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Robust guaranteed cost control for uncertain discrete delay systems via dynamic output feedback

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Abstract: The problem of robust guaranteed cost control for a class of time-varying uncertain discrete delay systems is studied. The guaranteed cost control law is implemented by using a dynamic output feedback compensator. The proposed methods are given in terms of linear matrix inequalities (LMIs). A numerical example is given to demonstrate the effectiveness of the proposed methods.

Key words: guaranteed cost control; uncertainty; discrete delay system; LMI

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不确定离散时滞系统的动态输出反馈鲁棒保性能控制

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摘要: 研究不确定离散时滞系统的动态输出反馈保性能控制问题,通过引入动态输出反馈补偿器,采用线性矩阵不等式的方法,导出了系统存在保性能控制律的充分条件.数值算例说明了其有效性.

关键词: 保性能控制; 不确定性; 离散时滞系统; 矩阵不等式

1 Introduction

Many physical systems contain inherent time delays and uncertainties. Since time delays always result in instability of control systems, there has been increasing interest in the research of robust stabilization for uncertain time-delay systems[1,2].

Recently, with the development of robust control theory, the robust guaranteed cost control approach to the design of state feedback control laws for uncertain systems has been a subject of intensive research. Quadratic guaranteed cost control for uncertain non-delay systems was dealt with in [3 ~ 5] and robust guaranteed cost control for continuous-time uncertain systems with delay was considered in [6 ~ 9]. However, all those researches have been done on uncertain systems without time-delay or continuous-time delay systems. Little attention has been paid towards discrete-time uncertain systems with delay. And furthermore, most of the work is based on the assumption that the system states can be measured such that a memoryless state-feedback control law can be constructed to stabilize the proposed systems. In the case that the system states are non-measurable, these methods will fail.

In this paper, we consider the problem of robust guaranteed cost control for a class of linear discrete delay systems with time-varying uncertainties. By using the discrete Lyapunov function technique, based on a dynamic output feedback compensator we develop the robust guaranteed cost control for the uncertain system, which makes the closed-loop system quadratically stable for all admissible uncertainties and guarantees an adequate level of performance. The proposed methods are given in terms of linear matrix inequalities (LMIs). We show that the feasibility of the LMI ensures the existence of the output feedback compensator which solve the robust guaranteed cost control problem.

2 Problem description and some preliminaries

Consider a discrete-time uncertain system with delays in the states described as

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$$\begin{cases} x(k+1) = [A + \Delta A(k)]x(k) + [A_d + \\ \qquad \Delta A_d(k)]x(k-h) + Bu(k), \\ y(k) = [C + \Delta C(k)]x(k) + \\ \qquad [C_d + \Delta C_d(k)]x(k-h), \\ x(k) = \phi(k), \forall k = -h, -h+1, \cdots, 0, \end{cases} \tag{1}$$

where h is a positive integer for delay time, $x(k) \in \mathbb{R}^n$ is the state vector, and $\phi(k)$ is an initial value vector at k. A, A_d and B represent constant matrices with appropriate dimensions, and $\Delta A(k)$, $\Delta A_d(k)$, $\Delta C(k)$ and $\Delta C_d(k)$ denote parameter uncertainties of the following form:

$$\begin{aligned} [\Delta A(k) \quad \Delta A_d(k)] &= H_1 F(k)[E_1 \quad E_2], \\ [\Delta C(k) \quad \Delta C_d(k)] &= H_2 F(k)[E_1 \quad E_2], \end{aligned} \tag{2}$$

where H_i and $E_i (i = 1,2)$ are known constant matrices with appropriate dimensions, and properly dimensional matrix $F(\cdot)$ is unknown, time-varying but norm-bounded as

$$F^T(k)F(k) \leqslant I.$$

Because we will consider the stabilization problem and controller design problem independent of the size of time delays, it is necessary to present the following assumption:

Assumption 1 The matrix $A + A_d$ is Schur stable.

Indeed, the stability of the nominal system of (1) without uncertainties is a necessary condition, but is not sufficient. Note also that if $A + A_d$ is unstable, the system cannot be stabilized independent of the delay size.

Choose the controlled output $z(k)$ as

$$z(k) = C_z x(k) + C_{zd} x(k-h).$$

Let a quadratic cost functional be

$$J \triangleq \sum_{k=0}^{\infty} z^T(k) z(k), \tag{3}$$

where J is called the guaranteed cost function. We are interested in finding output feedback compensator as

$$\begin{aligned} \tilde{x}(k+1) &= A_c \tilde{x}(k) + B_c(k), \\ u(k) &= C_c \tilde{x}(k), \ \tilde{x}(0) = 0, \end{aligned} \tag{4}$$

where $\tilde{x}(k) \in \mathbb{R}^n$ is the compensator state vector; A_c, B_c and C_c are the system matrices of the compensator to be designed such that the closed-loop system with this compensator is robustly stable and achieves an upper bound δ for the cost function J for all admissible uncertainties. The closed-loop system is of the following form:

$$\begin{cases} \xi(k+1) = \tilde{A}\xi(k) + \tilde{A}_d \xi(k-h), \\ z(k) = \bar{C}_z \xi(k) + \bar{C}_{zd} \xi(k-h), \end{cases} \tag{5}$$

where

$$\xi(k) = \begin{pmatrix} x(k) \\ \tilde{x}(k) \end{pmatrix},$$

$$\tilde{A} = \begin{pmatrix} A + \Delta A(k) & BC_c \\ B_c C + B_c \Delta C & A_c \end{pmatrix},$$

$$\tilde{A}_d = \begin{pmatrix} A_d + \Delta A_d(k) & 0 \\ B_c C_d + B_c \Delta C_d(k) & 0 \end{pmatrix},$$

$$\bar{C}_z = (C_z \quad 0), \ \bar{C}_{zd} = (C_{zd} \quad 0).$$

In view of the form of closed-loop system matrices, we can guarantee the Schur stability of matrix $\tilde{A} + \tilde{A}_d$ by choosing appropriate matrix A_c, B_c and C_c.

The proof of our main results needs the following Lemma.

Lemma 1[10] The inequality $Y + HFE + E^T F^T H^T < 0$ holds for all F satisfying $\| F \| \leqslant 1$ if there exists $\varepsilon > 0$ such that $Y + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$, where Y is symmatric matrix and H, E and F are real matrices of appropriate dimensions.

3 Guaranteed cost control for delay systems without uncertainties

In this section, we provide an approach based on LMI to construct guaranteed cost output feedback compensator for linear discrete-time systems with delays without any uncertainty.

For convenience of description, we use the following notations:

$$\bar{A} = \begin{pmatrix} A & BC_c \\ B_c C & A_c \end{pmatrix}, \ \bar{A}_d = \begin{pmatrix} A_d & 0 \\ B_c C_d & 0 \end{pmatrix},$$

$$\Omega_{11} = \begin{pmatrix} -S & -I + SR_1 \\ -I + R_1 S & -Q + R_1 \end{pmatrix},$$

$$\Omega_{13} = \begin{pmatrix} SA^T + \delta_1^T B^T & \delta_3^T \\ A^T & A^T Q + C^T \delta_2^T \end{pmatrix},$$

$$\Omega_{14} = \begin{pmatrix} SC_z^T \\ C_z^T \end{pmatrix}, \ \Omega_{15} = \begin{pmatrix} S & M \\ 0 & 0 \end{pmatrix},$$

$$\Omega_{22} = \begin{pmatrix} -R_1 & 0 \\ 0 & -R_2 \end{pmatrix},$$

$$\Omega_{23} = \begin{pmatrix} A_d^T & A_d^T Q + C_d^T \delta_2^T \\ 0 & 0 \end{pmatrix},$$

$$\Omega_{24} = \begin{pmatrix} C_{zd}^{\mathrm{T}} \\ 0 \end{pmatrix},\ \Omega_{33} = \begin{pmatrix} -S & -I \\ -I & -Q \end{pmatrix},$$

$$\Omega_{55} = \begin{pmatrix} -R_1^{-1} & 0 \\ 0 & -R_2^{-1} \end{pmatrix},$$

$$T = \mathrm{diag}\{X_2^{\mathrm{T}}, I, X_2^{\mathrm{T}}, I\},$$

$$\bar{T} = \mathrm{diag}\{X_2^{\mathrm{T}}, I, X_2^{\mathrm{T}}, I, I, I\}.$$

Define Lyapunov function as

$$V(\xi(k), \xi(k-1), \cdots, \xi(k-h)) =$$

$$\xi^{\mathrm{T}}(k) P \xi(k) + \sum_{i=1}^{h} \xi^{\mathrm{T}}(k-i) R \xi(k-i). \quad (6)$$

For simplicity, we denote $V(\xi(k), \xi(k-1), \cdots, \xi(k-h))$ as $V(k)$, and then $V(\xi(0), \xi(-1), \cdots, \xi(-h)) = V(0)$. Also define $V(\infty)$ as the denotion of $V(\xi(K), \xi(K-1), \cdots, \xi(K-h))$ when $K \to \infty$.

Then we have the following theorem.

Theorem 1 Consider the system (1) with the output compensator (4), the closed-loop system (5) is stable if there exist symmetric positive definite matrices Q, S and real matrices $\delta_1, \delta_2, \delta_3$ for some given positive definite matrices R_1 and R_2 such that the following LMI is feasible:

$$\Pi_0 = \begin{pmatrix} \Omega_{11} & 0 & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 \\ * & * & \Omega_{33} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Omega_{55} \end{pmatrix} < 0, \quad (7)$$

where " * " denotes the symmetric blocks of the symmetric matrix and

$$\begin{cases} MN^{\mathrm{T}} = I - SQ, \\ \delta_1 = C_c M^{\mathrm{T}},\ \delta_2 = NB_c, \\ \delta_3 = NA_c M^{\mathrm{T}} + QAS + QB\delta_1 + \delta_2 CS. \end{cases} \quad (8)$$

Moreover, the parameters of output feedback compensator A_c, B_c, C_c can be computed based on (8), and the cost function J satisfies the following upper bound

$$\sigma = \phi^{\mathrm{T}}(0) Q \phi(0) + \sum_{i=1}^{h} \phi^{\mathrm{T}}(-i) R_1 \phi(-i).$$

Proof Taking the forward difference of (6) we get

$$\Delta V(k) = \begin{pmatrix} \xi(k) \\ \xi(k-h) \end{pmatrix}^{\mathrm{T}} \Pi_1 \begin{pmatrix} \xi(k) \\ \xi(k-h) \end{pmatrix} - z^{\mathrm{T}}(k) z(k), \quad (9)$$

where

$$\Pi_1 = \begin{pmatrix} \bar{A}^{\mathrm{T}} P \bar{A} - P + R + \bar{C}_z^{\mathrm{T}} \bar{C}_z & \bar{A}^{\mathrm{T}} P \bar{A}_d + \bar{C}_z^{\mathrm{T}} \bar{C}_{zd} \\ \bar{A}_d^{\mathrm{T}} P \bar{A} + \bar{C}_{zd}^{\mathrm{T}} \bar{C}_z & \bar{A}_d^{\mathrm{T}} P \bar{A}_d - R + \bar{C}_{zd}^{\mathrm{T}} \bar{C}_{zd} \end{pmatrix}. \quad (10)$$

Noticing the manipulation as

$$\begin{pmatrix} \Pi_1 & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -I \end{pmatrix} = \Sigma^{\mathrm{T}} \Pi_2 \Sigma, \quad (11)$$

where

$$\Sigma^{\mathrm{T}} = \begin{pmatrix} I & 0 & \bar{A}^{\mathrm{T}} & \bar{C}_z^{\mathrm{T}} \\ 0 & I & \bar{A}_d^{\mathrm{T}} & \bar{C}_{zd}^{\mathrm{T}} \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix},$$

$$\Pi_2 = \begin{pmatrix} -P+R & 0 & \bar{A}^{\mathrm{T}} P & \bar{C}_z^{\mathrm{T}} \\ * & -R & \bar{A}_d^{\mathrm{T}} P & \bar{C}_{zd}^{\mathrm{T}} \\ * & * & -P & 0 \\ * & * & * & -I \end{pmatrix},$$

we can conclude that $\Pi_1 < 0$ holds if $\Pi_2 < 0$. And furthermore we can conclude that

$$\Delta V(k) < -z^{\mathrm{T}}(k) z(k),$$

and $V(\infty) \to 0$.

Hence, it follows from the Lyapunov stability theorem that the system (6) is asymptotically stable. On the other hand, under the initial conditions $x(k) = \phi(k)$, $u(k) = 0$ for $-h \leqslant k \leqslant 0$, we have

$$J < \sum_{k=0}^{\infty} (-\Delta V(k)) =$$

$$\psi^{\mathrm{T}}(0) P \psi(0) + \sum_{i=1}^{h} \psi^{\mathrm{T}}(-i) R \psi(-i).$$

Up to now, we should prove that (7) is the sufficient condition for $\Pi_2 < 0$ via LMI tool.

Without loss of generality we can always choose invertible matrices P, X_1, X_2 as

$$P = \begin{pmatrix} Q & N \\ N^{\mathrm{T}} & U \end{pmatrix} = \begin{pmatrix} S & M \\ M^{\mathrm{T}} & V \end{pmatrix}^{-1},$$

$$X_1 = \begin{pmatrix} I & Q \\ 0 & N^{\mathrm{T}} \end{pmatrix},\ X_2 = \begin{pmatrix} S & I \\ M^{\mathrm{T}} & 0 \end{pmatrix},$$

and furthermore

$$QS + NM^{\mathrm{T}} = I,$$

$$N^{\mathrm{T}} S + UM^{\mathrm{T}} = 0.$$

Therefore, it is easy to see that $PX_2 = X_1$ which is equivalent to

$$P = X_1 X_2^{-1}. \tag{12}$$

Premultiply T^{T} and postmultiply T to Π_2, we have

$$\Pi_3 = \begin{pmatrix} -X_2^{\mathrm{T}}X_1 + X_2^{\mathrm{T}}RX_2 & 0 & X_2^{\mathrm{T}}\bar{A}^{\mathrm{T}}X_1 & X_2^{\mathrm{T}}\bar{C}_z^{\mathrm{T}} \\ * & -R & \bar{A}_d^{\mathrm{T}}X_1 & \bar{C}_{zd}^{\mathrm{T}} \\ * & * & -X_2^{\mathrm{T}}X_1 & 0 \\ * & * & * & -I \end{pmatrix} < 0. \tag{13}$$

If we choose R in (13) as

$$R = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix},$$

and substitute correspondingly the system matrices and the decompositions shown in (12) into (13), it is obtained (7) based on notations in (8) by using Schur complement lemma. Furthermore it follows $J < \sigma$.

Procedure:

Step 1 Given $R_1 > 0$ and $R_2 > 0$, solve LMIs (8) to obtain $Q > 0, S > 0, \delta_1, \delta_2$ and δ_3;

Step 2 Compute a pair of invertible matrices M and N based on the first equality in (9) by svd function in Matlab;

Step 3 Substitute Q, S, M and N into X_1 and X_2, and we obtain P. If $P > 0$, next to Step 4, else to Step 1;

Step 4 Solve A_c, B_c and C_c via the other three equations in (9).

4 Robust guaranteed cost control for delay systems with uncertainties

In this section we consider how to determine an output compensator for delay system with uncertainties described by (1) and (2) such that the closed-loop system is robustly stable with guaranteed cost σ. The following result casts this robust guaranteed cost control problem into one which can be solved by the approach proposed in Section 3.

Theorem 2 Given positive definite matrices R_1 and R_2, the closed-loop delay system with uncertainties (6) is robustly stable if there exist positive definite matrices Q, S and real matrices $\delta_1, \delta_2, \delta_3$ for any admissible uncertainties satisfying (14) for a given scalar $\varepsilon > 0$, and the cost function (3) satisfies the following bound

$$\sigma = \phi^{\mathrm{T}}(0)Q\phi(0) + \sum_{i=1}^{h}\phi^{\mathrm{T}}(-i)R_1\phi(-i),$$

$$\bar{\Pi}_0 = \begin{pmatrix} \Pi_0 & \bar{\Omega}_{12} & \bar{\Omega}_{13} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon^{-1}I \end{pmatrix} < 0, \tag{14}$$

where

$$\bar{\Omega}^{\mathrm{T}} = (E_1 S^{\mathrm{T}} \quad E_1 \quad E_2 \quad 0 \quad 0 \quad 0),$$

$$\Omega_{13}^{\mathrm{T}} = (0 \quad 0 \quad 0 \quad 0 \quad H_1^{\mathrm{T}} \quad H_1^{\mathrm{T}}Q + H_2^{\mathrm{T}}\delta_2^{\mathrm{T}}).$$

Proof Similar to Theorem 1, $\Delta V(k) + z^{\mathrm{T}}(k)z(k) < 0$ can be guaranteed by

$$\tilde{\Pi}_1 = \begin{pmatrix} -P+R & 0 & \tilde{A}^{\mathrm{T}}P + \bar{C}_z^{\mathrm{T}} & \bar{C}_z^{\mathrm{T}} \\ * & -R & \tilde{A}_d^{\mathrm{T}}P & \bar{C}_{zd}^{\mathrm{T}} \\ * & * & -P & 0 \\ * & * & * & -I \end{pmatrix} < 0. \tag{15}$$

It is obvious, based on (2) and (5), that

$$(\tilde{A} \quad \tilde{A}_d) = (\bar{A} \quad \bar{A}_d) + \bar{H}F(\bar{E}_1 \quad \bar{E}_2),$$

where

$$\bar{H} = \begin{pmatrix} H_1 \\ B_c H_2 \end{pmatrix},\ \bar{E}_1 = (E_1 \quad 0),\ \bar{E}_2 = (E_2 \quad 0).$$

Using Lemma 1 it can be obtained $\tilde{\Pi}_1 < 0$ if and only if there exist $P > 0, R > 0$ and $\varepsilon > 0$ satisfying

$$\tilde{\Pi}_2 = \begin{pmatrix} -P+R & 0 & \bar{A}^{\mathrm{T}}P & \bar{C}_z^{\mathrm{T}} & \bar{E}_1 & 0 \\ * & -R & \bar{A}_d^{\mathrm{T}}P & \bar{C}_{zd}^{\mathrm{T}} & \bar{E}_2 & 0 \\ * & * & -P & 0 & 0 & P\bar{H} \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon^{-1}I \end{pmatrix} < 0. \tag{16}$$

The following proof procedure is based on the same method via (12) to (13).

Premultiply $\tilde{T}^{\mathrm{T}}$ and postmultiply $\tilde{T}$ to $\tilde{\Pi}_2$, we have

$$\tilde{\Pi}_3 = \begin{pmatrix} -X_2^{\mathrm{T}}X_1 + X_2^{\mathrm{T}}RX_2 & 0 & X_2^{\mathrm{T}}\bar{A}^{\mathrm{T}}X_1 & X_2^{\mathrm{T}}\bar{C}_z^{\mathrm{T}} & X_2^{\mathrm{T}}\bar{E}_1 & 0 \\ * & -R & \bar{A}_d^{\mathrm{T}}X_1 & C_{zd}^{\mathrm{T}} & \bar{E}_2 & 0 \\ * & * & -X_2^{\mathrm{T}}X_1 & 0 & 0 & X_1^{\mathrm{T}}\bar{H} \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon^{-1}I \end{pmatrix} < 0. \tag{17}$$

If substitute R in (13) and the decompositions in (12) into (17), we have (14) by using Schur Complements lemma once more. On the other hand, we can see (14) indicate the following inequality

$$\Delta V(k) + z^{\mathrm{T}}(k)z(k) < 0, \qquad (18)$$

so $J < \sigma$ can be obtained easily if we conduct add from $k = 0$ to ∞ at the two hands of (18) apparently.

Remark 1 Theorem 2 provides an LMI based method for the design of an output feedback compensator that robustly stabilizes uncertain discrete delay system (1) and guarantees an adequate level of performance. It should be pointed out that the feasibility of LMI (14) does not provide a unique solution Q, S, δ_1, δ_2 and δ_3. However, the non-uniqueness is in fact an advantage since we can construct a family of controllers using this property. The optimal quadratic guaranteed cost controller can be determined by a search over all parameters Q, S, δ_1, δ_2 and δ_3 satisfying LMI (14). The optimal cost is found as follows.

Remark 2 The advantage of LMI approach is that the problem of finding the optimal cost can be easily solved without requiring the tuning of any parameters. Note that the performance bound σ depends on the initial conditions $\psi(i)(i = -h, -h+1, \cdots, -1, 0)$. To remove this dependence of the initial conditions, we assume that the initial state is a random variable satisfying the following set

$$\Theta = \{\phi(i) \in \mathbb{R}^n : x(0) = U_0 V_0,\ x(-i) = U_i V_0,\ V_0^{\mathrm{T}} V_0 < I\}. \qquad (19)$$

In the case, we can get

$$J < \delta \leqslant \lambda_{\max}\Big(U_0^{\mathrm{T}} Q U_0 + \sum_{i=1}^{h} U_i^{\mathrm{T}} R_1 U_i\Big).$$

Then, the optimal quadratic performance can be computed by solving the following quasi-convex optimization problem:

$$\text{minimize } \lambda$$

$$\text{s.t.} \begin{cases} \bar{\Pi}_0 < 0, \\ \begin{pmatrix} -\lambda I + \sum_{i=1}^{h} U_i^{\mathrm{T}} R_1 U_i & U_0^{\mathrm{T}} Q \\ Q U_0 & -Q \end{pmatrix} < 0. \end{cases}$$

Remark 3 The dynamic output feedback compensator used in this paper is the discrete case of the one proposed by V. Kapila (1998). It should be noted that the results of discrete time case are more useful than the corresponding continuous time results since the use of digital hardware invariably requires that the designing of robust controller be implemented in discrete time.

5 Illustrative example

In this section we present an example to illustrate the theory developed in preceding sections. Given the time-discrete delay system (1) with the following parameters

$$A = \begin{pmatrix} 0.13 & 0.63 \\ 0 & 0.8 \end{pmatrix},\ A_d = \begin{pmatrix} 0.18 & 0 \\ 0.22 & 0.4 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.25 \\ 1.4 \end{pmatrix},\ C = \begin{pmatrix} 1 & -1 \\ 0 & 0.5 \end{pmatrix},$$

$$C_d = \begin{pmatrix} 0.11 & 0 \\ 0.1 & 0.13 \end{pmatrix},\ R_1 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix},$$

$$C_{zd} = (0.2 \quad 0.2),\ F(k) = \frac{1}{2}(\sin(10k) + 1),$$

$$H_1 = \begin{pmatrix} 0.2 \\ 0.12 \end{pmatrix},\ H_2 = \begin{pmatrix} -0.01 \\ 0.22 \end{pmatrix},$$

$$E_1 = (0.3 \quad -0.1),\ E_2 = (0.12 \quad 0.3),$$

$$C_z = (0.1 \quad 0.1),\ h = 4,\ R_2 = 1,\ \varepsilon = 1.$$

The initial state matrices in (19) are such as

$$V_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ U_0 = \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix},$$

$$U_1 = \begin{pmatrix} 0.6 \\ 0.1 \end{pmatrix},\ U_2 = \begin{pmatrix} 0 \\ 0.3 \end{pmatrix},$$

$$U_3 = \begin{pmatrix} 0.83 \\ -0.5 \end{pmatrix},\ U_4 = \begin{pmatrix} -0.78 \\ 0.12 \end{pmatrix}.$$

And the cost function is

$$J \triangleq \sum_{k=0}^{\infty} z^{\mathrm{T}}(k) z(k).$$

Consider system (1) without uncertainties, and by solving the LMI (8) based on Theorem 1, we obtain that

$$A_c = \begin{pmatrix} -0.0554 & -3.8061 \\ 0.0366 & -0.6072 \end{pmatrix},$$

$$B_c = \begin{pmatrix} -0.0835 & 3.5801 \\ 0.1274 & 2.2820 \end{pmatrix},$$

$$C_c = (-0.0263 \quad -0.4518).$$

With the above control parameters, the control law (5) robustly stabilizes uncertain discrete delay system (1) and guarantees the optimal quadratic performance $J(k) \leqslant 1.3444$.

Consider the uncertain system whose parameters are given above, and by using the optimization technique and the svd function in Matlab, we have

$$A_c = \begin{pmatrix} -0.4997 & -1.3206 \\ -0.0710 & -0.4715 \end{pmatrix},$$

$$B_c = \begin{pmatrix} 0.3525 & 6.9151 \\ 0.3007 & 5.8423 \end{pmatrix},$$

$$C_c = (-0.0815 \quad -0.1467),$$

$$J(k) \leqslant 1.2399.$$

From the example we can see that our method to design the output feedback compensator is feasible and simple enough.

6 Conclusion

An output feedback compensator which can robustly stabilize the discrete delay uncertain systems has been obtained by solving the LMIs. We have obtained a sufficient condition for guaranteeing not only the quadratic stability of the closed-loop system but also the cost function bound constraint. Moreover, we have presented the cost bound by assuming the initial values to be a zero mean random variable. The LMI has the advantage that it can be solved very efficiently by the computer software.

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