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Robust Almost Disturbance Decoupling Problem for a Class of Uncertain Time-Delay Systems

LU Guoping[1] and ZHENG Yufan[2]

(1. Department of Automation, Nantong Institute of Technology · Jiangsu Nantong, 226007, P. R. China;
2. Institute of Systems Science, East China Normal University · Shanghai, 200062, P. R. China)

Abstract: This paper addresses the robust H_∞-almost disturbance decoupling problem with stability (RADDPS) for a class of uncertain time-delay systems. The paper presents a sufficient condition for the feasibility of RADDPS via linear matrix inequality and algebraic Riccati equation, respectively. In addition, the corresponding state feedback controllers are constructed to solve RADDPS by the proposed approach.

Key words: time-delay systems; uncertainty; linear matrix inequality; algebraic Riccati equation; robustness; H_∞ almost disturbance decoupling

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一类不确定时滞系统的鲁棒几乎干扰解耦问题

陆国平[1] 郑毓蕃[2]

(1. 南通工学院自动化系·江苏南通, 226007; 2. 华东师范大学系统科学研究所·上海, 200062)

摘要: 讨论了一类不确定时滞系统的具有稳定性的鲁棒 H_∞ 几乎干扰解耦问题(RADDPS), 利用线性矩阵不等式和代数 Riccati 方程方法分别得到了 RADDPS 可解的充分条件, 并且相应给出鲁棒静态状态反馈控制器的设计.

关键词: 时滞系统; 不确定性; 线性矩阵不等式; 代数 Riccati 方程; 鲁棒性; H_∞ 几乎干扰解耦

1 Introduction

Robust almost disturbance decoupling problem with internal stability (RADDPS) is to find a controller such that the closed-loop system is asymptotically stable, and satisfies any given L_2 gain constraint in the presence of uncertainties. Recently, [1] discusses the RADDPS for a class of smooth systems with structural uncertainty. However the RADDPS for systems with uncertainty has not been fully solved in the literature and there are few results on the ADDPS for time-delay systems. In this paper we address the RADDPS for a class of systems with time-delay and uncertainties.

Almost disturbance decoupling problem with internal stability (ADDPS)[2] has been an interesting and practical topic in control theory for many years. In [3], ADDPS is solved in terms of necessary and sufficient geometric conditions, which involve almost controlled and almost conditionally invariant subspace. Since then, various extensions to nonlinear systems have been made. These works in [4] solve ADDPS for a class of SISO nonlinear systems. In a different formulation, [5] discusses the ADDPS of linear systems subject to input saturation and input additive disturbance. It is well known in [6] that the sufficient conditions for solving robust H_∞ control problem for time-delay systems are often realized as linear matrix inequalities (LMIs) or algebraic Riccati equations (AREs). However, these LMIs or AREs are dependent on the prescribed disturbance attenuation level γ, and may not always be feasible when γ varies, especially when γ is very small. Hence, the traditional techniques developed in the literature for robust H_∞ control can hardly be applied in solving the RADDPS for time-delay systems. In this paper, a design of state feedback controller is developed to solve the RADDPS in terms of LMIs, which is independent of γ. In addition, further discussion is made for the feasibility of

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the LMIs in terms of the equivalent modified ARE.

2 Problem statement

In this paper, we consider the following class of systems with time-delays and parameter uncertainties:

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^{r}[A_i + \Delta A_i(t)]x(t-d_i) + [B + \\ \quad \Delta B(t)]u(t) + Dw(t) + [A_{r+1} + \Delta A_{r+1}(t)]\cdot \\ \quad g[t, x(t), x(t-d_1), \cdots, x(t-d_r)], \\ z(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^p$ and $z(t) \in \mathbb{R}^s$ are respectively the state, the control input, the disturbance input and the controlled output; $d_0 = 0$ and $d_i \geqslant 0$ $(i = 1, 2, \cdots, r)$ are the time-delays; A_i, B, C, D are known constant matrices with appropriate dimensions $(i = 0, 1, \cdots, r+1)$; $\Delta B(t), \Delta A_i(t)$ are appropriately dimensional matrices representing time-varying parameter uncertainties which are sometimes denoted as ΔB and ΔA_i for $i = 0, 1, \cdots, r+1$; $g[t, x(t), x(t-d_1), \cdots, x(t-d_r)]$ is known vector-valued continuous function and can also be regarded as a perturbation for systems (1). For convenience, we denote $x_{d_i} = x(t-d_i)(i = 0, 1, \cdots, r)$, x_{d_0} as x, $g = g[t, x(t), x(t-d_1), \cdots, x(t-d_r)]$.

The purpose of this paper is to discuss the robust H_∞ almost disturbance decoupling problem with internal stability (RADDPS) for systems (1). That is, this paper presents a sufficient condition under which for any given $\gamma > 0$, a linear state feedback law can always be found such that the resulting closed-loop systems for systems (1) are globally asymptotically stable with the disturbance attenuation constraint $\| z(t) \|_2 \leqslant \gamma \| w(t) \|_2$ for all allowable uncertainties.

The following assumptions will be used in the sequel.

Assumption 2.1 The uncertainties $\Delta A_i(t)$ and $\Delta B(t)$ satisfy the following conditions:

$$\begin{cases} \Delta A_i(t) = H_i F_i(t) N_i, \ \Delta B(t) = H_b F_b(t) N_b, \\ F_i(t) F_i'(t) \leqslant I, \ F_b(t) F_b'(t) \leqslant I, \end{cases} \tag{2}$$

where H_i, N_i, H_b, N_b are known constant matrices with appropriate dimensions. The matrix-valued functions $F_i(t), F_b(t)$ are time-varying Lebesgue integrable, $i = 0, 1, 2, \cdots, r+1$.

Assumption 2.2 Assume that D, B, H_b and g satisfy the following conditions:

i) There exist constant matrices D_0, H_{b_0} with appropriate dimensions such that $D = BD_0$ and $H_b = BH_{b_0}$;

ii) There exist a constant $c_i > 0$ and constant matrices $E_i (i = 0, 1, 2, \cdots, r)$ with appropriate dimensions such that

$$g'g \leqslant \sum_{i=0}^{r} c_i x_{d_i}' E_i' E_i x_{d_i}. \tag{3}$$

The following lemma will be used in the proof of the main results of this paper.

Lemma 2.1[7] Assume that A, H, N are real constant matrices with appropriate dimesions. If $F'(t)F(t) \leqslant I$, where $F(t)$ is a matrix-valued function with appropriate dimension, then the following matrix-inequalities hold.

i) $$HF(t)N + N'F'(t)H' \leqslant \varepsilon^{-1}HH' + \varepsilon N'N, \ \forall \varepsilon > 0.$$

ii) $$[A + HF(t)N][A + HF(t)N]' \leqslant A(I - \varepsilon N'N)^{-1}A' + \varepsilon^{-1}HH'$$

with $\varepsilon > 0$ satisfying $\varepsilon N'N < I$.

3 Main results

The following theorem is the main result of this paper, which shows that if two LMIs (5) and (6) are feasible, then the RADDPS is feasible and a state feedback control law be constructed by the LMIs, simultaneously.

Theorem 3.1 Under Assumptions 2.1 and 2.2, the RADDPS for systems (1) is feasible via the following form of state feedback

$$u(t) = -(1+\eta)B'X^{-1}x(t), \ \eta \geqslant 0, \tag{4}$$

if the following LMIs on X, Y and positive constants ε, $\varepsilon_i (i = 0, 1, \cdots, r+1)$ are feasible.

$$\begin{pmatrix} -2I + \varepsilon N_b' N_b & H_{b_0} \\ N_{b_0}' & -\varepsilon I \end{pmatrix} < 0, \tag{5}$$

$$\begin{pmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ \Gamma_2' & \Gamma_4 & 0 \\ \Gamma_3' & 0 & \Gamma_5 \end{pmatrix} < 0, \tag{6}$$

where

$\Gamma_1 := A_0 X + X A_0' - 2BB' + \varepsilon B N_b' N_b B' + \varepsilon_0 H_0 H_0'$,

$\Gamma_2 := X(I \ I \ N_0')$, $\Gamma_3 := (A_1 \cdots A_{r+1} \ H_1 \cdots H_{r+1} \ H_b)$,

$\Gamma_4 := \text{blockdiag}(-Y, -(\sum_{i=1}^{r} c_i E_i' E_i + rI)^{-1}, -\varepsilon_0 I)$,

$\Gamma_5 := \text{blockdiag}(-I + \varepsilon_1 N_1' N_1, \cdots, -I + \varepsilon_{r+1} N_{r+1}' N_{r+1}, -\varepsilon_1 I, \cdots, -\varepsilon_{r+1} I, -\varepsilon I)$.

Proof Suppose that X, Y are the solutions of LMIs (5) and (6). For convenience, let $P = X^{-1}, Q = Y^{-1}$. Choose a Lyapunov functional candidate

$$V = x'(T)Px(t) + \sum_{i=1}^{r} \int_{t-d_i}^{t} x'(s)(I + c_i E_i' E_i)x(s)\mathrm{d}s. \quad (7)$$

Thus, the derivative of V along the closed-loop systems (1) and (4) is

$$\dot{V} = 2x'P\sum_{i=0}^{r}(A_i + \Delta A_i)x_{d_i} + 2x'P(A_{r+1} + \Delta A_{r+1})g + 2x'P(B + \Delta B)u + 2x'PDw + x'\sum_{i=1}^{r}(I + c_i E_i' E_i)x - \sum_{i=1}^{r} x_{d_i}'(I + c_i E_i' E_i)x_{d_i}. \quad (8)$$

It follows from LMI (5) and the Schur Complement Lemma[8] that there exists an $\varepsilon > 0$ such that $R_\varepsilon := 2I - \varepsilon^{-1} H_{b_0} H_{b_0}' - \varepsilon N_b' N_b > 0$. It follows from Assumption 2.2 and Lemma 2.1 that for $\eta > 0$, we have

$$2x'PDw = 2x'PBR_\varepsilon^{1/2} R_\varepsilon^{-1/2} D_0 w \leqslant \eta x'PBR_\varepsilon B'Px + \eta^{-1} w'D_0' R_\varepsilon^{-1} D_0 w. \quad (9)$$

Using (3), (4) and (9), we have

$$\dot{V} \leqslant 2x'P\sum_{i=0}^{r}(A_i + \Delta A_i)x_{d_i} + 2x'P(A_{r+1} + \Delta A_{r+1})g - 2(1 + \eta)x'P(B + \Delta B)B'Px + x'\sum_{i=1}^{r}(I + c_i E_i' E_i)x - \sum_{i=1}^{r} x_{d_i}'(I + c_i E_i' E_i)x_{d_i} + \sum_{i=1}^{r} c_i x_{d_i}' E_i' E_i x_{d_i} - g'g + \eta x'PBR_\varepsilon B'Px + \eta^{-1} w'D_0' R_\varepsilon^{-1} D_0 w.$$

Let

$$\Omega := \begin{pmatrix} M_0 & M_1 \\ M_1' & -I \end{pmatrix}, \quad (10)$$

where

$$\begin{cases} M_0 := P(A_0 + \Delta A_0) + (A_0 + \Delta A_0)'P - (1+\eta)P[(B+\Delta B)B' + B(B+\Delta B)']P + \sum_{i=1}^{r}(I + c_i E_i' E_i) + Q + \eta PBR_\varepsilon B'P, \\ M_1 := (P(A_1 + \Delta A_1) \ \cdots \ P(A_{r+1} + \Delta A_{r+1})), \end{cases} \quad (11)$$

then

$$\dot{V} \leqslant -x'Qx + \eta^{-1} w'D_0' R_\varepsilon^{-1} D_0 w + Z\Omega Z', \quad (12)$$

where $Z := (x' \ x_{d_1}' \ x_{d_2}' \ \cdots \ x_{d_r}' \ g')$.

Next we shall show that

$$\Omega \leqslant 0. \quad (13)$$

For any $\varepsilon_0, \varepsilon > 0$, by Lemma 2.1, we have

$$\begin{cases} P\Delta A_0 + \Delta A_0' P \leqslant \varepsilon_0 P H_0 H_0' P + \varepsilon_0^{-1} N_0' N_0, \\ -(1 + \eta)P[(B + \Delta B)B' + B(B + \Delta B)']P = -2(1 + \eta)PBB'P - (1 + \eta)PB(H_{b_0} F_b N_b + N_b' F_b' H_{b_0}')B'P \leqslant -PBR_\varepsilon B'P - \eta PBR_\varepsilon B'P. \end{cases} \quad (14)$$

By (11) and (14), we obtain

$$M_0 \leqslant PA_0 + A_0'P - P(BR_\varepsilon B' - \varepsilon_0 H_0 H_0')P + \varepsilon_0^{-1} N_0' N_0 + \sum_{i=1}^{r} c_i E_i' E_i + rI + Q. \quad (15)$$

In addition, it follows from the Schur Complement Lemma[8] that LMI (6) implies that there exist constants $\varepsilon_i > 0$ such that $I - \varepsilon_i N_i' N_i > 0$, and

$$A_0 X + XA_0' - BR_\varepsilon B' + \sum_{i=1}^{r+1} T_i + \varepsilon_0 H_0 H_0' + X(\sum_{i=1}^{r} c_i E_i' E_i + \varepsilon_0^{-1} N_0' N_0 + rI)X + XY^{-1}X < 0, \quad (16)$$

where $T_i := A_i(I - \varepsilon_i N_i' N_i)^{-1} A_i' + \varepsilon_i^{-1} H_i H_i', \ i = 0, 1, \cdots, r + 1$.

Then it follows from (15) and (16) that we have

$$M_0 < -P\sum_{i=1}^{r+1} T_i P. \quad (17)$$

Furthermore, it follows from Lemma 2.1 that we have

$$P(A_i + \Delta A_i)(A_i + \Delta A_i)'P \leqslant PT_i P, \ i = 1, 2, \cdots, r+1. \quad (18)$$

Then (17) and (18) imply $M_0 + P\sum_{i=1}^{r+1}(A_i + \Delta A_i)(A_i + \Delta A_i)'P < 0$. By means of the Schur Complement Lemma[8], (13) holds.

Therefore (12) and (13) imply

$$\dot{V} \leqslant -x'Qx + \eta^{-1} w'D_0' R_\varepsilon^{-1} D_0 w. \quad (19)$$

If $w = 0$, then $\dot{V} \leqslant -x'Qx$, which implies that the resulting closed-loop system is globally asymptotically stable.

Next we shall show $\| z(t) \|_2 \leqslant \gamma \| w(t) \|_2$. Let $x(t)$ be the trajectories of the closed-loop system of (1) and (4) with initial condition $x(t) = 0 (t \leqslant 0)$. Integrating both sides of (19) form 0 to t, we have that $\lambda_{\min}(Q)\int_0^t \| x \|^2 ds \leqslant \eta^{-1} \| D_0' R_\epsilon^{-1} D_0 \| \int_0^t \| w \|^2 ds$.

That is, $\| x \|_2 \leqslant \sqrt{\dfrac{\| D_0' R_\epsilon^{-1} D_0 \|}{\eta \lambda_{\min}(Q)}} \| w \|_2$. For any $\gamma > 0$, choosing $\eta^*(\gamma) = \dfrac{\| D_0' R_\epsilon^{-1} D_0 \| \, \| C \|^2}{\gamma^2 \lambda_{\min}(Q)}$. If $\eta \geqslant \eta^*(\gamma)$, then we have $\| z \|_2 \leqslant \gamma \| w \|_2$ which completes the proof.

Remark 3.1 It can be seen from Theorem 3.1 that LMIs (5) and (6) are independent of the disturbance attenuation level γ. For any given $\gamma > 0$, the control law (4) can be easily constructed by means of MATLAB LMI Toolbox[9].

Theorem 3.1 implies that if LMIs (5) and (6) are feasible, then the RADDPS is feasible. It is usually hard to present a sufficient condition to guarantee the feasibility of LMI directly. In order to present sufficient conditions to guarantee the feasiblility of LMIs (5) and (6), from Theorem 3.1, we obtain similar result based on ARE as follows:

Theorem 3.2 Under Assumptions 2.1 and 2.2, the RADDPS for systems (1) can be solved by the feedback law $u(t) = -(1+\eta)B'Px(t)(\eta \geqslant 0)$ if there exists a positive definite solution P for the following modified ARE

$$PA_0 + A_0'P - P(BR_\epsilon B' - \sum_{i=1}^{r+1} T_i - \epsilon_0 H_0 H_0')P + \sum_{i=1}^{r} c_i E_i' E_i + \epsilon_0^{-1} N_0' N_0 + rI + Q = 0, \tag{20}$$

where $T_i := A_i(I - \epsilon_i N_i' N_i)^{-1} A_i' + \epsilon_i^{-1} H_i H_i'$, Q, ϵ and ϵ_i are properly chosen constant positive definite matrix and positive constants with $R_\epsilon = 2I - \epsilon^{-1} H_{b_0} H_{b_0}' - \epsilon N_b' N_b > 0$ and $I - \epsilon_i N_i' N_i > 0$, $i = 0,1,2,\cdots,r+1$.

Proof The proof is similar to that of Theorem 3.1, therefore omitted.

The following remark presents sufficient conditions under which the modified ARE (20) is feasible.

Remark 3.2 Suppose that (A_0, B) is stabilizable and there exist constant matrices $H_0 = BH_{00}$, A_{0i}, H_{0i}, and $\epsilon_i > 0$ such that $H_i = BH_{0i}$, $A_i = BA_{0i}(i = 1,2,\cdots,r+1)$ and

$$\epsilon_i N_i' N_i < I,$$

$$R_\epsilon - \sum_{i=1}^{r+1} [A_{0i}(I - \epsilon_i N_i' N_i)^{-1} A_{0i}' + \epsilon_i^{-1} H_{0i} H_{0i}'] > 0.$$

Let $R(\epsilon_0) := R_\epsilon - \sum_{i=1}^{r+1} [A_{0i}(I - \epsilon_i N_i' N_i)^{-1} A_{0i}' + \epsilon_i^{-1} H_{0i} H_{0i}'] - \epsilon_0 H_{00} H_{00}'$, noticing that the matrix-valued function $R(\epsilon_0)$ is continuous function of ϵ_0, then there exists a sufficiently small $\epsilon_0 > 0$ such that $R(\epsilon_0) > 0$. It further implies that the modified ARE (20) has a unique positive definite matrix solution P, see [10]. It follows from Theorem 3.2 that the RADDPS for the system (1) is feasible.

Remark 3.3 From the proof of Theorem 3.1, it is easy to see that the modified ARE (20) and LMIs (5) and (6) are equivalent. That is, both Theorems 3.1 and 3.2 give an equivalent relationship to the RADDPS. It can be seen that the modified ARE (20) is always feasible under some conditions according to Remark 3.2, then the feasibility of LMIs (5) and (6) can be guaranteed in this case. One of the advantages of using LMIs in Theorem 3.1 is that the solutions are obtained without tuning any parameters, while some parameters are required to be tuned to search for the solutions in the modified ARE (20).

4 Conclusions

This paper presents a new way to implement a robust controller for RADDPS for a class of time-delay systems with uncertainties. A state feedback controller has been constructed by means of LMIs. The necessary conditions for the RADDPS are under investigation, and further analyses are carried out to construct dynamic output feedback controller to solve the RADDPS for a wider class of systems with uncertainties.

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本文作者简介

陆国平 1965年生.1998年6月在华东师范大学系统科学研究所获博士学位,现为南通工学院应用数学系副教授.研究领域有非线性控制,时滞控制和智能控制. Email:gplu@pub.nt.jsinfo.net

郑毓蕃 1941年生.华东师范大学系统科学研究所教授,博士生导师,澳大利亚墨尔本大学电气工程系客座教授.目前研究领域为非线性控制.

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本文作者简介

李树勇 1964年生.教授,博士.现在西安交通大学做博士后研究工作.主要研究方向:时滞微分方程,偏泛函微分方程和神经网络理论. Email:shuyongli@263.net

徐道义 1948年生.教授,博士生导师.主要研究方向:稳定性理论,大系统理论,时滞微分方程,离散系统和区间动力系统.