Article ID: 1000 - 8152(2002)03 - 0345 - 04

On Robust Control of Mobile Manipulators *

DONG Wenjie and XU Wenli
(Department of Automation, Tsinghua University·Beijing, 100084, P. R. China)

Abstract: This paper studies the tracking control problem of mobile manipulators in which parameter uncertainty, system friction and disturbance are considered. A robust tracking controller is proposed based on the defined tracking errors, the structural properties of the system, and Barbalat's lemmas. The proposed controller ensures the system state to asymptotically track the desired trajectory in the presence of uncertainties. Simulations presented in the paper show the effectiveness of the proposed approach.

Key words: mobile manipulator; tracking control; nonholonomic system; robust control; uncertainty
Document code: A

移动机械手的鲁棒控制

董文杰 徐文立
(清华大学自动化系·北京, 100084)

摘要: 讨论了受摩擦力、外界扰动及参数不确定的移动机械手控制问题. 基于 Barbalat 引理、系统的结构性质和定义的跟踪误差, 提出了鲁棒控制器. 该控制器能使系统的状态在参数不确定情况下全部渐进趋于给定的期望轨迹. 仿真结果验证了本文所提出控制方法的有效性.

关键词: 移动机械手; 跟踪控制; 非完整系统; 鲁棒控制; 不确定性

1 Introduction

A mobile manipulator is composed of a manipulator arm and a mobile base on which the arm is mounted. Several researchers have studied the systems, see [1,2] for details. However, the study of the tracking problem of the mobile manipulator has been limited to partial states of the system without consideration of parameter uncertainty, and in some cases the interaction between the mobile base and the manipulator are neglected. In this paper, we discuss the tracking problem of the full state of the uncertain mobile manipulator. A robust controller is proposed. The proposed controller ensures the tracking errors of the full state to asymptotically globally tend to zero and is robust with respect to the parameter and friction uncertainty and disturbance.

2 Modeling and problem statement

The mobile manipulator shown in Fig. 1 consists of a mobile base and a multi-link manipulator. Generally, the mobile base may belong to any type of the nonholonomic wheeled mobile robots discussed in [3]. The robotic manipulator is a series-chain multi-link manipulator. Considering the constraints to which the base is subjected, the system motion equations can be represented in the following form:

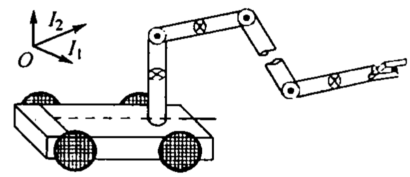

Fig. 1 A mobile manipulator

$$M_{11}(q)\ddot{q}_I + M_{12}(q)\ddot{q}_{II} + C_{11}(q,\dot{q})\dot{q} + G_{11}(q) + F_{11}(\dot{q}) + D_{11}(t) = J_1^{\mathrm{T}}(q_I)\lambda + B_{11}(q_I)\tau_I, \tag{1}$$

$$M_{21}(q)\ddot{q}_I + M_{22}(q)\ddot{q}_{II} + C_{21}(q,\dot{q})\dot{q} + G_{21}(q) + F_{21}(\dot{q}) + D_{21}(t) = \tau_{II}, \tag{2}$$

$$J_1(q_I)\dot{q}_I = 0, \tag{3}$$

where $q = [q_I^{\mathrm{T}}, q_{II}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^n$ is the generalized coordinates, $q_I \in \mathbb{R}^{n_1}$ and $q_{II} \in \mathbb{R}^{n_2}$ are the coordinates of the

* Foundation item: supported by the 985 Project of Tsinghua University (081100400), the Major Research Project of the Tenth-Five Plan of China (2001609A - 12).
Received date: 2000 - 08 - 10; Revised date: 2001 - 10 - 29.

mobile base and the manipulator respectively, $\tau_I \in \mathbb{R}^p$ and $\tau_{II} \in \mathbb{R}^{n_2}$ are the torques applied on the mobile base and the manipulator respectively, $B_{11} \in \mathbb{R}^{n_1 \times p}$ is the input matrix of the mobile base, $M_{ij}(1 \leqslant i,j \leqslant 2)$ denote the inertia matrix components, C_{11} and C_{21} are the Coriolis and centrifugal forces, G_{11} and G_{21} are the gravity vectors, F_{11} and F_{21} represent friction terms, D_{11} and D_{21} denote disturbance, λ is the vector of Lagrange multipliers corresponding to the constraints. $J_1 \in \mathbb{R}^{(n_1-m)\times n_1}$ is a full rank matrix qualifying the nonholonomic constraints. For clarity, we assume that J_1 contains the nonholonomic constraints only, otherwise a reduction procedure can be used to remove the holonomic constraints from (1) ~ (3). Let

$$M(q) = \begin{bmatrix} M_{11}(q) & M_{12}(q) \\ M_{21}(q) & M_{22}(q) \end{bmatrix},$$

$$C(q,\dot q) = \begin{bmatrix} C_{11}(q,\dot q) \\ C_{21}(q,\dot q) \end{bmatrix},\ G(q) = \begin{bmatrix} G_{11}(q) \\ G_{21}(q) \end{bmatrix},$$

$$F(\dot q) = \begin{bmatrix} F_{11}(\dot q) \\ F_{21}(\dot q) \end{bmatrix},\ D(t) = \begin{bmatrix} D_{11}(t) \\ D_{21}(t) \end{bmatrix},$$

$$B(q) = \begin{bmatrix} B_{11}(q_I) & 0 \\ 0 & I_{n_2\times n_2} \end{bmatrix},$$

$$J(q) = [J_1^{\mathrm T}(q_I),\ 0]^{\mathrm T},\ \tau = [\tau_I^{\mathrm T}, \tau_{II}^{\mathrm T}]^{\mathrm T}.$$

(1) ~ (3) can be written in the compact form:

$$M(q)\ddot q + C(q,\dot q)\dot q + G(q) + F(\dot q) + D(t) = J^{\mathrm T}(q)\lambda + B(q)\tau, \tag{4}$$

$$J(q)\dot q = 0. \tag{5}$$

In the system, we suppose $F(\dot q)$ and $D(t)$ are unknown. The inertia parameter vector $a \in \mathbb{R}^{d_1}$ of the system (4) is "uncertain", i.e., there exist $a_0 \in \mathbb{R}^{d_1}$ and a positive constant ρ such that $\|\tilde a\| := \|a - a_0\| \leqslant \rho$. Given a desired twice differentiable trajectory $q^*(t) = [q_I^{*\mathrm T}(t), q_{II}^{*\mathrm T}(t)]^{\mathrm T}$ which satisfies $J_1(q_I^*)\dot q_I^* = 0$, the tracking problem discussed here is finding a control law τ such that $\lim\limits_{t\to\infty}(q(t) - \dot q^*(t)) = 0$ and $\lim\limits_{t\to\infty}(\dot q(t) - \dot q^*(t)) = 0$.

It should be noted that the dynamics (4) possesses the following properties: 1) $M(q)$ is a bounded positive definite matrix, and $\forall q \in \mathbb{R}^n$, $\dot M(q) - 2C(q,\dot q)$ is skew-symmetric for a suitable definition of $C(q,\dot q)$; 2) $\forall \xi \in \mathbb{R}^n, M(q)\dot\xi + C(q,\dot q)\xi + G(q) = Y(q,\dot q,\xi,\dot\xi)a$, where the regressor matrix $Y(q,\dot q,\xi,\dot\xi)$ is a known function of $q,\dot q,\xi$ and $\dot\xi$; 3) There exist constants c_1 and c_2 such that $\|F(\dot q) + D(t)\| \leqslant c_1 + c_2\|\dot q\|$.

For Eq. (3), there exist linear independent vector fields $f_i(q_I)(1 \leqslant i \leqslant m)$ such that $\dot q_I = f_1(q_I)v_1 + \cdots + f_m(q_I)v_m = f(q_I)v_I$, where $f(q_I) = [f_1(q_I),\cdots, f_m(q_I)], v_I = [v_1,\cdots,v_m]^{\mathrm T}$ is suitably defined. Let $v_{II} = \dot q_{II}$, then $\dot q_{II} = v_{II}$. Thus

$$\dot q = g(q)v, \tag{6}$$

where $g(q) = \mathrm{diag}[f(q_I), I_{n_2\times n_2}]$ and $v = [v_I^{\mathrm T}, v_{II}^{\mathrm T}]^{\mathrm T}$. Differentiating (6), one obtains $\ddot q = \dot g v + g\dot v$. Substituting this equation into (4), and then multiplying both sides of (4) from left with $g^{\mathrm T}(q)$, one obtains

$$M_1(q)\dot v + C_1(q,\dot q)v + G_1(q) + F_1(q,\dot q) + D_1(q,t) = B_1(q)\tau, \tag{7}$$

where

$$M_1(q) = g^{\mathrm T}(q)M(q)g(q),$$
$$C_1(q,\dot q) = g^{\mathrm T}(q)M(q)\dot g(q) + g^{\mathrm T}(q)C(q,\dot q)g(q),$$
$$G_1(q) = g^{\mathrm T}(q)G(q),\ F_1(q,\dot q) = g^{\mathrm T}(q)F(\dot q),$$
$$D_1(q,t) = g^{\mathrm T}(q)D(t), B_1(q) = g^{\mathrm T}(q)B(q).$$

In what follows, it is assumed that $p \geqslant m$ and $f^{\mathrm T}B_{11}$ is a full rank matrix, so that the mobile manipulator is fully actuated.

Noting the results in [4], there exist diffeomorphic state and input transformations:

$$z_I = [z_1,\ z_{2,1},\cdots,z_{s_1,1};\cdots,\ z_{2,m-1},\cdots,z_{s_{m-1},m-1}]^{\mathrm T} = \phi_1(q_I), \tag{8}$$

$$u_I = [u_1,u_2,\cdots,u_m]^{\mathrm T} = \phi_2^{-1}(q_I)v_I,\ u_{II} = v_{II}, \tag{9}$$

such that the system (6),(7) can be put into the form

$$\begin{cases} \dot z_1 = u_1,\ \dot z_{s_j,j} = u_{j+1},\ \dot z_{i,j} = z_{i_1,j}u_1, \\ 2 \leqslant i \leqslant s_j - 1,\ 1 \leqslant j \leqslant m-1, \end{cases} \tag{10}$$

$$\dot z_{II} = u_{II}, \tag{11}$$

$$M_2(z)\dot{u}+C_2(z,\dot{z})u+G_2(z)+F_2(z,\dot{z})+D_2(z,t)=B_2(z)\tau, \tag{12}$$

where

$$z=[z_I^{\mathrm{T}},z_{II}^{\mathrm{T}}]^{\mathrm{T}},\ u=[u_I^{\mathrm{T}},u_{II}^{\mathrm{T}}]^{\mathrm{T}},$$

$$M_2(z)=\Omega^{\mathrm{T}}(q)M(q)\Omega(q)\mid_{\{q_I=\phi_1^{-1}(z_I),\ q_{II}=z_{II}\}},$$

$$C_2(z,\dot{z})=\Omega^{\mathrm{T}}(q)[M(q)\dot{\Omega}(q)+C(q,\dot{q})M(q)\Omega(q)]\mid_{\{q_I=\phi_1^{-1}(z_I),\ q_{II}=z_{II}\}}$$

$$G_2(z)=\Omega^{\mathrm{T}}(q)G(q)\mid_{\{q_I=\phi_1^{-1}(z_I),\ q_{II}=z_{II}\}},$$

$$F_2(z,\dot{z})=\Omega^{\mathrm{T}}(q)F(\dot{q})\mid_{\{q_I=\phi_1^{-1}(z_I),\ q_{II}=z_{II}\}},$$

$$D_2(z,t)=\Omega^{\mathrm{T}}(q)D(t)\mid_{\{q_I=\phi_1^{-1}(z_I),\ q_{II}=z_{II}\}},$$

$$B_2(z)=\Omega^{\mathrm{T}}(q)B(q)\mid_{\{q_I=\phi_1^{-1}(z_I),\ q_{II}=z_{II}\}},$$

$$\Omega(q)=\mathrm{diag}[f(q_I)\phi_2(q_I),I_{n_2\times n_2}].$$

It is easy to prove that the dynamics (12) retains the following two properties: 1) $M_2(z)$ is a positive definite matrix for any $z\in\mathbb{R}^n$ and $\dot{M}_2(z)-2C_2(z,\dot{z})$ is skew-symmetric; 2) For any differentiable $\xi\in\mathbb{R}^{m+n_2}$, $M_2(z)\dot{\xi}+C_2(z,\dot{z})\xi+G_2(z)=Y_2(z,\dot{z},\xi,\dot{\xi})a$ where $Y_2(z,\dot{z},\xi,\dot{\xi})$ is a known matrix of $z,\dot{z},\xi,\dot{\xi}$.

For the desired trajectory $q_I^*(t)$, it is easy to find $z_I^*=[z_1^*;z_{2,1}^*,\cdots,z_{s_1,1}^*;\cdots;z_{2,m-1}^*,\cdots,z_{s_{m-1},m-1}^*]^{\mathrm{T}}$, $u_I^*=[u_1^*,u_2^*,\cdots,u_m^*]^{\mathrm{T}}$, and $z_{II}^*=q_{II}^*$, $u_{II}^*=\dot{q}_{II}^*$ such that

$$\dot{z}_1^*=u_1^*,\ \dot{z}_{s_j,j}^*=u_{j+1}^*\dot{z}_{II}^*=u_{II}^*,$$

$$\dot{z}_{i,j}^*=z_{i+1,j}^*u_1^*,$$

$$2\leqslant i\leqslant s_j-1,1\leqslant j\leqslant m-1.$$

Therefore, the tracking problem can be restated as finding the control law τ of the system (10) ~ (12) such that

$$\lim_{t\to\infty}(z_I(t)-z_I^*(t))=0,\ \lim_{t\to\infty}(u_I(t)-u_I^*(t))=0,$$

$$\lim_{t\to\infty}(z_{II}(t)-z_{II}^*(t))=0,\lim_{t\to\infty}(u_{II}(t)-u_{II}^*(t))=0.$$

3 Controller design

Let $e_I=[e_1;\ e_{2,1},\cdots,e_{s_1,1};\cdots;\ e_{2,m-1},\cdots,e_{s_{m-1},m-1}]^{\mathrm{T}}=\Psi\cdot(z_I-z_I^*)$, where $\Psi=\mathrm{diag}[\Psi^1,\Psi_1^2,\cdots,\Psi_1^{m-1}]$, $\Psi_1^l(2\leqslant l\leqslant m-1)$ is the resulting matrix after eliminating the first row and the first column of the matrix Ψ^l. $\Psi^l=\{\psi_{i,j}^l\}\in\mathbb{R}^{s_l\times s_l}(1\leqslant l\leqslant m-1)$, and $\psi_{i,j}^l$ is defined as follows

$$\begin{cases}\psi_{i,i}^l=1\ (1\leqslant i\leqslant s_l),\ \psi_{i,j}^l=0\ (i<j;1\leqslant i,j\leqslant s_l),\\ \psi_{i,1}^l=0\ (2\leqslant i\leqslant s_l),\\ \psi_{i,j}^l=0\ (i,j \text{ are not odd or even at the same time}),\\ \psi_{i,j}^l=k_{i-3,l}\psi_{i-2,j}^l+\psi_{i-1,j-1}^l(4\leqslant i\leqslant s_l,\ 2\leqslant j\leqslant s_l)\end{cases}$$

and constants $k_{i,l}>0\ (1\leqslant i\leqslant s_l-3)$. Let

$$\eta_I=\begin{bmatrix}u_1^*+p\\ u_2^*-\mu_{3,1}e_{s_1,1}-k_{s_1-2,1}u_1^*e_{s_1-1,1}-u_1^*\sum_{i=2}^{s_1-1}\psi_{s_1,i}^1(z_{i+1,1}-z_{i+1,1}^*)\\ \vdots\\ u_m^*-\mu_{3,m-1}e_{s_{m-1},m-1}-k_{s_{m-1}-2,m-1}u_1^*e_{s_{m-1}-1,m-1}-u_1^*\sum_{i=2}^{s_{m-1}-1}\psi_{s_{m-1},i}^{m-1}(z_{i+1,m-1}-z_{i+1,m-1}^*)\end{bmatrix},$$

$$\eta_{II}=\dot{z}_{II}^*-K_{II}(z_{II}-z_{II}^*),$$

$$\dot{p}=-\mu_2p-\mu_1e_1-\sum_{l=1}^{m-1}\sum_{j=2}^{s_l-1}\left[\sum_{i=2}^{j}\frac{e_{j,l}\psi_{j,i}^lz_{i+1,l}}{k_{0,l}k_{1,l}\cdots k_{j-2,l}}+\frac{e_{s_l,l}\psi_{s_l,j}^lz_{j+1,l}}{k_{1,l}k_{2,l}\cdots k_{s_l-2,l}}\right],$$

where $k_{0,j}=1,k_{s_i-2,j}>0\ (1\leqslant j\leqslant m-1),\mu_1>0$, $\mu_2>0$, and $\mu_{3,j}>0\ (1\leqslant j\leqslant m-1)$, K_{II} is a constant positive definite matrix. Let $e_{II}=z_{II}-z_{II}^*$ and $e=[e_I^{\mathrm{T}},e_{II}^{\mathrm{T}}]^{\mathrm{T}}$, the following theorem can be obtained.

Theorem Assume that z_I^*, u_I^* and $\dot{u}_I^*$ are bounded, if u_1^* does not tend to zero, then the control law

$$\tau=B_2^{\#}(z)[Y_2(z,\dot{z},\eta,\dot{\eta})(a_0-\frac{\rho Y_2^{\mathrm{T}}(z,\dot{z},\eta,\dot{\eta})(u-\eta)}{\|Y_2^{\mathrm{T}}(z,\dot{z},\eta,\dot{\eta})(u-\eta)\|+\delta(t)})-\frac{\chi_1^2(u-\eta)}{\chi_1\|u-\eta\|+\delta(t)}-K_p(u-\eta)-\Lambda]$$

make $e,\dot{e}$ and p tend to zero, where # is any left inverse, $\eta=[\eta_I^{\mathrm{T}},\eta_{II}^{\mathrm{T}}]^{\mathrm{T}}$,

$$\Lambda = \begin{bmatrix} \mu_1 e_1 + \sum_{l=1}^{m-1}\sum_{j=2}^{s_l-1}\left[\sum_{i=2}^{j}\frac{e_{j,l}\psi_{j,i}^{l} z_{i+1,l}}{k_{0,l}k_{1,l}\cdots k_{j-2,l}} + \frac{e_{s_l,l}\psi_{s_l,j}^{l} z_{j+1,l}}{k_{1,l}k_{2,l}\cdots k_{s_l-2,l}}\right] \\ \frac{e_{s_1,1}}{k_{1,1}k_{2,1}\cdots k_{s_1-2,1}} \\ \vdots \\ \frac{e_{s_{m-1},m-1}}{k_{1,m-1}k_{2,m-1}\cdots k_{s_{m-1}-2,m-1}} \\ z_{II} - z_{II}^{*} \end{bmatrix},$$

$$\chi_1 = \| \Omega(z) \| \cdot [c_1 + c_2(\| \frac{\mathrm{d}}{\mathrm{d}t}\phi_1^{-1}(z_I) \| + \| \dot{z}_{II} \|)],$$

$$\Omega(z) = \Omega(q) \mid_{\{q_I = \phi_1^{-1}(z_I),\ q_{II} = z_{II}\}}$$

and $k_{0,j} = 1$, constants $k_{i,j} > 0$ $(1 \leqslant i \leqslant s_j - 2, 1 \leqslant j \leqslant m-1)$, $\mu_1 > 0$, $\mu_2 > 0$, $\mu_{3,j} > 0$ $(1 \leqslant j \leqslant m-1)$, K_{II}, and K_p are constant positive definite matrices, $\delta(t) > 0$ and such that $\int_0^\infty \delta(t)\mathrm{d}t < \infty$.

Proof Define $\tilde{u} = u - \eta$, let

$$V = \frac{1}{2}(p^2 + \mu_1 e_1^2 + e_{II}^{\mathrm{T}} e_{II} + \tilde{u}^{\mathrm{T}} M_2 \tilde{u}) + \frac{1}{2}\sum_{j=1}^{m-1}\sum_{i=2}^{s_j}\frac{e_{i,j}^2}{k_{0,j}k_{1,j}\cdots k_{i-2,j}}.$$

Differentiating V along the closed-loop system, with the help of the extended Barbalat's lemma, the theorem can be proved, which is omitted here for space limit.

The assumption about z_I^* can be relaxed, see [5,6]. In the controller, $\delta(t)$ could have different forms. For example, it may be $\mathrm{e}^{-\alpha t}$ $(\alpha > 0)$ or $1/(1+t)^{\alpha}$ $(\alpha > 1)$.

4 Simulations

Simulation of a typical mobile manipulator discussed in [7] is done. With the help of the result in the paper, the robust controller can be easily designed. Under certain parameters and initial conditions, responses of $(q - q^*)$ and $(\dot{q} - \dot{q}^*)$ are shown in Figs. 2 and 3. It is obvious that the tracking errors converge to zero, which shows the effectiveness of the proposed result.

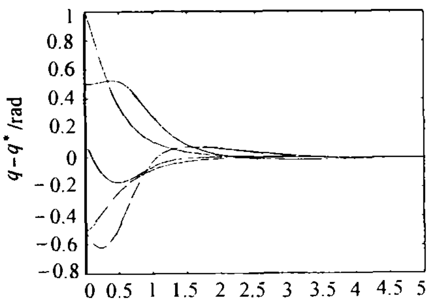

Fig. 2 Response of $(q-q^*)$

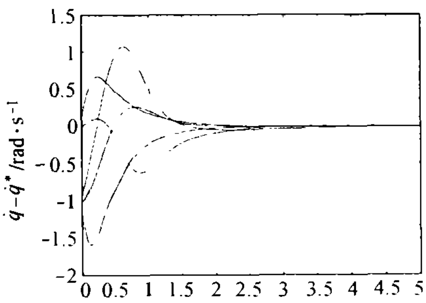

Fig. 3 Response of $(\dot{q}-\dot{q}^*)$

References

[1] Yamamoto Y and Yun X. Coordinating locomotion and manipulation of a mobile manipulator [J]. IEEE Trans. on Automatic Control, 1994, 39(6): 1326 - 1332

[2] Yamamoto Y and Yun X. Effect of the dynamic interaction on coordinated control of mobile manipulators [J]. IEEE Trans. on Robotics and Automation, 1996, 12(5): 816 - 824

[3] Campion G, Bastin G and d'Andrea-Novel B. Structure properties and classification of kinematic and dynamic models of wheeled mobile robots [J]. IEEE Trans. Robotics and Automation, 1996, 12(1): 47 - 62

[4] Dong Wenjie and Huo Wei. Tracking control of wheeled mobile robots with unknown dynamics [A]. Proc. of the IEEE Conf. Robotics and Automation [C], Detroit, Michigan, USA, 1999, 2645 - 2650

[5] Dong Wenjie, Huo Wei, Tso S K, et al. Tracking control of uncertain dynamic nonholonomic system and its application to wheeled mobile robots [J]. IEEE Trans. on Robotics and Automation, 2000, 16(6): 870 - 874

[6] Dong Wenjie and Xu W L. Adaptive tracking control of uncertain nonholonomic dynamic system [J]. IEEE Trans. on Automatic Control, 2001, 46(3): 450 - 454

[7] Dong Wenjie, Xu Yangsheng and Wang Qi. On tracking control of mobile manipulators [A]. Proc. of IEEE Conf. on Robotics and Automation [C], San Francisco, CA, USA, 2000, 3455 - 3560

本文作者简介

董文杰 在清华大学自动化系作博士后研究工作.研究领域为机器人动力学与控制,非完整系统控制,自适应控制等. Email: dongwenjie@yahoo.com

徐文立 清华大学自动化系博士生导师.研究领域为运动控制,计算机视觉等.