) $K(W)[\frac{\partial W}{\partial x}QQ^{\mathrm T}\frac{\partial^{\mathrm T}W}{\partial x}]\leqslant\beta\alpha(1+\frac{1}{\eta})^{-1}$;

vi) $\frac{1}{2}(1-\sqrt{1-\beta})<\varepsilon<\frac{1}{2}(1+\sqrt{1-\beta})$;

vii) $V(W)=\frac{1}{\varepsilon}\int_0^W K(t)\mathrm dt$.

M 由式(9)定义.

证 容易验证式(7)可以写成:

$$\frac{\partial V}{\partial x}f+\frac{1}{4}\frac{\partial V}{\partial x}G_1G_1^{\mathrm T}\frac{\partial^{\mathrm T}V}{\partial x}-(\frac{1}{2}\frac{\partial V}{\partial x}G_2+H^{\mathrm T}K)(K^{\mathrm T}K)^{-1}(\frac{1}{2}\frac{\partial V}{\partial x}G_2+H^{\mathrm T}K)^{\mathrm T}+H^{\mathrm T}H\leqslant0.$$

上式中: $G_2=[g_2\quad0\quad0\quad0],H=\begin{bmatrix}h_1\\\lambda_fm_f\\0\end{bmatrix},K=[K_1\quad K_2]$, K_2 满足条件: $H^{\mathrm T}K_2=0,K_1^{\mathrm T}K_2=0,K_2^{\mathrm T}K_2=I$. 从而根据定理条件以及定理2,我们可以构造序列以满足HJI不等式(8),取 $G_1=G_2P+Q,P=\begin{bmatrix}P_e\\0\\0\\0\end{bmatrix}$, 即 $G_1=g_2P_e+Q$. 注意到 $KP=[K_1\quad K_2]\begin{bmatrix}P_e\\0\\0\\0\end{bmatrix}=K_1P_e$,并以 $\gamma=1$ 代入定理2的结论即可得结论. 证毕.

注 尽管约束 K_2 的3个矩阵方程是容易满足的,但实际上本定理证明中 K_2 所应满足的这些条件仅仅是形式上的,因为在后面的计算中——如我们所见——K_2 与零矩阵相乘而消失了.

4 动态输出反馈(Dynamic output feedback)

考虑系统

$$\begin{cases}\dot x(t)=f(x)+\Delta f(x)+g_1(x)\omega+(g_2(x)+\Delta g_2(x))u,\\ z=h_1(x)+k_{12}(x))u,\\ y=h_2(x)+\Delta h_2(x)+k_{21}(x)\omega.\end{cases}\tag{10}$$

y 是观测输出向量,设如下标准假设成立:

假设 2

$$\|\Delta h_2\|\leqslant\|m_h\|,$$

$$k_{12}[k_{12}\quad g_1]=[I\quad0].$$

给出动态输出反馈控制器如下:

$$\begin{cases}\dot\xi=f_c(\xi)+g_c(\xi)y,\\ u=h_c(\xi).\end{cases}\tag{11}$$

我们有如下结论:

定理 5 设系统(10)满足假设1,2,取形如式(11)的动态输出反馈,若存在 $K(x,\xi),P(x,\xi)$ 使得

$$N_1N_1^{\mathrm T}=\frac{1}{\gamma^2}PP^{\mathrm T}-(K^{\mathrm T}K)^{-1},$$

$$N_2^{\mathrm T}N_2=I-K(K^{\mathrm T}K)^{-1}K^{\mathrm T}.$$

并且存在 $W(x,\xi)>0,\alpha(x,\xi)>0$ 使得

i) $\frac{\partial W}{\partial\hat x}(\hat f(x)+[B\quad G_c]N_1^{-1}(K^{\mathrm T}k)^{-1}K^{\mathrm T}N_2^{-1}\cdot\begin{bmatrix}C\\Dh_c\end{bmatrix})\leqslant-\alpha(x,\xi)$;

ii) $\frac{\begin{bmatrix}C\\Dh_c\end{bmatrix}^{\mathrm T}(N_2N_2^{\mathrm T})^{-1}\begin{bmatrix}C\\Dh_c\end{bmatrix}}{\alpha}<\infty$,当 $\hat x\to0$,

则动态反馈输出控制器(11)使得系统满足 $\mathrm H_\infty$ 指标且闭环稳定. 条件i),ii)中:

$$\hat x=\begin{bmatrix}x\\\xi\end{bmatrix},\ B=\begin{bmatrix}\frac{1}{\gamma}g_1&\frac{1}{\lambda_1}e_f&\frac{1}{\lambda_2}e_u&0\\0&0&0&0\end{bmatrix},$$

$$G_c=\begin{bmatrix}0&0\\0&\sqrt{\frac{1}{\gamma_2}+\frac{1}{\lambda_1^2}}g_c\end{bmatrix},$$

$$C = \begin{bmatrix} h_1 \\ \lambda_2 m_f \\ \lambda_1 m_h \end{bmatrix}, \ D = \begin{bmatrix} I \\ \lambda_2 m_u \end{bmatrix}.$$

证　考虑式(11),则闭环系统为

$$\begin{cases} \dot{\hat{x}}(t) = \hat{f}(\hat{x}) + \Delta\hat{f}_1(\hat{x}) + \Delta\hat{f}_2(\hat{x}) + \hat{g}(\hat{x})\omega, \\ z = \hat{h}(\hat{x}). \end{cases} \tag{12}$$

上式中:

$$\hat{f}(\hat{x}) = \begin{bmatrix} f(x) + g_2(x)h_c(\xi) \\ f_c(\xi) + g_2(x)h_2(x) \end{bmatrix},$$

$$\Delta\hat{f}_1(\hat{x}) = \begin{bmatrix} \Delta f(x) \\ g_c(\xi)\Delta h_2(x) \end{bmatrix} = \begin{bmatrix} e_f & 0 \\ 0 & g_c(\xi) \end{bmatrix}\begin{bmatrix} \delta_f \\ \Delta h_2(x) \end{bmatrix} = e_1\delta_1,$$

$$\Delta\hat{f}_2(\hat{x}) = \begin{bmatrix} \Delta g_2(x)h_c(\xi) \\ 0 \end{bmatrix} = \begin{bmatrix} e_g & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_g h_c \\ 0 \end{bmatrix} = e_2\delta_2,$$

$$\hat{g}(\hat{x}) = \begin{bmatrix} g_1(x) \\ g_c(\xi)k_{21}(x) \end{bmatrix}, \ \hat{h}(\hat{x}) = h_1 + k_{12}h_c.$$

容易验证,若如下的 HJI 不等式有正定解,则定理成立:

$$\frac{\partial U}{\partial \hat{x}}\hat{f} + \frac{1}{4}\frac{\partial U}{\partial \hat{x}}\left(\frac{1}{\gamma^2}\hat{g}\hat{g}^{\mathrm T} + \frac{1}{\lambda_1^2}e_1e_1^{\mathrm T} + \frac{1}{\lambda_1^2}e_2e_2^{\mathrm T}\right)\frac{\partial^{\mathrm T} U}{\partial \hat{x}} + h_1^{\mathrm T}h_1 + \lambda_1^2 m_f^{\mathrm T}m_f + \lambda_1^2 m_h^{\mathrm T}m_h + h_c^{\mathrm T}h_c + \lambda_2^2 h_c^{\mathrm T}m_g^{\mathrm T}m_g h_c \leqslant 0,$$

令

$$\frac{1}{\gamma^2}\hat{g}\hat{g}^{\mathrm T} + \frac{1}{\lambda_1^2}e_1e_1^{\mathrm T} + \frac{1}{\lambda_2^2}e_2e_2^{\mathrm T} = BB^{\mathrm T} + G_cG_c^{\mathrm T},$$

$$h_1^{\mathrm T}h_1 + \lambda_1^2 m_f^{\mathrm T}m_f + \lambda_1^2 m_h^{\mathrm T}m_h + h_c^{\mathrm T}h_c + \lambda_2^2 h_c^{\mathrm T}m_g^{\mathrm T}m_g h_c = C^{\mathrm T}C + h_c^{\mathrm T}D^{\mathrm T}Dh_c.$$

为了构造形如式(2)的 HJI 不等式

$$\frac{\partial V}{\partial x}F + \frac{1}{4\gamma^2}\frac{\partial V}{\partial x}G_1G_1^{\mathrm T}\frac{\partial^{\mathrm T} V}{\partial x} - \left(\frac{1}{2}\frac{\partial V}{\partial x}G_2 + H^{\mathrm T}K\right)(K^{\mathrm T}K)^{-1}\left(\frac{1}{2}\frac{\partial V}{\partial x}G_2 + H^{\mathrm T}K\right)^{\mathrm T} + H^{\mathrm T}H \leqslant 0,$$

我们令:

$$\hat{f} = F - G_2(K^{\mathrm T}K)^{-1}K^{\mathrm T}H,$$

$$BB^{\mathrm T} + G_cG_c^{\mathrm T} = \frac{1}{\gamma^2}G_1G_1^{\mathrm T} - G_2(K^{\mathrm T}K)^{-1}G_2^{\mathrm T},$$

$$C^{\mathrm T}C + h_c^{\mathrm T}D^{\mathrm T}Dh_c = H^{\mathrm T}(I - K(K^{\mathrm T}K)^{-1}K^{\mathrm T})H.$$

同时考虑匹配条件:

$$G_1 = G_2P.$$

取

$$N_1N_1^{\mathrm T} = \frac{1}{\gamma^2}PP^{\mathrm T} - (K^{\mathrm T}K)^{-1},$$

$$N_2^{\mathrm T}N_2 = I - K(K^{\mathrm T}K)^{-1}K^{\mathrm T},$$

可得:

$$G_2 = [B \quad G_c]N_1^{-1}, \ H = N_2^{-1}\begin{bmatrix} C \\ Dh_c \end{bmatrix}.$$

由推论 1 可知,若存在 $W(x,\xi) > 0, \alpha(x,\xi) > 0$ 使得

i') $\dfrac{\partial W}{\partial \hat{x}}F(x,\xi) \leqslant -\alpha(x,\xi)$;

ii') $\dfrac{H^{\mathrm T}H}{\alpha} < \infty$,当 $\hat{x} \to 0$.

将 G_2, H 代入上式,由推论 1 知 HJI 不等式解存在,稳定性证明则与定理 1 类似.　　证毕.

5 数值例子(Numerical example)

设系统(1)中

$$f(x) = \begin{bmatrix} -x_1^3 & -x_1 \\ 1 & \end{bmatrix}, \ g_1 = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{12} \end{bmatrix},$$

$$g_2 = k_g I, \ h_1 = \begin{bmatrix} x_1^2 \\ \frac{1}{2}x_2 \end{bmatrix}, \ k_{12} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix},$$

$$e_f = e_u = I, \ e_\omega = 0, \ m_f = \begin{bmatrix} x_1 \\ 0 \end{bmatrix},$$

$$m_u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ m_\omega = 0,$$

则

$$M = \frac{1}{\gamma}I, \ Q = 0.$$

g_{11}, g_{12}, k_g 为常数,使得当 $G_1 = g_2P_e$ 时,$P_eP_e^{\mathrm T} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$;又 $\| K_1P_e \| = \dfrac{\sqrt{3}}{2} < 1$(这里取的是迹范数).由条件 iii)可取 $\eta = \dfrac{1}{4}$.

由 $[h_1^{\mathrm T} \quad m_f^{\mathrm T}]\begin{bmatrix} h_1 \\ m_f \end{bmatrix} = x_1^4 + x_1^2 + \dfrac{1}{4}x_2^2$,可取 $\alpha(x) = x_1^4 + x_1^2 + \dfrac{1}{2}x_2^2e^{x_2}$, $W(x) = \dfrac{1}{2}x_1^2 + x_2^2e^{x_2}$ 易验证条件 i),ii)成立.

由 iv),可得

$$K(W(x)) \geqslant \frac{\frac{3}{8}x_1^4 + x_1^2 + \frac{11}{64}x_2^2}{x_1^4 + x_1^2 + \frac{1}{2}x_2^2e^{x_2}}.$$

同时由于 $Q = 0$,条件 v)自然成立.所以可取 $K(W) = 1$;我们取 $\beta = 0.36 < 1$,由 vi)可取 $\varepsilon = 0.5$;最后,由 vii)可得 $V(W) = 2W(x) = x_1^2 + 2x_2^2e^{x_2}$.

6 结论(Conclusion)

非线性控制理论的发展远没有达到线性系统理论的成熟的地步,一个突出的表现就在于它的理论成果很难转化为现实的应用.其主要原因就在于所谓理论上有意义的成果往往过于复杂,而导致实际应用效能的下降,以至于根本无法投入实际操作.可喜的是,这一现象目前正在逐步改变.本文的结果是这一方面的一个结果的自然延伸.

本文根据已有文献的结果,结合HJI不等式,也仅仅是通过HJI不等式的联系,给出了一类非线性系统的 H_∞ 控制问题的解的构造,直接拓宽了现有的结果.我们进一步考虑的动态输出反馈问题的结论,不再依赖一个难解的HJI不等式,而是两个简明的判据,改进了以往的结果.

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