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Reconfigurable Control System Design by Output Feedback Eigenstructure Assignment

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Abstract: A new method for designing reconfigurable control system by using eigenstructure assignment is proposed. Under the condition $m + r - 1 \geqslant n$, the eigenvalues of the reconfigured closed-loop fault system can completely recover those of the original system by resynthesizing a new output feedback gain matrix, and the corresponding eigenvectors of the former is assigned in assignable subspace as close to those of the latter as possible. So the performance of the reconfigured system may recover the original one to a maximum extent. The advantage of the proposed method is that the stability of the reconfigured system can be guaranteed, and the algorithm for calculating the output feedback gain matrix is relatively simple. The illustrative example and simulation results demonstrate the effectiveness of the proposed method.

Key words: stable reconfigurable control; eigenstructure assignment; output feedback

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基于输出反馈特征结构配置的重构控制系统设计

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摘要: 推出了一种利用特征结构配置设计重构控制系统的新方法.当故障后的系统满足 $m + r - 1 \geqslant n$ 的条件时,通过重新设计一输出反馈增益阵,使重构后的闭环故障系统的特征值全部恢复到故障前的位置,特征向量也在可配置子空间中尽量接近故障前相应的闭环系统特征向量,从而使重构后的系统性能最大程度地恢复到故障前的系统性能.这种方法的优点是重构系统的稳定性可得到保证,且计算输出反馈增益阵的算法相对简单.应用实例和仿真结果说明本文方法的有效性.

关键词: 稳定重构控制; 特征结构配置; 输出反馈

1 Introduction

With the development of the automatic control theory, especially its wide-ranging application in almost all engineering fields, the control system becomes more and more complex. To design a complex control system, its safety and reliability must be considered as the first priority, especially for the safety-critical control systems, such as aircraft, underwater navigators and nuclear plants etc. Safety and reliability is more important than performance for this kind of systems in some sense. To meet the extremely high safety and reliability requirement, the best way is that the control system should be able to retain stability and regain the acceptable performance in the event of system component failures or drastic variation in operating conditions (Here, whether the system component failures or drastic variation in operating conditions are called outstanding variations, which would make the control systems designed by using conventional or adaptive method unstable or its performance becomes too bad to satisfy engineering requirement. In the sequel, we briefly call the outstanding variation as variation). This is where reconfigurable control system proves to be use-

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ful. A reconfigurable control system is one that can adjust the parameters and/or even the structure of the controller automatically on-line, maintain overall system stability in real time and recover the performance of the original system to a maximum extent after the outstanding variations.

The study on recofigurable control system is a challenging subject. Many new methods and schemes were proposed in the references [1 ~ 8]. In addition to linear quadratic regulator method (Looze et al, 1985) and pseudo inverse method (Gao Z. and Antsaklis P. J. et al, 1991), eigenstructure assignment method (EAM) (J. Jiang et al, 1994) become more and more attractive. Based on the fact that the performance of a control system is mainly determined by its eigenstructure, for the post-variation system, a new feedback gain matrix was synthesized to recover the eigenvalues of the original system and made corresponding eigenvectors as close to those of the original system as possible. If the full state feedback is feasible, the eigenvalues of the original system can be recovered completely, and the corresponding eigenvectors of the reconfigured control system can be made as close to those of the original system as possible in a least square sense. But if only output feedback is feasible, only the dominant eigenvalues can be recovered, so the stability of the reconfigured control system may not be guaranteed. In this paper, on the basis of the EAM, we emphasize the design of reconfigurable control system only with output feedback in a more general sense. The proposed method can easily be extended to the full state feedback case by simply setting the output matrix C to be an identity matrix.

The paper is organized as follows: in Section 2, the problem in study is mathematically described. Section 3 concentrates on how to assign the eigenstructure of the reconfigurable control system and calculation on output feedback gain matrix; to demonstrate the effectiveness of the proposed method, an illustrative example and simulation results are given in Section 4; Finally in Section 5, we give a conclusion and remarks on the method.

2 The problem description

Consider a linear time invariant(LTI) MIMO system:

$$\begin{cases} \dot{X}(t) = AX(t) + BU(t), \\ Y(t) = CX(t), \end{cases} \quad (2.1)$$

where $X(t) \in \mathbb{R}^n$, $U(t) \in \mathbb{R}^{m_n}$, and $Y(t) \in \mathbb{R}^{r_n}$ are the states, inputs and outputs of the system, respectively; $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m_n}$ and $C \in \mathbb{R}^{r_n \times n}$ are the state transition, input and output matrices, respectively. Further, assume $\{C, A, B\}$ to be controllable, and observable; B and C to be of full rank, i.e. $\text{rank}[B] = m_n$, $\text{rank}[C] = r_n$. Meanwhile, without loss of generality, assume $m_n \leqslant n$, $r_n \leqslant n$. System (2.1) satisfying the conditions above is called the normal system.

Based on the fact that the internal behavior of a LTI system can be determined by its eigenstructure, and the performance of the closed-loop system can be improved by modifying the eigenstructure with outputs feedback. For the normal system (2.1), let:

$$U(t) = -K_o Y(t). \quad (2.2)$$

The closed-loop normal system

$$\dot{X}(t) = [A + BK_o C)X(t) = A_{cl}X(t) \quad (2.3)$$

can be made to have a set of desired eigenvalues λ_i, $i = 1, 2, \cdots, n$, and corresponding eigenvectors V_i, $i = 1, 2, \cdots, n$.

Suppose that because of the outstanding variation, the normal system (2.1) becomes:

$$\begin{cases} \dot{X}_f(t) = A_f X_f(t) + B_f U_f(t), \\ Y_f(t) = C_f X_f(t), \end{cases} \quad (2.4)$$

where $X_f(t) \in \mathbb{R}^n$, $U_f(t) \in \mathbb{R}^m$, and $Y_f(t) \in \mathbb{R}^r$ are the states, inputs and outputs of the fault system, respectively; $A_f \in \mathbb{R}^{n \times n}$, $B_f \in \mathbb{R}^{n \times m}$ and $C_f \in \mathbb{R}^{r \times n}$ are the state transition, input and output matrices, respectively. System (2.4) is called the fault system.

The task for reconfiguring a control system is to re-synthesize a new output feedback gain matrix, noted by K_{fo}, to make the performance of the closed-loop fault system as close to that of the closed-loop normal system as possible. Based on the eigenstructure assignment method, a new K_{fo} must be re-synthesized, such that the eigenvalues, λ_{fi}, $i = 1, 2, \cdots, n$, of the closed-loop fault system:

$$\dot{X}_f(t) = [A_f + B_f K_{fo} C_f] X_f(t) = A_{fcl} X_{(}t) \quad (2.5)$$

will be arbitrarily close to those of the original system. This can be mathematically described as:

$$\lambda_{fi} = \lambda[A_f + B_f K_{fo} C_f] = \lambda[A + BK_o C] = \lambda_i, \quad i = 1, 2, \cdots, n. \quad (2.6)$$

But for the eigenvector, according to [9], V_i and V_{fi} must lie in the subspace spanned by:

$$L_i = (\lambda_i I - A)^{-1}B,\ i = 1,2,\cdots,n, \tag{2.7}$$

$$L_{fi} = (\lambda_{fi} I - A_f)^{-1}B_f,\ i = 1,2,\cdots,n, \tag{2.8}$$

respectively. Even if λ_{fi} can be made as identical as λ_i, V_{fi} and V_i may not lie in the same subspace. So V_{fi}, $i = 1,2,\cdots,n$ can be chosen only in its assignable space to make them as close to $V_i, i = 1,2,\cdots,n$ as possible in some sense.

3 Eigenstructure assignment by output feedback

Assume $\Delta = \{\lambda_1,\lambda_2,\cdots,\lambda_n\} \in D$ to be the eigenvalues of the closed-loop normal system.

Where D is the class of all sets of n complex numbers satisfying the following two conditions:

1) Each complex number λ in Δ must be accompanied by its conjugate $\bar{\lambda}$;

2) All numbers in Δ are assumed to be distinct.

Without loss of generality, further assume $\lambda_i \in \Delta$ to be different with those of the open-loop system.

For fault system, if $\{C_f, A_f, B_f\}$ is controllable and observable, $\mathrm{rank}[B_f] = m$, $\mathrm{rank}[C_f] = r$, and $m + r - 1 \geqslant n$, according to [7,10], there always exists an output feedback gain matrix K_{fo}, which makes all the n eigenvalues of the closed-loop fault system, noted by $\Delta_f = \{\lambda_{f1},\lambda_{f2},\cdots,\lambda_{fn}\} \in D$, assigned arbitrarily close to Δ. According to [11], divide Δ_f into $\Delta_f = \{\Delta_{fm}, \Delta_{fn-m}\}$. $\Delta_{fm} = \{\lambda_1,\lambda_2,\cdots,\lambda_m\} \in D$ and $\Delta_{fn-m} = \{\lambda_{m+1},\lambda_{m+2},\cdots,\lambda_n\} \in D$.

Assume $W_m = [w_1, w_2,\cdots,w_m]^{\mathrm{T}}$ to be the matrix which consists of right eigenvectors corresponding to Δ_{fm}. By the definition of the eigenvector:

$$W_m A_{cl} = \Pi_m W_m, \tag{3.1}$$

where $\Pi_m = \mathrm{diag}\{\lambda_1,\lambda_2,\cdots,\lambda_m\}$.

Theorem 1[4] For the fault system (2.4) with the output feedback gain matrix K_{fo}, if the following conditions:

1) $\{C_f, A_f, B_f\}$ is controllable and observable and $\mathrm{rank}[B_f] = m$, $\mathrm{rank}[C_f] = r$;

2) $m + r - 1 \geqslant n$

are satisfied, such that:

1) All n eigenvalues of the closed-loop fault system can be assigned arbitrarily close to those of the closed-loop normal system;

2) The eigenvalues of the closed-loop fault system consists of m eigenvalues determined by Π_m, and $(n-m)$ eigenvalues determined by the following substate feedback system:

$$\dot{\bar{X}}(t) = [\bar{A} + \bar{B}\bar{K}]\bar{X}(t), \tag{3.2}$$

where $\bar{A} = S_2 A_f B_c$, $\bar{B} = S_2 A_f B_f$, $\bar{K} = -[W_m B_f]^{-1} W_m B_c$. $B_c \in \mathbb{R}^{n\times(n-m)}$ is a matrix that makes $[B_f, B_c]$ non-singular, and $\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = [B_f B_c]^{-1}$.

According to Theorem 1, if $m + r - 1 \geqslant n$, all n eigenvalues of the closed-loop fault system can be assigned arbitrarily close to those of the closed-loop normal system. But for the eigenvector assignment, it is not so easy as the eigenvalue assignment. Generally speaking, even if λ_{fi} can be made arbitrarily close to λ_i, because of the system component failure or operating conditions variations, the corresponding eigenvectors V_{fi} does not lie in the same subspace as V_i. V_{fi} can be chosen only in its assignable subspace to make V_{fi} as close to the corresponding V_i as possible in a least square sense, which can be formulated by the following minimization index:

$$\min J_i(V_{fi}) = \min(V_i - V_{fi})^{\mathrm{T}} W_i (V_i - V_{fi}),\quad i = 1,2,\cdots,n, \tag{3.3}$$

where $W_i \in \mathbb{R}^{n\times n}$ is a positive definite weighting matrix. V_{fio} that is closest to the corresponding V_i can be obtained by using orthogonal projection:

$$V_{fio} = L_{fi}[L_{fi}^{\mathrm{T}} W_i^{\mathrm{T}} W_i L_{fi}]^{-1} L_{fi}^{\mathrm{T}} W_i^{\mathrm{T}} V_i,\ i = 1,2,\cdots,n. \tag{3.4}$$

It is clear that the minimization index is nothing but the orthogonal projection. If the eigenvector of the closed-loop normal system happens to be nearly perpendicular to the assignable eigen-subspace of the fault system, there will be a large projection error. The error bound estimation was discussed in [6].

The following task is to calculate the output feedback gain matrix in two steps. In the first step, according to the subsystem (3.2), a state feedback gain matrix $\bar{K}$ is calculated by using any common algorithm to assign the eigenvalues Δ_{fn-m} and corresponding eigenvectors V_{fi}, $i = m+1, m+2,\cdots,n$. Considering the relationship between the subsystem (3.2) and the fault system, the

$\bar{K}$ can be determined in the following way:

1) Determine V_{fio}, $i = m+1, m+2, \cdots, n$, in terms of (3.4);

2) Let $U_i = S_2 V_{fio}$, $i = m+1, m+2, \cdots, n$;

3) Choose a proper $\bar{K}$ to make $(\bar{A} + \bar{B}\bar{K})U_i = \lambda_i U_i$.

The following theorem will prove that if $U_i = S_2 V_{fio}$ were an eigenvector of the subsystem (3.2), V_{fio} would be the eigenvector of the closed-loop fault system (2.5).

Theorem 2 If $U_i = S_2 V_{fio}$, $i = m+1, m+2, \cdots, n$, were assigned as right eigenvector of subsystem (3.2) by choosing a proper state feedback gain matrix $\bar{K}$, V_{fio} would be the eigenvector of the closed-loop fault system (2.5).

Proof If $U_i = S_2 V_{fio}$ were a eigenvector of the subsystem (3.2), there must be: $(\bar{A} + \bar{B}\bar{K})U_i = \lambda_i U_i$, and for the subsystem (3.2).

$$\bar{A}_{cl}\begin{bmatrix} U_i \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{A} + \bar{B}\bar{K} & \bar{B}M_{-1} \\ 0 & \Pi_m \end{bmatrix}\begin{bmatrix} U_i \\ 0 \end{bmatrix} = \lambda_i \begin{bmatrix} U_i \\ 0 \end{bmatrix}.$$

So that $[U_i^{\mathrm{T}} \quad 0]^{\mathrm{T}}$ is the right eigenvector of $\bar{A}_{cl}$. Noted it by $\bar{V}_{fi}$, and then:

$$\bar{V}_{fi} = \begin{bmatrix} U_i \\ 0 \end{bmatrix} = \begin{bmatrix} S_2 V_{fi} \\ W_m V_{fi} \end{bmatrix} = \begin{bmatrix} S_2 \\ MS_1 - NS_2 \end{bmatrix} V_{fi} = TV_{fi}.$$

Meanwhile:

$$A_{cl}V_{fi} = T^{-1}\bar{A}_{cl}TV_{fi} = T^{-1}\bar{A}_{cl}\bar{V}_{fi} = \lambda_i T^{-1}\bar{V}_{fi} = \lambda_i V_{fi}.$$

It is clear that V_{fi} is the right eigenvector of the closed-loop fault system (2.5). The conclusion is as follows.

To recover the performance of the closed-loop normal system to a maximum extent, without loss of generality, all the eigenvalues can be reordered in the following way: $\mathrm{Re}[\lambda_1] \leqslant \mathrm{Re}[\lambda_2] \leqslant \cdots \leqslant \mathrm{Re}[\lambda_n]$, which make the eigenvectors corresponding to the eigenvalues with the maximum negative real parts become assignable. After finishing the assignment of the $(n-m)$ eigenvalues and the corresponding eigenvectors, the following theorem shows a way to assign the other m eigenvalues.

Theorem 3 Assume $\bar{K}$ is a state feedback gain matrix, which makes the subsystem (3.2) recover the $(n-m)$ eigenvalues and corresponding eigenvectors of the closed-loop normal system. Suppose the conditions in Theorem 1 be met, if the output feedback gain matrix were chosen as : $K_{fo} = (W_m B_f)^{-1} Z_m$, then all n eigenvalues of the closed-loop fault system can recover those of the closed-loop normal system, where

$$\begin{cases} Z_m = [z_1, z_2, \cdots, z_m]^{\mathrm{T}}, \\ z_i^{\mathrm{T}} C_f[\lambda_i I - A_f]^{-1}[B_f\bar{K} + B_c] = 0, \ i = 1,2,\cdots,m, \end{cases} \tag{3.5}$$

$$\begin{cases} W_m = [w_1, \ w_2, \cdots, w_m]^{\mathrm{T}}, \\ w_i^{\mathrm{T}} = z_i^{\mathrm{T}} C_f[\lambda_i I - A_f]^{-1}, \ i = 1,2,\cdots,m. \end{cases} \tag{3.6}$$

Proof According to [11], all the n eigenvalues of the closed-loop fault system are divided into two subsets, $\Delta_f = \{\Delta_1, \Delta_2\}$, $\Delta_1 \in D_m$, $\Delta_2 \in D_{n-m}$. From (3.6), $w_i^{\mathrm{T}} = z_i^{\mathrm{T}} C_f[\lambda_i I - A_f]^{-1}$, $\lambda_i \in \Delta_1$, is equal to:

$$w_i^{\mathrm{T}}[\lambda_i I - A_f] = z_i^{\mathrm{T}} C_f, \ i = 1,2,\cdots,m. \tag{3.7}$$

Write the above formula in a compact form:

$$\Pi_m W_m - W_m A_f = Z_m C_f. \tag{3.8}$$

Construct a transformation matrix T in the following way,

$$T = \begin{bmatrix} 0 & I_{n-m} \\ M & -N \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} S_2 \\ MS_1 - NS_2 \end{bmatrix},$$

where $M \in \mathbb{R}^{m \times m}$ is non-singular, $N \in \mathbb{R}^{m \times n-m}$, $S_1 \in \mathbb{R}^{m \times n}$, $S_2 \in \mathbb{R}^{(n-m) \times n}$.

$$T^{-1} = [B_f \quad B_c]\begin{bmatrix} M^{-1}N & M^{-1} \\ I_{n-t} & 0 \end{bmatrix} =$$

$$[B_c + B_f M^{-1} N \quad B_f M^{-1}]$$

and let $M = (W_m B_f)^{-1}$, $N = W_m B_c$, then:

$$\bar{A}_{cl} = TA_{fcl}T^{-1} =$$

$$\begin{bmatrix} S_2 \\ MS_1 - NS_2 \end{bmatrix}[A_f + B_f K_{fo} C_f][B_c + B_f\bar{K} \quad B_f M^{-1}] =$$

$$\begin{bmatrix} \bar{A}_{cl11} & \bar{A}_{cl12} \\ \bar{A}_{cl21} & \bar{A}_{cl22} \end{bmatrix}, \tag{3.9}$$

where

$$\bar{A}_{cl11} = S_2 A_f B_c + S_2 A_f B_f \bar{K},$$

$$\bar{A}_{cl12} = S_2 A_f B_f M^{-1} + S_2 B_f K_{fo} C_f M^{-1} N = S_2 A_f B_f M^{-1},$$

$$\bar{A}_{cl21} = W_m[A_f + B_f K_{fo} C_f][B_c + B_f\bar{K}] = \Pi_m W_m[B_c + B_f\bar{K}] = 0,$$

$$\bar{A}_{cl22} = W_m[A_f + B_f K_{fo} C_f]B_f M^{-1} = \Pi_m W_m B_f M^{-1} = \Pi_m.$$

The above results are easily gotten by simply using the relations $K_{fo} = (W_m B_f)^{-1} Z_m$, $\bar{K} = (W_m B_f)^{-1} W_m B_c$ and $Z_m C_f = \Pi_m W_m - W_m A_f$. So that:

$$\bar{A}_{cl} = \begin{bmatrix} S_2A_fB_c + S_2A_fB_f\bar{K} & S_2A_fB_fM^{-1} \\ 0 & \Pi_m \end{bmatrix}. \tag{3.10}$$

Properly choosing $\bar{K}$ can make the $(n-m)$ eigenvalues of the subsystem $S_2A_fB_c + S_2A_fB_f\bar{K}$ to recover those of the closed-loop normal system Δ_{n-m}. So it is obvious that if $K_{fo} = (W_mB_f)^{-1}Z_m$, all the n eigenvalues of the closed-loop fault system can recover those of the closed-loop normal system. Hence the end of the proof.

From Theorem 3, the algorithm for calculating the output feedback gain matrix will be:

1) Rearrange the eigenvalues of the closed-loop normal system in the order: $\mathrm{Re}[\lambda_1] \leqslant \mathrm{Re}[\lambda_2] \leqslant \cdots \leqslant \mathrm{Re}[\lambda_n]$;

2) Construct the m corresponding eigenvector V_{fi}, $i = 1,2,\cdots,m$, in its assignable subspace in terms of (3.4);

3) Properly choose a B_c, calculate $\bar{K}$ in terms of Theorem 2;

4) Calculate z_i in terms of (3.5);

5) Calculate w_i in terms of (3.6);

6) Calculate output feedback gain matrix $K_{fo} = (W_mB_f)^{-1}Z_m$.

4 An illustrative example and simulation results comparison

The aircraft longitudinal control system is illustrated by an example to demonstrate the effectiveness of the reconfigured control system by the proposed method. The linear dynamic model of the normal system is given by (2.1). Where

$$A = \begin{bmatrix} -0.0582 & 0.0651 & 0.0 & -0.171 \\ -0.303 & -0.685 & 1.109 & 0.0 \\ -0.0715 & -0.658 & -0.947 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0 & 1.0 \\ -0.0541 & 0.0 \\ -1.11 & 0.0 \\ 0.0 & 0.0 \end{bmatrix},\ C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}.$$

The state, input and output variables as well as their meaning can be referred to [6]. For the normal system, the output feedback gain matrix K_o is synthesized as:

$$K_o = \begin{bmatrix} -0.00031 & 4.77044 & 1.70457 \\ -2.01504 & -1.13002 & 0.02904 \end{bmatrix}.$$

The corresponding eigenvalues and eigenvectors of the closed-loop normal system are:

$$\Delta = \{-0.5973,\ -1.5 + \mathrm{j}2,\ -1.5 - \mathrm{j}2,\ -2\}$$

and

$$V = \begin{bmatrix} -0.01 & 0.15+\mathrm{j}0.10 & 0.15-\mathrm{j}0.10 & 0.97 \\ 0.11 & 0.23-\mathrm{j}0.25 & 0.23+\mathrm{j}0.25 & 0.14 \\ -0.26 & 0.38+\mathrm{j}0.60 & 0.38-\mathrm{j}0.60 & 0.09 \\ 0.09 & 0.10-\mathrm{j}0.27 & 0.10+\mathrm{j}0.27 & -0.05 \end{bmatrix}.$$

The dynamical response of three outputs of closed-loop normal system is shown in Fig. 1.

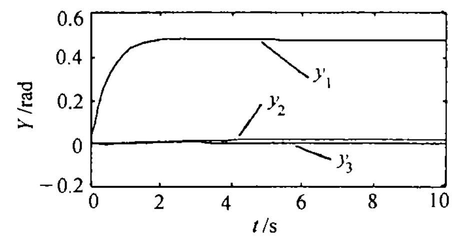

Fig. 1 Dynamical response of the normal system

Now, suppose that the system dynamics have changed into the following model due to outstanding variation in operating condition.

$$A_f = \begin{bmatrix} -0.0582 & 0.1 & 0.0 & -0.171 \\ -0.103 & -0.685 & 1.109 & 0.0 \\ -0.0715 & -0.658 & 1.98 & 0.0 \\ 0.0 & 0.0 & 1.5 & 0.0 \end{bmatrix},$$

$$B_f = \begin{bmatrix} 0.0 & 0.9 \\ -0.09 & 0.0 \\ -1.11 & 0.0 \\ 0.0 & 0.0 \end{bmatrix},\ C_f = \begin{bmatrix} 0.9 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.7 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}.$$

The fact must be noted that if the output feedback gain matrix K_o were not resynthesized and the original one is kept in use after the outstanding variation of the system dynamics, the performance of the closed-loop system would become too bad to satisfy the engineering requirement. The dynamical response of three outputs of fault system with the original controller is shown in Fig. 2. It is obvious that the performance becomes too worse to satisfy the engineering requirements. In fact, the eigenvalues of the fault system with the original feedback gain matrix become:

$$\Delta_f = \lambda(A_f + B_fK_oC_f) = \{-0.0153 + \mathrm{j}2.4684,\ -0.0153 - \mathrm{j}2.4684,\ -0.5935,\ -1.6633\}.$$

Because of the drastic decrease of the real parts of two complex eigenvalues in comparison with those of the closed-loop normal system, they can provide very little damping, so the response of the system would oscillate.

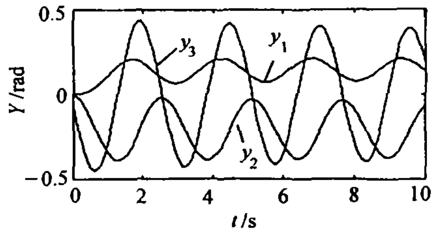

Fig. 2 Dynamical response of the fault system

It is easy to prove that $\{C_f, A_f, B_f\}$ is controllable and observable, rank $[B_f] = 2$, rank $[C_f] = 3$, and $m+r-1=2+3-1=4=n$. The conditions in Theorem 3 are satisfied. The proposed method in this paper can be used to design the reconfigurable control system. A new output feedback gain matrix K_{fo} resynthesized by the proposed method is:

$$K_{fo} = \begin{bmatrix} -1.669 & 32.9568 & 5.1557 \\ -1.3642 & 21.1606 & 21.8292 \end{bmatrix}.$$

The corresponding eigenvalues and eigenvectors of the reconfigured closed-loop fault system are:

$$\Delta_r = \{-0.5973,\ -1.4969 + j1.9958,\ -1.4969 - j1.9958,\ -2.0\}$$

and

$$V_r = \begin{bmatrix} 0.20 & -0.07+j0.98 & -0.07-j0.98 & -0.99 \\ -0.98 & 0.05+j0.07 & 0.05-j0.07 & -0.12 \\ -0.11 & 0.11-j0.06 & 0.11+j0.06 & 0.09 \\ 0.04 & -0.01+j0.64 & -0.11-j0.07 & -0.06 \end{bmatrix}.$$

Because the eigenvalues of the closed-loop normal system can be completely recovered by the reconfigured closed-loop fault system, and the corresponding eigenvectors of the latter can be made as close to those of the former as possible in least square sense, then the performance of the latter can recover that of the former to the maximum extent. The dynamical response of three outputs of fault system with resythsized controller is shown in Fig. 3.

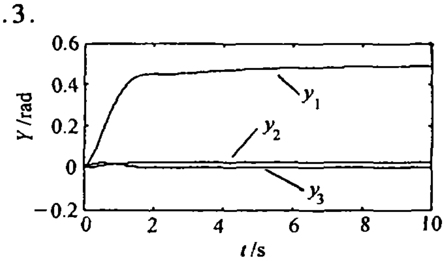

Fig. 3 Dynamical response of the reconfigured system

Comparing the results with those in[6], we can easily find that the proposed method have the advantage over the method in [6]. In [6], the reconfigured closed-loop fault system has the eigenvalues:

$$\Delta_r 6 = \{-0.5973,\ -1.5+j2.0,\ -1.5-j2.0,\ -0.64235\}.$$

It is clear that one eigenvalue λ_4 was not recovered in [6]. Its moving near to the image axle will cause the performance of the reconfigured closed-loop fault system to become worse than that of the closed-loop normal system to a certain extent.

5 Conclusions and remarks

Based on the fact that, for a LTI system, its internal behavior can be determined by its eigenstructure, the performance of the closed-loop system can be improved by modifying the eigenstructure with outputs feedback, a new method for designing reconfigurable control system by using eigenstructure assignment is proposed in this paper. Under the condition $m + r - 1 \geqslant n$, the performance of the reconfigured closed-loop fault system by resynthesizing a new output feedback gain matrix can recover those of the closed-loop normal system to a maximum extent. Because all eigenvalues of the closed-loop normal system can be recovered by the reconfigured closed-loop fault system, so the stability of the reconfigured system can be guaranteed. Another advantage of the method is that the algorithm for calculating the output feedback gain matrix is relatively simple. The illustrative example and simulation results indicate the effectiveness of the proposed method in this paper.

To reconfigure a fault system by using the method proposed in this paper, the fault system must satisfy the conditions:

1) (C_f, A_f, B_f) is controllable and observable and rank$[B_f] = m$, rank$[C_f] = r$;

2) $m + r - 1 \geqslant n$.

For some engineering system, the conditions are too restrictive to be satisfied, especially for condition 2). Even if the system is normal, it is difficult to satisfy the conditions. So the application of the proposed method is limited to a class of system in engineering. For a class of system, if the condition 2), $m + r - 1 \geqslant n$, is not satisfied, we will propose another new method to resynthesize the output feedback gain matrix to recover the performance of the closed-loop normal system and guarantee the stability of the reconfigured closed-loop system. The study is still in progress.

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