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Output Feedback Robust Stabilization of Linear System via Controller Switching *

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Abstract: The output feedback robust stabilization problem is studied in this paper. It is addressed in the case that only finite pre-designed output feedback controllers are allowed to be used. By using single and multiple Lyapunov function techniques, we present sufficient conditions for the problem to be solved. Also, the switching laws are explicitly designed. Numerical example shows the effectiveness of the proposed approach.

Key words: robust stabilization; hybrid output feedback; controller switching
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通过控制器切换实现线性系统输出反馈鲁棒镇定

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摘要: 研究了带有不确定性的线性系统的输出反馈镇定问题.假定事先给定有限个备选的输出反馈控制器,分别利用单李雅普诺夫函数和多李雅普诺夫函数方法给出了两种切换律的设计方案.仿真实例说明本文所给方法的有效性.

关键词: 鲁棒镇定; 混杂输出反馈; 控制器切换

1 Introduction

Hybrid control is not only applicable to hybrid systems, but also applicable to ordinary systems. It is commonly understood that hybrid control often provides better robustness than conventional control, and is suitable for dealing with problems with state and control constraints[1]. In some situations it is possible to design several controllers and then switch between them to provide a performance improvement over a fixed controller. In other situations the choice of linear or nonlinear controllers available to the designer is always limited and the design task is to use the available set of controllers in a stabilizable fashion.

In paper [2], Daniel Liberzon discussed the problem of stabilization with finite-state hybrid output feedback, but no uncertainty is involved. This paper studies the robust stabilization problem of uncertain linear systems by means of output feedback controller switching. Two sufficient conditions in terms of Riccati algebraic inequalities are given by using single and multiple Lyapunov function techniques respectively for the problem to be solved, and the switching laws are explicitly designed.

2 Problem statement

Consider the linear time-invariant control system

$$\begin{aligned} \dot{x} &= (A + \Delta A)x + Bu, \\ y &= Cx, \end{aligned} \tag{1}$$

where x, u, y are the state, control, and output respectively, ΔA is the uncertainty.

Now suppose that we are given a collection of gain matrices $K_1, K_2, \cdots, K_m$ of suitable dimensions. Setting $u = K_i y$ for some $i \in \{1, 2, \cdots, m\}$, we obtain the system

$$\dot{x} = (A + \Delta A + BK_iC)x. \tag{2}$$

Denoting $A + BK_iC$ by A_i for each $i \in \{1, 2, \cdots, m\}$, we are faced with the following problem: using the measurements of the output $y = Cx$, can we find a switching signal: $\sigma: [0, +\infty) \to \{1, 2, \cdots, m\}$ such that the switched system $\dot{x} = (A_\sigma + \Delta A)x$ is asymptotically stable?

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3 Main results

For simplicity, we assume that $m = 2$. Given two matrices A and B, the matrix pencil $\gamma_\alpha(A, B)$ is defined as the one-parameter family of matrices $\alpha A + (1-\alpha)B$, $\alpha \in [0,1]$. Suppose that uncertain matrix ΔA has the following structure

$$\Delta A = EFD, \tag{3}$$

where E and D are constant matrices, F satisfies the following condition

$$F^{\mathrm{T}}F \leqslant I, \tag{4}$$

where I is a unit matrix.

In this paper, we will use single and multiple Lyapunov function techniques respectively to construct a switching signal $\sigma: [0, +\infty) \to \{1,2\}$ such that the switched system

$$\dot{x} = (A_\sigma + \Delta A)x \tag{5}$$

is asymptotically stable.

Method 1 Single Lyapunov function technique.

Theorem 1 If there exist a matrix $\bar{A} \in \gamma_\alpha[A_1, A_2]$, symmetric positive definite matrices P, Q and a real number $\varepsilon > 0$, such that

$$\bar{A}^{\mathrm{T}}P + P\bar{A} + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P = -Q, \tag{6}$$

then there exists a switching signal $\sigma: [0, +\infty) \to \{1,2\}$ such that system (5) is robustly asymptotically stable.

Proof Since $\bar{A} \in \gamma_\alpha(A_1, A_2)$, there exists a number $\alpha \in (0,1)$ such that $\bar{A} = \alpha A_1 + (1-\alpha)A_2$. The equation (6) can be rewritten as

$$\alpha(A_1^{\mathrm{T}}P + PA_1) + (1-\alpha)(A_2^{\mathrm{T}}P + PA_2) + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P = -Q, \tag{7a}$$

or equivalently,

$$\alpha(A_1^{\mathrm{T}}P + PA_1 + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P) + (1-\alpha) \times (A_2^{\mathrm{T}}P + PA_2 + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P) = -Q, \tag{7b}$$

which implies

$$\alpha x^{\mathrm{T}}(A_1^{\mathrm{T}}P + PA_1 + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P)x + (1-\alpha)x^{\mathrm{T}} \times (A_2^{\mathrm{T}}P + PA_2 + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P)x = -X^{\mathrm{T}}Qx < 0 \quad (\forall x \in \mathbb{R}^n \setminus \{0\}). \tag{8}$$

Let

$$\Omega_i = \{x \mid x^{\mathrm{T}}(A_i^{\mathrm{T}}P + PA_i + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P)x < 0\}, \quad i = 1,2. \tag{9}$$

Then from (8) we know $\mathbb{R}^n \setminus \{0\} = \Omega_1 \cup \Omega_2$.

Let

$$\sigma(t) = \begin{cases} 1, & x(t) \in \Omega_1, \\ 2, & x(t) \in \Omega_2 \setminus \Omega_1, \end{cases} \tag{10}$$

and $V(x) = x^{\mathrm{T}}Px$. Differentiating of V along system (5) yields

$$\begin{aligned} \dot{V}(x) &= \\ & x^{\mathrm{T}}[(A_\sigma + \Delta A)^{\mathrm{T}}P + P(A_\sigma + \Delta A)]x = \\ & x^{\mathrm{T}}(A_\sigma^{\mathrm{T}}P + PA_\sigma + D^{\mathrm{T}}F^{\mathrm{T}}E^{\mathrm{T}}P + PEFD)x \leqslant \\ & x^{\mathrm{T}}[A_\sigma^{\mathrm{T}}P + PA_\sigma + \varepsilon D^{\mathrm{T}}F^{\mathrm{T}}FD + \varepsilon^{-1}PEE^{\mathrm{T}}P)x \leqslant \\ & x^{\mathrm{T}}[A_\sigma^{\mathrm{T}}P + PA_\sigma + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}PEE^{\mathrm{T}}P)x. \end{aligned} \tag{11}$$

So, by (9),(10) we have

$$\dot{V}(x) < 0. \tag{12}$$

Therefore V decreases along solutions of system (5), which implies asymptotic stability.

Method 2 Multiple Lyapunov function technique.

Theorem 2 If there exist two numbers β_1 and β_2, both nonnegative or both nonpositive, two positive definite matrices P_1, P_2 and a positive number $\varepsilon > 0$, such that the following inequalities (13),(14) are satisfied, then there exists a switching signal $\sigma: [0, +\infty) \to \{1,2\}$ such that system (5) is robustly asymptotically stable.

$$-P_1A - A^{\mathrm{T}}P_1 + \beta_1(P_2 - P_1) - P_1BK_1C - C^{\mathrm{T}}K_1^{\mathrm{T}}B^{\mathrm{T}}P_1 - \varepsilon D^{\mathrm{T}}D - \varepsilon^{-1}P_1EE^{\mathrm{T}}P_1 > 0, \tag{13}$$

$$-P_2A - A^{\mathrm{T}}P_2 + \beta_2(P_1 - P_2) - P_2BK_2C - C^{\mathrm{T}}K_2^{\mathrm{T}}B^{\mathrm{T}}P_2 - \varepsilon D^{\mathrm{T}}D - \varepsilon^{-1}P_2EE^{\mathrm{T}}P_2 > 0. \tag{14}$$

Proof Without loss of generality, we suppose that $\beta_1, \beta_2 \geqslant 0$. Substituting $A_1 = A + BK_1C$ and $A_2 = A + BK_2C$ into (13) and (14), results in

$$-P_1A_1 - A_1^{\mathrm{T}}P_1 + \beta_1(P_2 - P_1) - \varepsilon D^{\mathrm{T}}D - \varepsilon^{-1}P_1EE^{\mathrm{T}}P_1 > 0, \tag{15}$$

$$-P_2A_2 - A_2^{\mathrm{T}}P_2 + \beta_2(P_1 - P_2) - \varepsilon D^{\mathrm{T}}D - \varepsilon^{-1}P_2EE^{\mathrm{T}}P_2 > 0. \tag{16}$$

By virtue of the S-procedure in paper [3], we reach the following conclusion.

$$x^{\mathrm{T}}(P_1A_1 + A_1^{\mathrm{T}}P_1 + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}P_1EE^{\mathrm{T}}P_1)x < 0, \tag{17}$$

whenever $x^{\mathrm{T}}(P_1 - P_2)x \geqslant 0$ and $x \neq 0$, and

$$x^{\mathrm{T}}(P_2A_2 + A_2^{\mathrm{T}}P_2 + \varepsilon D^{\mathrm{T}}D + \varepsilon^{-1}P_2EE^{\mathrm{T}}P_2)x < 0, \tag{18}$$

whenever $x^{\mathrm{T}}(P_2 - P_1)x \geqslant 0$ and $x \neq 0$.

Let

$$\Omega_1 = \{x \mid x^{\mathrm{T}}(P_1 - P_2)x \geqslant 0 \text{ and } x \neq 0\}, \tag{19}$$

$$\Omega_2 = \{x \mid x^{\mathrm{T}}(P_2 - P_1)x \geqslant 0 \text{ and } x \neq 0\}, \tag{20}$$

and $V_1(x) = x^{\mathrm{T}}P_1x$, $V_2(x) = x^{\mathrm{T}}P_2x$.

Let $\sigma(t) = \begin{cases} 1, & x \in \Omega_1 \\ 2, & x \in \Omega_2 \setminus \Omega_1 \end{cases}$. Consider the system (5), when $x(t) \in \Omega_1$, applying (17), we have

$$\dot{V}_1(x) = x^T[(A_1 + \Delta A)^T P_1 + P_1(A_1 + \Delta A)]x \leqslant x^T[A_1^T P_1 + P_1 A_1 + \varepsilon D^T D + \varepsilon^{-1} P_1 E E^T P_1]x < 0. \tag{21}$$

When $x(t) \in \Omega_2 \setminus \Omega_1$, applying (18)

$$\dot{V}_2(x) = x^T[(A_2 + \Delta A)^T P_2 + P_2(A_2 + \Delta A)]x < x^T[A_2^T P_2 + P_2 A_2 + \varepsilon D^T D + \varepsilon^{-1} P_2 E E^T P_2]x < 0. \tag{22}$$

Note that (21) holds whenever $x^T(P_1 - P_2)x \geqslant 0$ and $x \neq 0$, and (22) holds whenever $x^T(P_2 - P_1)x \geqslant 0$, and $x \neq 0$, we know that at any switching time t_j, $V_{\sigma(t_j)}(x(t_j)) \leqslant \lim_{t \to t_j^-} V_{\sigma(t)}(x(t))$. Therefore, asymptotic stability of system (5) follows immediately from multiple Lyapunov function theory in [4].

Remark Theorems 1 and 2 show that systems which can not be stabilized by ordinary static output feedback controller could be stabilized by means of static output feedback controller switching. Equation (6) or bilinear matrix inequalities (13), (14) are easy to be solved by standard methods of matrix equation and inequality.

4 Numerical example

Consider the following system

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{23}$$

where $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $B = C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and the uncertainty

$$\Delta A = EF(t)D = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} F(t) \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}.$$

Suppose there are two controllers

$$u_1 = K_1 y = \begin{bmatrix} -8 & -1 \\ -1 & 0 \end{bmatrix} y \text{ and } u_2 = K_2 y = \begin{bmatrix} -1 & -1 \\ -1 & -7 \end{bmatrix} y.$$

Let $F(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}$. It is easy to see that neither controller u_1 nor controller u_2 can stabilize system (23) (see Fig.1 and Fig.2), but system (23) can be stabilized by using Method 1 and Method 2 (see Fig.3 and Fig.4). In fact, in (6), (13),(14), we can take $\varepsilon = \beta_1 = \beta_2 = 1, \alpha = 0.8$, and $P = \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix}$, $P_1 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$. Then by (6), the switching law is $\sigma(t) = 1$, whenever $-3x_1^2 + x_2^2 \leqslant 0$, and $\sigma(t) = 2$, whenever $-3x_1^2 + x_2^2 > 0$. By (13),(14), the switching law is $\sigma(t) = 1$, whenever $x_1^2 \geqslant x_2^2$, and $\sigma(t) = 2$, whenever $x_1^2 < x_2^2$.

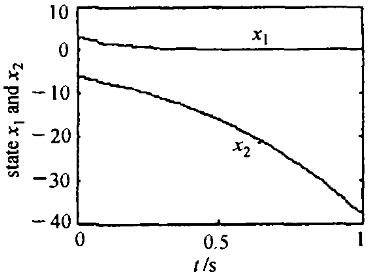

Fig. 1 The state response of system (23) under feedback u_1

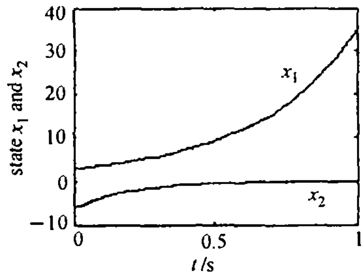

Fig. 2 The state response of system (23) under feedback u_2

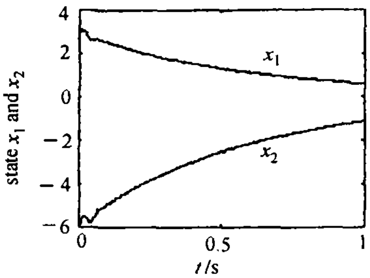

Fig. 3 The state response of systems (23) under switching feedback control law from Method 1

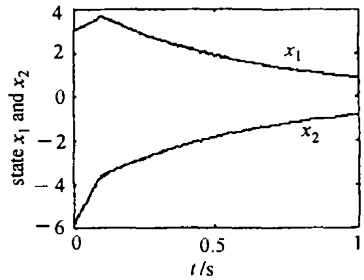

Fig. 4 The state response of systems (23) under switching feedback control law from Method 2

References

[1] Liberzon D and Morse S A. Basic problems in stability and design of switched systems [J]. Control Systems Magazine, 1999, 19(5): 59 - 70

[2] McClamroch N H, Rui C, Kolmanovsky I, et al. Hybrid closed loop systems: a nonlinear control perspective [A]. In Proc. 36th Conf. Decision and Control [C], San Diego, California, USA, 1997, 114 - 119

[3] Boyd S, Ghaoui L E, Feron E, et al. Linear matrix inequalities in system and control theory [A]. SIAM Studies in Applied Mathematics [M]. Philadelphia, PA: SIAM, 1994, Vol. 15

[4] Brannicky M S. Multiple Lyapunov functions and other analysis tools for switched and hybrid systems [J]. IEEE Trans. Automatic Control, 1998, 43(4): 475 - 482

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to Bayes optimal discriminant function [J]. IEEE Trans. Neural Networks, 1990, 1(4): 296 - 298

[3] Kanaya K and Miyaker S. Bayes statistical behavior and valid generalization of pattern classifying neural networks [J]. IEEE Trans. Neural Networks, 1991, 2(4): 471 - 475

[4] Barnard E and Casasent D. A comparison between criterion functions for linear classifiers with an application to neural nets [J]. IEEE Trans. Systems, Man, and Cybernetics, 1989, 19(6): 1030 - 1041

[5] Dimitris and Padons A. New nonleast-squares neural network learning algorithm for hypothesis testing [J]. IEEE Trans. Neural Networks, 1995, 6(3): 596 - 609

[6] Blum J R. Approximation methods which converge with probability one [J]. Annals of Mathematical Statistic, 1954, 25(4): 734 - 744

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