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Exponential Attraction Domain and Exponential Convergent Rate of Associative Memory Neural Networks with Delays *

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Abstract: Exponential stability of associative memory neural networks with delays, in which there are more equilibria corresponding to different modes, is discussed by employing the inequality techniques and the properties of nonnegative matrices. Some estimation results about the exponential attraction domain and the exponential convergent rate of equilibrium and sufficient conditions for associative memory neural networks with delays to be exponentially stable are obtained. These results can be used not only for evaluation of error-correction capability of associative memory neural networks with delays, but also for the synthesis of such networks.

Key words: delay; associative memory neural network; exponential attraction domain; exponential convergent rate

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含时延的联想记忆神经网络的指数吸引域和指数收敛速度

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摘要: 采用不等式技巧和非负矩阵性质, 给出了含时延的联想记忆神经网络平衡点的指数吸引域和指数收敛速度估计以及指数稳定的一些判断条件.

关键词: 时延; 联想记忆神经网络; 指数吸引域; 指数收敛速度

1 Introduction

Hopfield neural networks has attracted the attention of the scientists, due to their promising potential for the tasks of classification, associative memory and parallel computation, etc., and various results were reported (see [1 ~ 4]). However, in hardware implementation, time delays occur due to finite switching speeds of the amplifiers and can affect the stability of a network by creating oscillatory and unstable characteristics. It is important to investigate the dynamics of Hopfield neural networks with delays (see [5]). Some papers[5 ~ 8] discussed the stability of Hopfield neural networks with delays and established some sufficient conditions for global stability and some methods of estimating exponential convergent speed of equilibrium. But they did not consider the estimation about error-correction capability of such networks. In fact, there can be several equilibria in a Hopfield neural network with delays, these equilibria correspond to different modes. In the case of employing the networks for associative memories, it is desirable that each equilibrium be asymptotically stable and the domain of attraction of equilibrium be estimated because it is much relevant to evaluation of error-correction capability and the domain of attraction of equilibrium. The capacity of associative memory can be analyzed with the extent and the rate of error-correction. However, to the best of our knowledge, little has been reported about the results on estimation of the exponential attraction domain and exponential convergent rate and evaluations of error-correction capability for associative memory neural networks with delays. In this paper, we will discuss the asymptotic behavior of Hopfield associative memoryneuralne tworks with delays and obtain some sufficient criteria for exponential stability of such networks and some

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methods on estimation of the exponential attraction domain and exponential convergent rate of equilibrium of such networks by employing the inequality techniques and the properties of nonnegative matrices.

2 Main results

Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\Omega_j^+ = \{z_j \mid z = (z_1,\cdots,z_n)^T \in \Omega, z_j \geqslant 0\}$ where the set $\Omega \subset \mathbb{R}^n$. We define $[x]^+ = (|x_1|,\cdots,|x_n|)^T$ for $x = (x_1,\cdots,x_n)^T \in \mathbb{R}^n$. $x < y$ means $x \leqslant y$ and $x_i < y_i (i \in \Gamma)$ for $x, y \in \mathbb{R}_+^n$ where $\Gamma = \{i : x_i \neq 0, 1 \leqslant i \leqslant n\}$. The symbol $\rho(A)$ denotes the spectral radius of a square matrix A and $W_\rho(A)$ the characteristic space associated with $\rho(A)$.

In the paper, we consider the following Hopfield associative memory neural networks with delays

$$\dot{u}_i(t) = -b_i u_i(t) + \sum_{j=1}^n T_{ij} g_j(u_j(t-\tau_{ij}(t))) + c_i,$$
$$i = 1,\cdots,n,\ t \geqslant 0, \tag{1}$$

where $u_i(t)$ is the state of i-th neuron, $b_i > 0$, T_{ij} and $c_i (i,j = 1,\cdots,n)$ are constants, $0 \leqslant \tau_{ij}(t) \leqslant \tau (i = 1,\cdots,n)$ where τ is a nonnegative constant and $g_j : \mathbb{R} \to \mathbb{R} (j = 1,\cdots,n)$ is a continuous function. The initial conditions associated with system (1) are of the form

$$u_i(s) = \Psi_i(s),\ i = 1,\cdots,n,\ s \in [-\tau, 0], \tag{2}$$

where $\Psi_i : [-\tau, 0] \to \mathbb{R}$ is continuous. We assume that the solutions of the system (1) and (2) exist[9] and $u^* = (u_1^*,\cdots,u_n^*)^T$ is an equilibrium of the system (1), that is,

$$-b_i u_i^* + \sum_{j=1}^n T_{ij} g_j(u_j^*) + c_i = 0\ (i = 1,\cdots,n). \tag{3}$$

Definition A set $D \subset \mathbb{R}^n (D - \{u^*\}$ is nonempty) is said to be the exponential attraction domain of the equilibrium u^* of the system (1), if there are constants $M \geqslant 1$ and $\alpha > 0$ such that, for all $\Psi = (\Psi_1,\cdots,\Psi_n)^T : [-\tau, 0] \to D$, the solutions $u(t)$ of the system (1) satisfy

$$\| u(t) - u^* \| \leqslant M \| \Psi - u^* \|_\tau e^{-\alpha t},\ \forall t \geqslant 0, \tag{4}$$

where $\| \cdot \|$ denotes some norm and $\| \Psi - u^* \|_\tau = \max_{-\tau \leqslant s \leqslant 0} \| \Psi(s) - u^* \|$. α is called as the exponentially convergent rate.

Let $x_i(t) = u_i(t) - u_i^*$ and $f_j(x_j(t - \tau_{ij}(t))) = g_j(u_j(t - \tau_{ij}(t))) - g_j(u_j^*)$, and the system (1) can be written as

$$\dot{x}_i(t) = -b_i x_i(t) + \sum_{j=1}^n T_{ij} f_j(x_j(t - \tau_{ij}(t))),$$
$$i = 1,\cdots,n, \tag{5}$$

and the initial condition (2) as

$$x_i(s) = \Psi_i(s) - u_i^* \equiv \varphi_i(s),\ s \in [-\tau, 0], i = 1,\cdots,n. \tag{6}$$

Thus the system (5) has equilibrium 0 and the stability analysis of the trivial solution of the systems (5) and (6) is equal to the equilibrium u^* of systems (1) and (2). So, we will only discuss the stability of the trivial solution of the system (5) in the paper.

Theorem 1 Assume that the system (5) satisfies

H1) $f_j : \mathbb{R} \to \mathbb{R} (j = 1,\cdots,n)$ is continuous function;

H2)

$$|f_j(x_j(t - \tau_{ij}(t)))| \leqslant$$
$$p_j(\| x_j(t) \|_\tau) |(x_j(t - \tau_{ij}(t)))| (i,j = 1,\cdots,n),$$

where $p_j(\cdot) : \Omega_j^+ \to \mathbb{R}$ is continuous and monotonically nondecreasing in Ω_j^+ and $\| x_j(t) \|_\tau = \max_{-\tau \leqslant s \leqslant 0} |x_j(t+s)|$;

H3) There exists a positive vector $K = (k_1,\cdots,k_n)^T$ such that $\rho(P(K)) < 1$, where $P(k) = (p_{ij}(k_j))_{n \times n}$ is $n \times n$ matrix and $p_{ij}(k_j) = \dfrac{|T_{ij}| p_j(k_j)}{b_i}$, then there are vector $d = (d_1,\cdots,d_n)^T > 0$ and constant $k' = \max_{1 \leqslant i \leqslant n} \{d_i^{-1} k_i\}$ such that the set $H_{K,d} = \{x \in \Omega \mid [x]^+ < (d_1,\cdots,d_n)^T k'\}$ is an exponential attraction domain of the trivial solution of system (5) and the exponentially convergent rate α is estimated by $\max_{1 \leqslant i \leqslant n} \left\{ \rho(P(K)) \dfrac{b_i e^{\alpha\tau}}{b_i - \alpha} \right\} < 1$.

Proof Since $\rho(P(K)) < 1$, there are constants $d_i > 0 (i = 1,\cdots,n)$ such that, for $i = 1,\cdots,n$,

$$\sum_{j=1}^n d_i^{-1} d_j p_{ij}(k_j) < 1, \text{ or } \sum_{j=1}^n d_i^{-1} d_j \frac{|T_{ij}| p_j(k_j)}{b_i} < 1.$$

By continuity of $p_j(\cdot)$, there are a sufficient small positive constant δ and a positive constant $\alpha < \max_{1 \leqslant i \leqslant n} \{b_i\}$ such that

$$\max_{1 \leqslant i \leqslant n} \sum_{j=1}^n d_i^{-1} d_j \frac{|T_{ij}| p_j(k_j + \delta d_j)}{b_i - \alpha} e^{\alpha\tau} < 1. \tag{7}$$

Let $x(t) = (x_1(t),\cdots,x_n(t))^T$ be the solution of the system (5), then, by constant variable formula, we have

$$x_i(t) = \begin{cases} \varphi_i(0)e^{-b_i t} + \int_0^t \sum_{j=1}^n T_{ij} f_j(x_j(s-\tau_{ij}(s)))e^{-b_i(t-s)}ds, & t > 0, \\ \varphi_i(t), & -\tau \leqslant t \leqslant 0. \end{cases} \tag{8}$$

Setting

$$z_i(t) = \begin{cases} d_i^{-1} \mid x_i(t) \mid e^{at}, & t \geqslant 0, \\ d_i^{-1} \mid \varphi_i(t) \mid, & -\tau \leqslant t \leqslant 0, \end{cases}$$

then $\| x_i(t) \|_\tau \leqslant d_i \| z_i(t) \|_\tau (t \geqslant 0)$. Taking absolute value and multiplying both sides by $d_i^{-1}e^{at}$ in (8) and using assumption H_2), we have

$$z_i(t) \leqslant z_i(0)e^{-(b_i-a)t} + \int_0^t \sum_{j=1}^n \mid T_{ij} \mid d_i^{-1}d_j p_j(d_j \| z_j(s) \|_\tau) z_j(s - \tau_{ij}(s))e^{a\tau}e^{-(b_i-a)(t-s)}ds. \tag{9}$$

Noticing, when $\Phi(s):[-\tau,0] \to H_{K,d}$, there are positive constant $k \leqslant k'$ and $r \in \{1,\cdots,n\}$ such that $d_r^{-1} \| \varphi_r \|_\tau = k$ and $d_i^{-1} \| \varphi_i \|_\tau \leqslant k (i \neq r)$. We first show that, for $\Phi(s):[-\tau,0] \to H_{K,d}$, the above k and $\varepsilon \in (0,\delta)$,

$$z(t) < (k+\varepsilon)E \leqslant (k'+\varepsilon)E, \ \forall t \geqslant 0, \tag{10}$$

where $z(t) = (z_1(t),\cdots,z_n(t))^{\mathrm{T}}$ and $E = (1,\cdots,1)^{\mathrm{T}}$ is an n-dimensional column vector, of which each component is 1.

If (10) is not true, there must be some $t_1 > 0$ and $l \in \{1,\cdots,n\}$, such that

$$z_l(t_1) = k+\varepsilon, \text{ and } z(t) \leqslant (k+\varepsilon)E, \ \forall t \in [0,t_1]. \tag{11}$$

By(7), (9) and (11), it holds

$$\begin{aligned} k+\varepsilon = z_l(t_1) \leqslant \\ (k+\varepsilon)e^{-(b_l-a)t_1} + \int_0^{t_1} \sum_{j=1}^n \mid T_{lj} \mid d_l^{-1}d_j p_j(d_j(k+\varepsilon))(k+\varepsilon)e^{a\tau}e^{-(b_l-a)(t_1-s)}ds \leqslant \\ (k+\varepsilon)e^{-(b_l-a)t_1} + \sum_{j=1}^n \mid T_{lj} \mid d_l^{-1}d_j p_j(k_j + d_j\varepsilon)(k+\varepsilon)e^{a\tau}\frac{1-e^{-(b_l-a)t_1}}{b_l - a} < \\ (k+\varepsilon)e^{-(b_l-a)t_1} + (k+\varepsilon)(1-e^{-(b_l-a)t_1}) = k+\varepsilon, \end{aligned}$$

which is a contradiction and so (10) holds. Let $\varepsilon \to 0$, and we get

$$z(t) \leqslant kE = d_r^{-1} \| \varphi_r \|_\tau E, \ \forall t \geqslant 0.$$

Thus there is constant $M' \geqslant 1$ such that

$$\| z(t) \| \leqslant M' \| \phi_r \|_\tau, \ t \geqslant 0. \tag{12}$$

This implies that, for $\Phi:[-\tau,0] \to H_{K,d}$, there are constant $M \geqslant 1$ and $a > 0$ such that

$$\| x(t) \| \leqslant M \| \Phi \|_\tau e^{-at}, \ t \geqslant 0. \tag{13}$$

So the set $H_{K,d} = \{x \in \Omega \mid [x]^+ < (d_1,\cdots,d_n)^{\mathrm{T}}k'\}$ is an exponential attraction domain of the trivial solution of system (5).

Because δ is sufficiently small in (7), the exponentially convergent rate a is easily estimated by $\max_{1\leqslant i\leqslant n}\left\{\rho(P(K))\frac{b_i e^{a\tau}}{b_i - a}\right\} < 1$ using the spectral properties of nonnegative matrix.

From Theorem 1 and its proof, we can get the following corollaries.

Corollary 1 If the assumptions H_1) and H_2) hold and there exists a nonnegative constant k such that, for $i = 1,\cdots,n$, $\sum_{j=1}^n \frac{\mid T_{ij} \mid p_j(k_j)}{b_i} < 1$, then the set $D = \{x \in \mathbb{R}^n \mid x = (x_1,\cdots,x_n)^{\mathrm{T}} \in \Omega, \mid x_i \mid < k\}$ is an exponential attraction domain of the trivial solution of system (5) and the exponentially convergent rate a is estimated by $\max_{1\leqslant i\leqslant n}\left\{\sum_{j=1}^n \mid T_{ij} \mid P_j(k)\frac{e^{a\tau}}{b_i - a}\right\} < 1$.

Corollary 2 If the assumptions H_1) and H_2) hold and $\rho(P(0)) < 1$, then the system(5)is exponentially stable about the trivial solution.

Corollary 3 If the assumptions H_1) and H_2) hold and for any nonnegative vector K, there is $\rho(P(K)) < 1$, then the system (5) is globally exponentially stable about the trivial solution and the exponentially convergent rate a is estimated by $\sup_K \rho(P(K)) \times \max_{1\leqslant i\leqslant n}\left\{\frac{e^{a\tau}b_i}{b_i - a}\right\} < 1$.

Similarly, by properties of nonnegative matrices, we obtain still the following theorem.

Theorem 2 If the system (5)satisfies H_1) and H_2) and the set $H - \{0\}$ is nonempty where $H = \{x \in \Omega \mid [x]^+ < K, [x]^+ \in W_p(P(k)), \rho(P(K)) < 1\}$, then the set H is an exponential attraction domain of the trivial solution of system (5) and the exponentially convergent rate a is estimated by $\rho(P(K))\frac{b_i e^{a\tau}}{b_i - a} < 1$.

(Continued on page 449)

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(Continued from page 444)

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