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Application and Study of Optimal Control Theory in Control System of Active Magnetic Bearings

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Abstract: A state equation for a radical 4-degree-of-freedom active magnetic bearing is built, and the approach on how to use linear quadratic method of optimal control theory to design a centralized and decentralized parameters control system is introduced. Simulations have conducted within MATLAB. The results of simulations and experiments show that decentralized controllers designed from optimal state feedback theory at the speed of 60 000rpm meet the requirement of the active magnetic bearing system.

Key words: active magnetic bearing; state equation; optimal control theory; controller

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最优控制理论在磁轴承控制系统中的应用研究

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摘要: 建立了径向四自由度磁轴承系统状态方程,介绍了采用最优控制理论中线性二次型方法设计磁轴承集中和分布参数控制器的方法和步骤,并用 MATLAB 语言进行了仿真比较. 仿真和实验结果表明:磁轴承转子在 60 000rpm 以下运行,基于最优状态反馈控制理论设计的控制器,忽略各自由度之间的耦合,用分散控制策略设计的磁轴承控制系统,基本满足磁轴承性能要求.

关键词: 磁轴承; 状态方程; 最优控制理论; 控制器

1 Introduction

The system of active magnetic bearings (AMB's) is an innovative, high performance bearing system that suspends a rotating shaft with attractive magnetic force in a contact-free manner. It is superior, with no mechanical contact, no wear, no need of lubrication, non-pollution and long service life. It is applied in many areas such as energy resource, aeronautics and astronautics, high-speed precision machine tools, robot and so on. Controller is a key part in AMB technology, and the performance of the controller will not only determine whether the rotor can be suspended steadily, but also affect dynamic capability of the system (such as bearing stiffness, damping) and turning precision of the rotor. It is very important to design and optimize the controller in AMB system design[1~4]. In this paper, based on setting up a state equation of AMB, the controller of an AMB system is optimized by applying optimal control theory and using MATLAB toolboxes. The digital controller realizes AMB suspension steadily.

2 Mathematical model of radial 4-degree-of-freedom active magnetic bearings

2.1 Geometrical configuration of radial active magnetic bearings

Figure 1 shows the basic structure of rotor system of an active magnetic bearing. The couplings of mechanism, magnetic circuits and sensors of any directions must be avoided in designing the structure of AMB system. AMB can gain favorable performance only after reasonable configuration. In the reference frame as showed in Fig. 1, 0 is the rotor mass center. Movement of the rotor in radial 4-free directions is controlled by AMB A and B. The force produced by each AMB goes in the directions of X and Y. The AMB of each direc-

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tion is controlled by two opposite electromagnets, as shown in Fig.2.

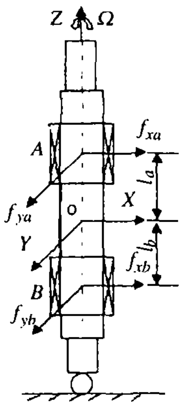

Fig. 1 Sketch of rotor structure of an active magnetic bearing

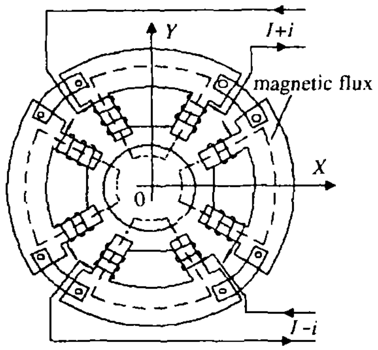

Fig. 2 Sketch of a radial active magnetic bearing

2.2 State equation of radial 4-degree-of-freedom active magnetic bearings

Configuration of AMB and practical working situations are very complicated. So, for the mathematical model, we assume that 1) the rotor is rigid and symmetrical, functionary forces are independent of each other in two perpendicular directions; 2) the functionary forces are counteract for the rotor when AMB suspend steadily; 3) the structures and parameters of 4-degree-of-freedom are almost consistent, and 4) the displacements measured by sensors are considered as the displacement of the rotor moving in corresponding direction.

Define m as the rotor mass, Ω as the rotational speed. J_x, J_y and J_Z are moments of inertia about spinning X, Y and Z axis, respectively and $J_x = J_y$. The displacements of the rotor at AMB A and B are $x_{xa}, x_{xb}, x_{ya}, x_{yb}$ in the directions X and Y, respectively. The control currents in corresponding electric coils are $i_{xa}, i_{xb}, i_{ya}, i_{yb}$.

Assuming state parameters of the system: $X = (x_{xa}, x_{xb}, x_{ya}, x_{yb}, \dot{x}_{xa}, \dot{x}_{xb}, \dot{x}_{ya}, \dot{x}_{yb})^{\mathrm{T}}$.

Input parameters: $U = (i_{xa}, i_{xb}, i_{ya}, x_{yb})^{\mathrm{T}}$.

Output parameters: $Y = (y_{xa}, y_{xb}, y_{ya}, y_{yb})^{\mathrm{T}}$.

We can write the state equations:

$$\begin{cases} \dot{X} = AX + BU, \\ Y = CX, \end{cases} \tag{1}$$

where the square matrix A is state matrix of 8×8, matrix B is control matrix of 8×4, the matrix C is output matrices of 4×8, matrix A and B are obtained from equations based on Newton second law, formula of magnetic attractive force and momentum law. See detailed process in reference [1].

One example of the parameters in an AMB experimental system[1] are: $m = 1\mathrm{kg}$, $J_z = 1.238 \times 10^{-4}\mathrm{kg \cdot m^2}$, $J_x = J_y = 1.744 \times 10^{-3}\ \mathrm{kg \cdot m^2}$, $l_a = 59\mathrm{mm}$, $l_b = 57\mathrm{mm}$, $K_x = 3.77 \times 10^5\mathrm{N/m}$, $K_i = 113.16\mathrm{N/A}$.

3 Design of an optimal state-variable feedback controller

3.1 Design of a centralized parameters controller

The system of a radial 4-degree-of-freedom AMB is a multivariate linear constant system, which is both controllable and observable. A multivariate optimal state-variable feedback controller can be designed by using linear quadratic method[5].

Since the control algorithm is realized by digital signal processor (DSP), discrete-time state-variable feedback controller must be designed. Dispersing (1) leads to the following equation:

$$X(k+1) = FX(k) + GU(k),\ k = 0,1,\cdots,N-1, \tag{2}$$

where $F = \mathrm{e}^{AT}$, $G = (\int_0^T \mathrm{e}^{AT}\mathrm{d}t)B$, T — sampling cycle. Discrete quadratic performance index:

$$J = \frac{1}{2}X^{\mathrm{T}}(N)PX(N) + \frac{1}{2}\sum_{k=0}^{N-1}[x^{\mathrm{T}}(k)Q(k)X(k) + U^{\mathrm{T}}(k)R(k)U(k)], \tag{3}$$

where matrices P, Q and R are symmetrical matrices. R is positive stable matrix; Q and P are positive half-stable matrices.

The proportions of state variable and control variable in the performance index J are determined by matrices Q and R. Choosing Q and R, we must consider the balance between energy and dynamic performance. Generally R is a unit matrix, and Q is diagonal matrix. The elements on the cross line of matrix Q are corresponding to the power rate of each state variable, which chooses "a" as displacement and "b" as velocity of each freedom of AMB. In this way, increasing "a" may improve stiffness of the system, and increasing "b" may improve the damping of the system, and reduce corresponding excess value range. According to (1) and (2), solve the Riccati equation[5], we can obtain K of discrete state feedback matrix, and gain the control value in discrete-time "k": $U(k) = -KX(k)$.

In the following analysis we assume that the rotational velocity is 30 000 rpm, and expound the course of designing controller and simulation curves using MATLAB tools.

For a continuous system, we adopt discrete-time $T = 100\mu s$, and solve F and G from (2). Furthermore, take matrix $R = I$, $a = 3000$ and $b = 100$ in matrix Q, feedback K can be expressed as follows

$$K = \begin{bmatrix} 42.0125 & 2.6238 & 0.5376 & -0.5323 & 17.7783 & 3.1428 & -0.0493 & 0.0497 \\ 2.6240 & 42.3875 & -0.5368 & 0.5315 & 3.1428 & 18.2285 & 0.0479 & 0.0483 \\ -0.5376 & 0.5323 & 42.0125 & 2.6239 & 0.0493 & -0.0497 & 17.7783 & 3.1428 \\ 0.5368 & -0.5313 & 2.6239 & 42.3876 & -0.0479 & 0.0483 & 3.1427 & 18.2285 \end{bmatrix}. \quad (4)$$

Figure 3 shows unit step respond in X_a direction when $R = I$, $a = 1000, 2000, 3000$, $b = 100$. It shows that increasing "a" will make the responding speed quicker, the time of ascending and adjusting shorter, and we can achieve appropriate performance of unit step response through selecting "a" value. At the same time, we can see that unit step input in direction X_a has little coupling to other three directions.

Figure 4 shows unit step response in X_a direction when $R = I$, $a = 1000$, $b = 100, 150, 200$. It shows that changing "b" has less influence on step response of system, the response of the other directions is also less when unit signal inputs in the direction X_a, at the same time, but when we reduce the value of "b", the coupling degree is also reduced in two perpendicular directions at the same AMB. Finally, we gain a controller which has fine performance and causes less coupling when $a = 3000$, $b = 100$, and AMB works in the speed range from 0 to 60 000rpm[1].

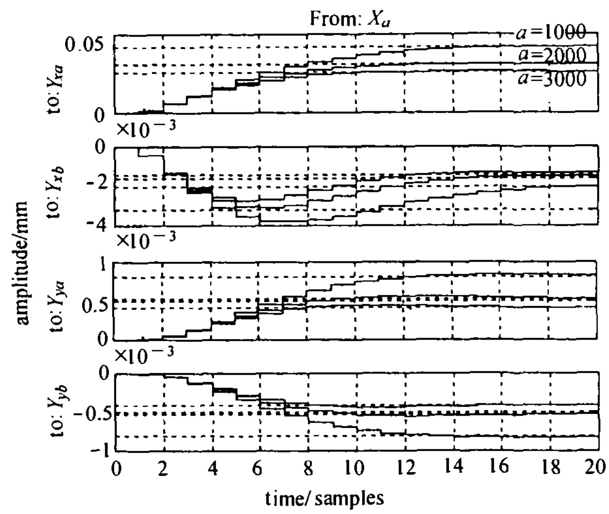

Fig. 3 Unit step respond in X_a direction when "a" chose different value, b=100

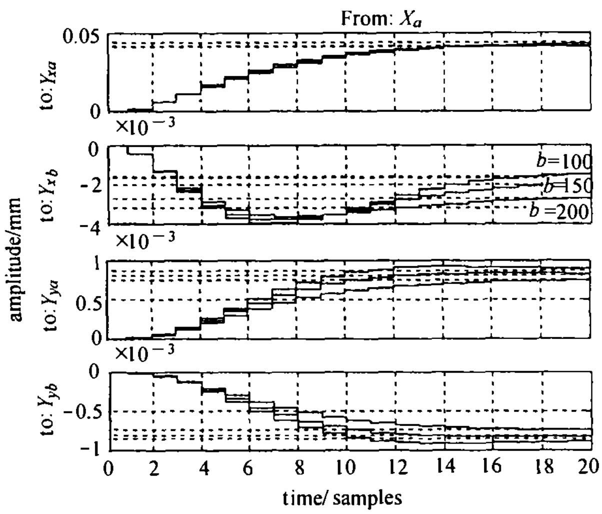

Fig. 4 Unit step respond in X_a direction when "b" chose different value, a=1000

3.2 Design of a decentralized parameters controller

The decentralized parameters control system is relative to the centralized parameters control system. The decentralized parameters control system is that the whole AMB system is divided into independent subsystems, and coupling between the subsystems is not considered so that the control system is considerably simplified. We regard feedback matrix "K" in Eq. (4) as two submatrixes of the order 4×4, further regard the two submatrixes as being on the cross matrix. So a decentralized control can be realized in the way that each-freedom feedback control signal is independent without other-freedom signal interaction. However, the state feedback matrix "K" designed with optimal control theory is two submatrixes of the order 4 × 4 and is not on the cross line matrix, which means that there is coupling between directions. According to the simulation curves Fig. 3 and Fig. 4, the coupling is less between directions. Furthermore the elements on the cross line of the two submatrixes of feedback matrix "K" are greater than the elements which are not on the cross line. So it is a feasible way for us to directly adopt the state feedback control matrix "K", and avoid the elements being not on the cross line, and take it for feedback matrix of discrete controller. In order to explain the feasibility of this conclusion, we avoid the elements of matrix "K" being not on the cross line at the speed of 30 000rpm, $a = 1000, 2000, 3000$, $b = 100$, we gain the simulation curve Fig. 5 in direction X_a. It shows that Fig. 5 is almost the same as Fig. 3 in unit step respondse of X_a direction and in the influence of the other three directions. Therefore we can design the decentralized parameters controller by using decentralized control strategy.

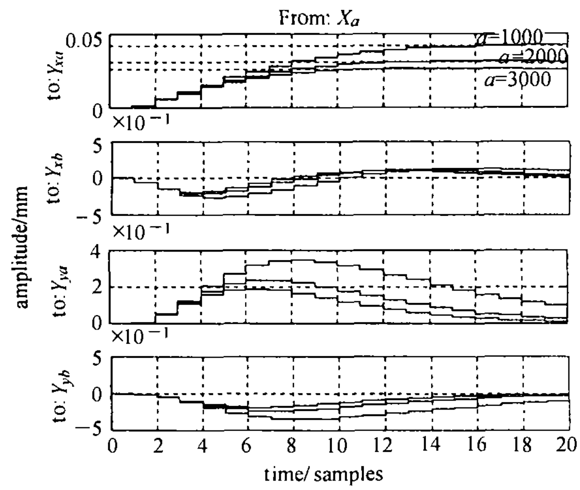

Fig. 5 Unit step respond in X_a direction when "a" chose different value, b=100, coupling is ignored

3.3 Digital realization of a decentralized parameters controller[1]

In the following simulation we set the rotational velocity = 30 000 rpm, $a = 3000$, $b = 100$, and the optimal feedback matrix "K" is obtained, and discrete control matrix is obtained as follows when only the elements on the cross lines of submatrixes are selected:

$$K' = \begin{bmatrix} 42.0125 & 0 & 0 & 0 & 17.7783 & 0 & 0 & 0 \\ 0 & 42.3875 & 0 & 0 & 0 & 18.2285 & 0 & 0 \\ 0 & 0 & 42.0125 & 0 & 0 & 0 & 17.7783 & 0 \\ 0 & 0 & 0 & 42.3876 & 0 & 0 & 0 & 18.2285 \end{bmatrix}.$$

Take subsystem of AMB A in direction X_a for example, the feedback control value may be written as:

$$U_{xa}(k) = 42.0125X_{xa}(k) + 17.7783\dot{X}_{xa}(k). \tag{5}$$

In the system of the experiment, the power amplifier's gain $k_c = 1$, sensor's gain $K_s = 20000$, we can obtain the expression of U_{xa} through transforming from (5) differential equation into difference equation, and using the same approach. The control arithmetic expressions of the other freedoms can also be obtained. From the control arithmetic expression of a controller (5), we can see that it is a proportional and differential control, and can be realized easily but some input static error. In general, we need integral part to make a whole PID controller. Finally a closed loop control can be implemented by using four independent PID controllers in 4-degree-of-freedom[1,3,4]. According to the above method, we design a discrete controller algorithm, and use a digital controller with digital signal processor (DSP), and realize AMB suspend steadily. Furthermore AMB can always suspend steadily at different speed range by adjusting the parameters in a small range based on the results of the simulations. In the situation of unload, the rotor displacements are well controlled within an acceptable small range that the maximum vibration amplitude is no more than 20μm in the speed range of 0 to 60 000rpm. The study outcome has passed the approval of the National Defense Technology Committee and the Technology Committee of Jiangsu Province[1].

4 Conclusion

In this paper, by setting up a state equation of 4-degree-of-freedom active magnetic bearings, centralized and decentralized parameters controllers are designed by using LQ control. The results of the simulations and experiments have showed that the decentralized controller meets the demands of AMB when decentralized control strategy replaces centralized control strategy under the speed 60 000rpm. The study outcome has great reference value for developing controllers of AMB's.

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