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Optimal Fuzzy PID Controller with Incomplete Derivation and Its Simulation Research on Application of Intelligent Artificial Legs *

TAN Guanzheng[1], XIAO Hongfeng[1] and WANG Yuechao[2]
(1. Institute of Robotics, Central South University · Changsha, 410083, P. R. China;
2. Robotics Laboratory, Academia Sinica · Shengyang, 110015, P. R. China)

Abstract: A new kind of optimal fuzzy PID controller is proposed, which is composed of an on-line fuzzy inference mechanism and a conventional PID controller with incomplete derivation. In the fuzzy inference mechanism, three adjustable factors x_p, x_i, and x_d are introduced. Their function is to further modify and optimize the result of fuzzy inference so that the controller has the optimal control effect on a given object. The optimal values of these factors are determined based on the ITAE criterion and the Nelder and Mead's flexible polyhedron search algorithm. The PID controller has been used to control a D.C. motor of the intelligent artificial leg designed by the authors. The result of simulation indicates that the controller is very effective and can be used to control different kinds of objects and processes.

Key words: incomplete derivation; fuzzy PID controller; adjustable factors; flexible polyhedron search algorithm; intelligent artificial leg

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具有不完全微分的最优模糊 PID 控制器及其在智能人工腿中应用的仿真研究

谭冠政[1] 肖宏峰[1] 王越超[2]
(1. 中南大学机器人研究所·长沙,410083; 2. 中国科学院机器人学开放研究实验室·沈阳,110015)

摘要: 提出了一种新的最优模糊 PID 控制器,它由两部分组成,即在线模糊推理机构和带有不完全微分的常规 PID 控制器.在模糊推理机构中,引入了三个可调节因子 x_p, x_i 和 x_d,其作用是进一步修改和优化模糊推理的结果,以使控制器对一个给定对象具有最优的控制效果.可调节因子的最优值采用 ITAE 准则及 Nelder 和 Mead 提出的柔性多面体最优搜索算法加以确定.这种 PID 控制器被用来控制由作者设计的智能人工腿中的一个直流电机.仿真结果表明该控制器的设计是非常有效的,它可被用于控制各种不同的对象和过程.

关键词: 不完全微分; 模糊 PID 控制器; 可调节因子; 柔性多面体搜索算法; 智能人工腿

1 Introduction

Proportional-integral-derivative (PID) controllers are most frequently adopted in practical cases due to their simple structure and algorithm. They can provide acceptable performance for a large range of processes. Fuzzy inference has been recognized to be very appropriate in implementing the operator experience in designing control systems. It has been widely adopted in the last decade[1,2], and some applications to PID controllers have been devised. Tzafestas and Papanikolopoulos proposed an approach in which the performance of PID controllers is enhanced based on a fuzzy matrix that contains the experience of a human-controller[3]. S.Z. He devised a method that contains a Ziegler-Nichols-like formula with a single parameter. This parameter can be self-tuned by means of an on-line fuzzy inference mechanism[4]. Zhao et al developed a fuzzy gain scheduling scheme where PID parameters are determined based on fuzzy rules, depending on the values of error signal and its time derivative[5]. Visioli proposed a method on the basis of the fuzzification of the set-point weighting in which to achieve both the aims of reducing the overshoot

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and decreasing the rise time, a fuzzy module is used to modify the weight depending on the current output error and its time derivative[6].

In this paper, a new kind of optimal fuzzy PID controller is proposed, which contains two parts. One is an on-line fuzzy inference mechanism, and the other is a conventional PID controller with incomplete derivation. In the fuzzy inference mechanism, three adjustable factors x_p, x_i, and x_d are introduced. Their function is to further modify and optimize the result of fuzzy inference so as to make the controller have the optimal control effect on a given object. This kind of optimal fuzzy PID controller has been used to control a D.C. motor of the intelligent artificial leg designed by the authors. Section 5 gives the result of computer simulation for the motor system.

2 PID formula with incomplete derivation

Introducing derivation control can improve the transient performance of a system but the system output is very sensitive to disturbances in this case. To overcome the drawback, a first-order inertia term can be added to the PID formula, which is a low-pass filter and its transfer function is $G_f(s) = 1/(1 + T_f s)$. The derivation control with the first-order inertia term is called incomplete derivation control. The transfer function of PID controllers with incomplete derivation is expressed as:

$$U(s) = (K_p + \frac{K_p}{T_i s} + \frac{K_p T_d s}{1 + T_f s})E(s). \quad (1)$$

In discrete-time domain, the above formula can be expressed as:

$$u(k) = K_p e(k) + K_i \sum_{j=0}^{k} e(j) + u_d(k), \quad (2)$$

$$u_d(k) = K_d(1-\lambda)[e(k)-e(k-1)]+\lambda u_d(k-1), \quad (3)$$

where $K_i = K_p T/T_i$, $K_d = K_p T_d/T$, and $\lambda = T_f/(T_f + T) < 1$. T is the sampling period.

3 Optimal fuzzy PID controller

3.1 Structure of the fuzzy PID controller

The structure of the optimal fuzzy PID controller is shown in Fig. 1, in which the on-line fuzzy inference mechanism is used to adjust PID parameters K_p, K_i, and K_d in real time according to the system error $e(t)$ and its time derivative $\dot{e}(t)$.

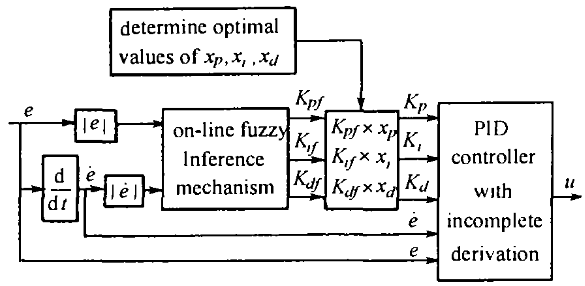

Fig. 1 Structure of the optimal fuzzy PID controller

The inference mechanism implements the following mapping:

$$\underset{\sim}{E} \times \underset{\sim}{EC} \rightarrow \underset{\sim}{K_{pf}} \times \underset{\sim}{K_{if}} \times \underset{\sim}{K_{df}}, \quad (4)$$

where $\underset{\sim}{E}$ and $\underset{\sim}{EC}$ are the fuzzy quantities of $e(t)$ and $\dot{e}(t)$ respectively. Another part of the controller is a PID controller with incomplete derivation. Its output can be computed by formulas (2) and (3). In Fig. 1, three adjustable factors x_p, x_i, and x_d are introduced. For a given system, their optimal values can be determined by means of ITAE criterion and the Nelder and Mead's flexible polyhedron search algorithm[7], which will be introduced in Section 4. After these factors are determined, the PID parameters K_p, K_i, and K_d can be obtained by the following formulas:

$$\begin{cases} K_p = K_{pf} \times x_p, \\ K_i = K_{if} \times x_i, \\ K_d = K_{df} \times x_d, \end{cases} \quad (5)$$

where

$$\begin{cases} 0 \leqslant x_p \leqslant 1, \\ 0 \leqslant x_i \leqslant 1, \\ 0 \leqslant x_d \leqslant 1. \end{cases} \quad (6)$$

From the formulas (5), one finds the effect adjusting the values of factors x_p, x_i, and x_d is equivalent to modifying the fuzzy inference rules.

3.2 Idea of adjustment of PID parameters

1) K_p controller: In the initial stage of a regulation process, K_p should be assigned to a suitably smaller value to reduce the impact on the system which results from the initial variations of physical quantities of the system. In the middle stage, K_p should be increased to raise the response speed and dynamic precision of the system. In the final stage, K_p should be decreased to reduce the overshoot and raise the system's static stability.

2) K_i controller: In the initial stage of a regulation process, K_i should be assigned to a very small value to prevent the integral saturation phenomenon which results from the saturation non-linearity characteristic and other affections of the system and may cause the increase of system overshoot. In the middle stage, K_i should be assigned to a moderate value and not be too large to avoid bringing about any effect on the dynamic stability of the system. In the final stage, K_i should be increased to reduce the steady-state error and thus raise the control precision of the system.

3) K_d controller: In the initial stage of a regulation process, K_d should be assigned to a larger value to reduce overshoot and overcome oscillations. In the middle stage, K_d should be assigned to a smaller value than that of the initial stage and keep invariable because the regulating characteristics are sensitive to the variation of K_d in this stage. In the final stage, K_d should be reduced to enhance the system capability of rejecting disturbances and compensate for the increase of regulating time, which results from the larger K_d value in the initial stage of the regulation process.

3.3 Rules for fuzzy inference mechanism

The fuzzy sets of $\underset{\sim}{E}$ and $\underset{\sim}{EC}$ are all defined as {Z, S, M, B}. Also the fuzzy sets of PID parameters $\underset{\sim}{K_{pf}}$, $\underset{\sim}{K_{if}}$ and $\underset{\sim}{K_{df}}$ are all defined as {Z, S, M, B}. According to the above mentioned adjustment idea, sixteen fuzzy rules can be obtained as shown in Table 1.

Table 1 Fuzzy rule table of $\underset{\sim}{K_{pf}}/\underset{\sim}{K_{if}}/\underset{\sim}{K_{df}}$

$\underset{\sim}{EC}$	$\underset{\sim}{E}$			
	B	M	S	Z
B	M/Z/S	S/S/M	M/M/Z	M/B/Z
M	B/Z/M	M/S/M	B/B/S	B/B/Z
S	B/Z/B	M/Z/B	B/B/S	B/B/Z
Z	B/Z/B	M/Z/B	B/B/S	Z/B/Z

3.4 Control inquiry table

The Mamdani inference method is used as the fuzzy inference mode. The method to achieve the greatest degree of membership is adopted for defuzzification. By these methods, the control inquiry table for PID parameters $\underset{\sim}{K_{pf}}$, $\underset{\sim}{K_{if}}$ and $\underset{\sim}{K_{df}}$ is obtained as shown in Table 2.

Table 2 Control inquiry table for $\underset{\sim}{K_{pf}}/\underset{\sim}{K_{if}}/\underset{\sim}{K_{df}}$

$\underset{\sim}{EC}$	$\underset{\sim}{E}$						
	0	+1	+2	+3	+4	+5	+6
0	6/6/0	6/6/1	6/6/1	4/0/6	4/0/6	6/0/6	6/0/6
+1	6/6/0	6/6/1	6/6/1	4/0/6	4/0/6	6/0/6	6/0/6
+2	6/6/0	6/6/1	6/6/1	4/0/6	4/0/6	6/0/6	6/0/6
+3	6/6/0	6/6/1	6/6/1	4/0/4	4/0/4	6/0/4	6/0/4
+4	4/6/0	4/6/1	4/6/1	1/1/4	1/1/4	4/0/4	4/0/4
+5	4/6/0	4/6/0	4/6/0	1/1/4	1/1/4	4/0/1	4/0/1
+6	4/6/0	4/6/0	4/6/0	1/1/4	1/1/4	4/0/1	4/0/1

4 Determination of the optimal values of adjustable factors

The Nelder and Mead's flexible polyhedron search algorithm is used to determine the optimal adjustable factors x_p^*, x_i^*, and x_d^* for a given system. First, ITAE criterion is taken as the objective function $F(X)$, which is expressed as follows:

$$F(X) = \text{ITAE} = \int_0^\infty t \mid e(t) \mid \mathrm{d}t, \qquad (7)$$

where $X = [x_p, x_i, x_d]$ is a three dimensional vector and the values of x_p, x_i and x_d are hidden in $e(t)$.

To obtain the optimal vector $X^* = [x_p^*, x_i^*, x_d^*]$, first an initial flexible polyhedron with four vertices X_1^0, X_2^0, X_3^0, and X_4^0 must be formed. Among the four vertices, X_1^0 can be selected randomly but each element of which must satisfy the constraints given in (6). X_2^0, X_3^0, and X_0^4 can be determined according to the construction principle of regular polyhedron[7].

Definition A vertex is called feasible point if it satisfies constraints (6). Otherwise, the vertex is called infeasible point.

In order to ensure that X_i^0, $i = 1,2,3,4$, satisfies constraints (6), a program has been written which can convert an infeasible point into a feasible one. The program is called "the feasible point converter (FPC)". The converting rules of FPC are defined as follows:

1) If x_p, x_i or $x_d > 1$, then set x_p, x_i or $x_d = 1$.

2) If x_p, x_i or $x_d < 0$, then set x_p, x_i or $x_d = 0$.

Among the four vertices of the initial polyhedron, let X_g and X_s have the greatest and smallest values of objective function $F(X)$ respectively, and let X_l have the second greatest value of $F(X)$. Let X_c be the centroid of all the vertices excluding X_g.

The basic formulas of flexible polyhedron search algorithm are summarized as follows[7]:

$$X_c = (\sum_{i=1}^{4} X_i^0 - X_g)/3, \quad (8)$$

$$X_r = X_c + \alpha(X_c - X_g), \quad (9)$$

$$X_e = X_c + \gamma(X_r - X_c), \quad (10)$$

$$X_p = X_c + \beta(X_g - X_c), \quad (11)$$

$$X_i \leftarrow X_s + 0.5(X_i - X_s),\ i = 1,2,3,4, \quad (12)$$

where X_r, X_e, and X_p are reflection point, expansion point, and contraction point, respectively; α, γ and β are reflection, expansion, and contraction coefficient, respectively. Formula (12) is utilized to reduce the size of the flexible polyhedron.

According to the idea of Nelder and Mead, after the initial flexible polyhedron is formed, the algorithm should enter into an iterative process, in which the size of the initial polyhedron will be reduced step by step. When $\sum_{i=1}^{4} \| X_i^k - X_s^k \| < \varepsilon_1$ (ε_1 is a preselected small number and k denotes stage number), the polyhedron will become very small and the solution is obtained at a local optimum point. To search further the global optimum solution, X_s^k will be used as the first vertex to set up another new initial polyhedron, and then another cycle of iterative process that reduces the size of the new initial polyhedron will be started again. In the following algorithm, when the iterative process reaches Step 12, it completes one iterative cycle; when the process reaches Step 11, it completes one iterative stage of that particular cycle. kk is the cycle number that starts from zero, and k is the stage number that starts from zero for each cycle. If the difference of the solutions from two consecutive cycles is less than a preselected small number ε_2, the optimal solution X^* is obtained and the iterative process will stop. The optimization algorithm for X^* is given as follows.

Optimization algorithm for X^*

Step 1 Set $kk = 0, k = 0$, and $X^* = [0, 0, 0]$. Select α, γ and β in (9), (10), and (11). Also select ε_1 and ε_2.

Step 2 Select X_1^0 randomly. Convert X_1^0 to become feasible by FPC if it is originally infeasible.

Step 3 Determine $X_i^0, i = 2,3,4$, according to the construction principle of regular polyhedron[7]. Convert $X_i^0, i = 2,3,4$, to become feasible by FPC if it is originally infeasible.

Step 4 From the four vertices X_1^k, X_2^k, X_3^k, and X_4^k, determine X_g^k, X_l^k, and X_s^k.

Step 5 Compute the centroid X_c^k by (8).

Step 6 Reflect to obtain X_r^k by (9). Convert X_r^k to become feasible by FPC if X_r^k is originally infeasible.

Step 7 If $F(X_r^k) \leqslant F(X_s^k)$, then do a) and b) specified below; else go to Step 8.

a) Expand to obtain X_e^k by (10). Convert X_e^k to become feasible by FPC if X_e^k is originally infeasible.

b) Set

$$X_g^k = \begin{cases} X_r^k, & \text{if } F(X_r^k) \leqslant F(X_e^k), \\ X_e^k, & \text{if } F(X_r^k) > F(X_e^k), \end{cases}$$

then set $k = k + 1$, and go to Step 10.

Step 8 If $F(X_s^k) < F(X_r^k) < F(X_l^k)$, set $X_g^k = X_r^k$, then set $k = k + 1$, and go to Step 11. Otherwise go to Step 9.

Step 9 If $F(X_l^k) \leqslant F(X_r^k) < F(X_g^k)$, contract to obtain X_p^k by (11). Convert X_p^k to become feasible by FPC if X_p^k is originally infeasible. Otherwise go to Step 10. If $F(X_p^k) < F(X_g^k)$, set $X_g^k = X_p^k$, then set $k = k + 1$, and go to Step 11. Otherwise go to Step 10.

Step 10 Reduce the size of the polyhedron by (12), then find the new best vertex X_s^k with the smallest value of the objective function $F(X)$, and set $k = k + 1$. Continue the next step.

Step 11 If $\sum_{i=1}^{4} \| X_i^k - X_s^k \| < \varepsilon_1$, then output X_s^k and go to Step 12; else go to Step 4.

Step 12 If $\| X_s^k - X^* \| < \varepsilon_2$, then output X_s^k as $X^* = [x_p^*, x_i^*, x_d^*]$ and output $F(X^*)$; else set $X_1^0 = X_s^k$ and $X^* = X_s^k$. Then set $kk = kk + 1$, $k = 0$, and go to Step 3.

The above optimization algorithm for X^* can be realized by the program written in Matlab. Fig.2 shows the simulation structure of a system with the optimal fuzzy PID controller.

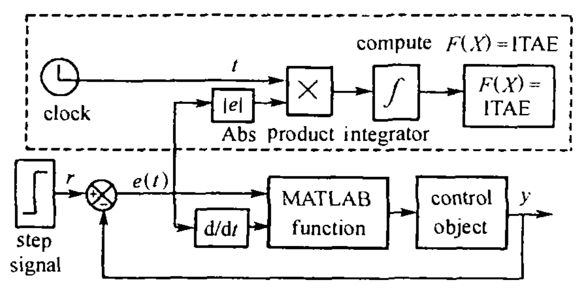

Fig. 2 Simulation structure diagram of a system with the optimal fuzzy PID controller

5 Computer simulation example

The above optimal fuzzy PID controller has been used in the intelligent artificial leg designed by the authors. This artificial leg consists of a knee joint, a shank and a foot. In the knee joint, there are a walking speed sensor, an air cylinder with a D.C. motor at its tail, a microprocessor and batteries. The walking speed sensor is used to measure the walking speed of the leg in real time. The air cylinder is the actuating mechanism used to control the bend and stretch movements of the knee joint. The motor is used to control the opening of a throttle valve in the cylinder. Regulating the opening can change the bend and stretch speeds of the knee joint and thereby change the walking speed of the leg. The microprocessor controls the motor's motion according to the measurement value of the walking speed. The power of the control system is supplied with small-size lithium batteries[8].

The optimal fuzzy PID controller has been used to control the motor. The transfer function of the motor is $G(s) = 6068/[s(s^2 + 110s + 6068)]$. For the given system, the relevant parameters are selected as $\lambda = 0.9$, $\alpha = 1.0, \gamma = 2.0, \beta = 0.5, \varepsilon_1 = 0.01$ and $\varepsilon_2 = 0.01$. The first vertex of initial flexible polyhedron is selected as $X_1^0 = [0.6,\ 0.6,\ 0.6]$. The unit step responses of this system are shown in Fig.3, in which one is the response obtained by means of the classical Ziegler-Nichols method, and another is the response obtained by means of the optimal fuzzy PID controller proposed in this paper.

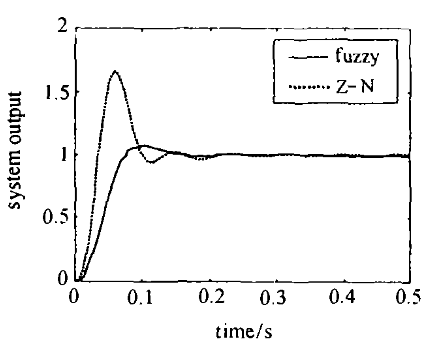

Fig. 3 Unit step responses of the given system

The result of the solution for the optimal adjustable factors X^* is shown in Table 3. From the table, the value of the object function $F(X^*)$ is reduced in each cycle. For this given system, the optimal adjustable factors are $x_p^* = 0.3073, x_i^* = 0.5199$, and $x_d^* = 0.2217$.

Table 3 Result solving for X^*

factors		cycle 1	…	cycle 21	cycle 22	cycle 23
X^*	x_p^*	0.2975	…	0.3073	0.3080	0.3073
	x_i^*	0.6031	…	0.5020	0.5166	0.5199
	x_d^*	0.0405	…	0.2217	0.2212	0.2217
$F(X^*)$		0.6721	…	0.0550	0.0547	0.0547

6 Conclusions

From the result of computer simulation example, it is concluded that the optimal fuzzy PID controller proposed in the paper is correct and very effective. It optimizes the performances of conventional PID controllers and can be widely used to control different kinds of objects and processes.

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本文作者简介

谭冠政 1962年生.分别于1988年和1992年在国防科技大学和南京航空航天大学获工学硕士和工学博士学位.现为中南大学机器人研究所所长,信息科学与工程学院教授,模式识别与智能系统学科博士生导师.主要研究方向:人工智能与机器人,仿生机器人与智能仿生系统,先进控制理论与先进算法. Email: tgz@mail.csu.edu.cn

肖宏峰 1971年生.工学硕士.主要研究方向:仿生机器人与智能仿生系统,先进控制理论与先进算法.

王越超 工学博士.现为中国科学院机器人学开放研究实验室主任,教授,国家"863计划"智能机器人主题专家组专家.主要研究领域:智能机器人系统及控制,先进控制理论与先进算法.