Article ID: 1000 - 8152(2003)02 - 0211 - 06

Robust H_∞ state estimation for linear state-delayed and measurement-delayed systems with uncertainties

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Abstract: The problem of robust H_∞ state estimation for a class of continuous-time systems with known state delay and measurement delay as well as with norm-bounded parameter uncertainties is concerned with. Sufficient conditions for the solutions of this problem are presented to ensure that there exists the asymptotically stable state estimator such that the transfer function from exogenous disturbance to output estimation error satisfies the prescribed H_∞ performance for all admissible perturbations in terms of two algebraic Riccati inequalities. The results extend to the case of the problem of robust H_∞ state estimation for a class of continuous-time systems with unknown state delay and measurement delay as well as with norm-bounded parameter uncertainties. For known state delay and measurement delay systems, the state estimator derived does not depend on parameter uncertainties, but depend on time delays, and for unknown state delay and measurement delay systems, the state estimator depends on neither parameter uncertainties nor time delays.

Key words: robust H_∞ state estimation; time delay; uncertain system; algebraic Riccati inequality

CLC number: TP13　　**Document code**: A

具有状态和测量时滞不确定系统的鲁棒 H_∞ 状态估计

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摘要: 考虑一类已知状态和测量时滞且范数有界参数不确定连续时间系统的鲁棒 H_∞ 状态估计问题.这个问题解的充分条件由二个代数 Riccati 不等式给出,它可以保证存在一个渐近稳定状态估计器使得对于所有不确定性从外界干扰到输出估计误差的传递函数满足指定的 H_∞ 指标.以上这些结果可以推广到一类未知状态和测量时滞且范数有界参数不确定连续系统的鲁棒 H_∞ 状态估计问题,对于已知状态和测量时滞系统,所得状态估计器与参数不确定性无关,而与时滞有关.对于未知状态和测量时滞系统,其状态估计器不仅与参数不确定性无关,而且与时滞也无关.

关键词: 鲁棒 H_∞ 状态估计; 时滞; 不确定系统; 代数 Riccati 不等式

1 Introduction

The dynamic behaviour of many physical processes contains inherent time delays and uncertainties, and can be modeled by time-delay system with uncertainties[1,2]. It is well known that time delays and/or parameter uncertainties are often the main cause of instability of control system. In the past few years, many researchers have investigated the robust control problem for time-delay systems without/with uncertainties using H_∞ approach and obtained many significant results[2~8]. Meanwhile, much attention has been paid to the problem of H_∞ state estimation[9~15]. But only a few papers address problem of H_∞ state estimation for time-delay systems[16~20]. The problem of H_∞ state estimation for linear time-invariant system with delayed measurements has been considered in [16], and a state space solution to the problem is characterized in terms of two algebraic Riccati equations and one finite-horizon Riccati differential equation. The problem of H_∞ filtering for linear time-varying systems with time-delayed measurement has been investigated in [17], and the solution to this problem involves a Riccati differential equation similar to the

Received date: 2001 - 09 - 30; Revised date: 2002 - 05 - 15.

Foundation item: supported by the National Natural Science Foundation of China(69974010); the Foundation for University Key Teacher by the Ministry of Education(DQ2000 - 14); the Fujian Provincial Natural Science Foundation(F97009).

one which arise in H_∞ filtering for systems without time-delay. [18,19,20] have considered the robust H_∞ filtering for uncertain linear systems with delayed states and outputs. In this paper we address the problem of robust H_∞ state estimation for linear systems with inherent time-delay in state and time-delay in measurement output as well as with norm-bounded parameter uncertainties. For known state delay and measurement delay systems, a state estimator with time-delay is constructed to ensure that the output estimation error satisfies the prescribed H_∞ performance for all admissible perturbations in terms of two algebraic Riccati inequalities. For unknown state delay and measurement delay systems, a state estimator without time-delay is constructed to ensure that the output estimation error satisfies the prescribed H_∞ performance for all admissible perturbations in terms of two algebraic Riccati inequalities.

The rest of this paper is organized as follows. In Section 2, the robust H_∞ state estimation problem for continuous-time linear systems with known time-delays and norm-bounded parameter uncertainties is formulated. Sufficient conditions for the solution of this problem are presented in Section 3 which involves two algebraic Riccati inequalities. The robust H_∞ state estimation problem for continuous-time linear systems with unknown time-delays and norm-bounded parameter uncertainties is considered in Section 4. An illustrative example is provided in Section 5. Concluding remarks are finally made in Section 6.

2 Problem formulation

Consider the following class of linear systems with time delays and parameter uncertainties:

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + A_d x(t - d_1) + D_1 w(t), \\ y(t) = (C + \Delta C)x(t) + C_d x(t - d_2) + D_2 w(t), \\ z(t) = Lx(t), \\ x(t) = \phi(t);\ t \in [-\max(d_1, d_2), 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the measured output, $z(t) \in \mathbb{R}^m$ is the output to be estimated, $w(t) \in \mathbb{R}^q$ is the square-integrable disturbance, $\phi(t)$ is the continuous initial-value function, d_1 and d_2 are the constant known time-delays, $A, A_d, C, C_d, D_1, D_2, L$ are known constant matrices that describe the nominal system. ΔA and ΔC are perturbation matrices which represent parameter uncertainties and assumed to be of the time-invariant form

$$\begin{bmatrix} \Delta A \\ \Delta C \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \Gamma N, \tag{2}$$

where M_1, M_2 and N are known constant matrices of appropriate dimensions, and $\Gamma \in \mathbb{R}^{i \times j}$ is a perturbation matrix which satisfies

$$\Gamma\Gamma^{\mathrm{T}} \leqslant I. \tag{3}$$

The perturbations ΔA and ΔC are said to be admissible if they satisfy (2) and (3). In the following, we will assume that the system (1) is quadratically stable for all admissible perturbations.

Consider the following state estimator for the system (1):

$$\begin{cases} \dot{\hat{x}} = F\hat{x}(t) + A_d\hat{x}(t - d_1) + G[y(t) - C_d\hat{x}(t - d_2)], \\ \hat{z}(t) = L\hat{x}(t), \end{cases} \tag{4}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state, $\hat{z}(t) \in \mathbb{R}^m$ is an estimate for $z(t)$, and F and G are state estimator parameters to be determined.

Define the state estimation error $e(t)$ and the output estimation error $e_z(t)$ by

$$e(t) = x(t) - \hat{x}(t), \tag{5}$$

$$e_z(t) = z(t) - \hat{z}(t). \tag{6}$$

Define an augmented state vector to be

$$x_e(t) = \begin{bmatrix} e(t) \\ x(t) \end{bmatrix}.$$

Combining the system (1) and the filter (4) into an augmented system gives

$$\begin{cases} \dot{x}_e(t) = (A_e + \Delta A_e)x_e(t) + A_{de}x_e(t - d_1) + \\ \qquad\qquad C_{de}x_e(t - d_2) + B_e w(t), \\ e_z(t) = C_e x_e(t), \end{cases} \tag{7}$$

where

$$\begin{cases} A_e = \begin{bmatrix} F & A - GC - F \\ 0 & A \end{bmatrix}, \\ \Delta A_e = \begin{bmatrix} M_1 - GM_2 \\ M_1 \end{bmatrix} \Gamma [0 \quad N] =: M_e \Gamma N_e, \end{cases} \tag{8a}$$

$$\begin{cases} A_{de} = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix},\ C_{de} = \begin{bmatrix} -GC_d & 0 \\ 0 & 0 \end{bmatrix}, \\ B_e = \begin{bmatrix} D_1 - GD_2 \\ D_1 \end{bmatrix},\ C_e = [L \quad 0]. \end{cases} \tag{8b}$$

The objective of this paper is to design the asymptotically stable state estimator (4) such that, for all admissible perturbations ΔA and ΔC, the H_∞ norm of the transfer function

$$H(s) = C_e(sI - A_e - \Delta A_e - A_{de}e^{-sd_1} - C_{de}e^{-sd_2})^{-1}B_e, \tag{9}$$

from $w(t)$ to $e_z(t)$ satisfies $\| H(s) \|_\infty < \gamma$ for a given positive constant $\gamma > 0$.

3 Robust H_∞ state estimation for known time-delays systems

Before giving the main results, we first present three lemmas.

Lemma 1 For any matrices of appropriate dimensions X, Y and any positive constant $\alpha > 0$,

$$XY^T + YX^T \leqslant \alpha XX^T + \frac{1}{\alpha}YY^T. \tag{10}$$

Lemma 2 For any matrices of appropriate dimensions X, Y and any symmetric positive definite matrix of appropriate dimensions $\Pi > 0$, we have

$$XY^T e^{j\omega d} + YX^T e^{-j\omega d} \leqslant X\Pi X^T + Y\Pi^{-1}Y^T \tag{11}$$

for all $\omega \in \mathbb{R}$.

Lemma 3 For any symmetric matrix

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix},$$

the following statements are equivalent.

1) $L < 0$;

2) $L_{11} < 0$, $L_{22} - L_{12}^T L_{11}^{-1} L_{12} < 0$;

3) $L_{22} < 0$, $L_{11} - L_{12} L_{22}^{-1} L_{12}^T < 0$.

In the following, we give a sufficient condition such that the augmented system (7) is asymptotically stable and simultaneously $\| H(s) \|_\infty < \gamma$ for all admissible perturbations ΔA and ΔC.

Theorem 1 If there exist symmetric positive definite matrices $Q > 0$, $\Pi > 0$ and $\Sigma > 0$ such that

$$(A_e + \Delta A_e)Q + Q(A_e + \Delta A_e)^T + \frac{1}{\gamma^2}QC_e^TC_eQ + B_eB_e^T + A_{de}\Pi^{-1}A_{de}^T + Q\Pi Q + C_{de}\Sigma^{-1}C_{de}^T + Q\Sigma Q < 0, \tag{12}$$

then the augmented system (7) is asymptotically stable and $\| H(s) \|_\infty < \gamma$.

Proof The results are readily obtained from Lyapunov method and algebraic operation. Q.E.D.

Theorem 2 If there exists a positive constant $\alpha > 0$ and symmetric positive definite matrices $Q > 0$, $\Pi > 0$ and $\Sigma > 0$ such that

$$A_eQ + QA_e^T + \frac{1}{\gamma^2}QC_e^TC_eQ + B_eB_e^T + A_{de}\Pi^{-1}A_{de}^T + Q\Pi Q + C_{de}\Sigma^{-1}C_{de}^T + Q\Sigma Q + \frac{1}{\alpha}QN_e^TN_eQ + \alpha M_eM_e^T < 0, \tag{13}$$

then the augmented system (7) is asymptotically stable and $\| H(s) \|_\infty < \gamma$ for all admissible perturbations ΔA and ΔC.

Proof Using Lemma 1 and $\Gamma\Gamma^T \leqslant I$, we have

$$\begin{aligned} &\Delta A_eQ + Q\Delta A_e^T = \\ &M_e\Gamma N_eQ + QN_e^T\Gamma^T M_e^T \leqslant \\ &\alpha M_e\Gamma\Gamma^T M_e^T + \frac{1}{\alpha}QN_e^TN_eQ \leqslant \\ &\alpha M_eM_e^T + \frac{1}{\alpha}QN_e^TN_eQ. \end{aligned} \tag{14}$$

From the above inequality, if Ineq.(13) is satisfied, then Ineq.(12) holds for all admissible perturbations ΔA and ΔC. Using Theorem 1, we can conclude that the augmented system (7) is asymptotically stable and $\| H(s) \|_\infty < \gamma$ for all admissible perturbations ΔA and ΔC. Q.E.D.

Ineq.(13) is not solvable in its present form as the augmented system matrices contain the state estimator parameters F and G. In the next theorem, we seek to make suitable choice of F and G and replace (13) by two algebraic Riccati inequalities which are solvable.

Theorem 3 Consider the system (1) and suppose there exist symmetric positive definite matrices $Q_1 > 0$, $Q_2 > 0$, $\Pi_1 > 0$, $\Pi_2 > 0$, $\Sigma_1 > 0$ and $\Sigma_2 > 0$, and a positive constant $\alpha > 0$ such that

$$A_1Q_1 + Q_1A_1^T + \frac{1}{\gamma^2}Q_1L^TLQ_1 + R_1 + Q_1\Pi_1Q_1 + A_d\Pi_1^{-1}A_d^T + Q_1\Sigma_1Q_1 - (Q_1C_1^T + R_{12})(R_2 + C_d\Sigma_1^{-1}C_d^T)^{-1}(Q_1C_1^T + R_{12})^T < 0, \tag{15}$$

$$A^TQ_2 + Q_2A + \frac{1}{\alpha}N^TN + Q_2R_1Q_2 + \Pi_2 + Q_2A_d\Pi_2^{-1}A_d^TQ_2 + \Sigma_2 < 0, \tag{16}$$

where

$$\begin{aligned} &A_1 = A + R_1Q_2,\ C_1 = C + R_{12}^TQ_2, \\ &R_1 = D_1D_1^T + \alpha M_1M_1^T,\ R_2 = D_2D_2^T + \alpha M_2M_2^T, \\ &R_{12} = D_1D_2^T + \alpha M_1M_2^T, \end{aligned} \tag{17}$$

then for all admissible perturbation ΔA and ΔC, there exists the asymptotically stable state estimator (4) with

$$F = A_1 - GC_1, \tag{18}$$

$$G = (Q_1C_1^{\mathrm{T}} + R_{12})(R_2 + C_d\Sigma_1^{-1}C_d^{\mathrm{T}})^{-1}, \tag{19}$$

such that $\| H(s) \|_\infty < \gamma$.

Proof By Theorem 2, we define S for the sufficient condition (13) as follows

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix} := $$

$$A_eQ + QA_e^{\mathrm{T}} + \frac{1}{\gamma^2}QC_e^{\mathrm{T}}C_eQ + B_eB_e^{\mathrm{T}} + A_{de}\Pi^{-1}A_{de}^{\mathrm{T}} + Q\Pi Q + C_{de}\Sigma^{-1}C_{de}^{\mathrm{T}} + Q\Sigma Q + \frac{1}{\alpha}QN_e^{\mathrm{T}}N_eQ + \alpha M_eM_e^{\mathrm{T}} < 0, \tag{20}$$

Q, Π and Σ are assumed to be block-diagonal positive definite matrices:

$$\begin{cases} Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2^{-1} \end{bmatrix} > 0, \\ \Pi = \begin{bmatrix} \Pi_1 & 0 \\ 0 & \Pi_2 \end{bmatrix} > 0,\ \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} > 0. \end{cases} \tag{21}$$

Substituting (8) and (21) into (20), the author has

$$S_{11} = FQ_1 + Q_1F^{\mathrm{T}} + \frac{1}{\gamma^2}Q_1L^{\mathrm{T}}LQ_1 + (D_1 - GD_2)(D_1 - GD_2)^{\mathrm{T}} + \alpha(M_1 - GM_2)(M_1 - GM_2)^{\mathrm{T}} + Q_1\Pi_1Q_1 + A_d\Pi_1^{-1}A_d^{\mathrm{T}} + Q_1\Sigma_1Q_1 + GC_d\Sigma_1^{-1}C_d^{\mathrm{T}}G^{\mathrm{T}}, \tag{22}$$

$$S_{12} = (A - GC - F)Q_2^{-1} + (D_1 - GD_2)D_1^{\mathrm{T}} + \alpha(M_1 - GM_2)M_1^{\mathrm{T}}, \tag{23}$$

$$S_{22} = AQ_2^{-1} + Q_2^{-1}A^{\mathrm{T}} + \frac{1}{\alpha}Q_2^{-1}N^{\mathrm{T}}NQ_2^{-1} + D_1D_1^{\mathrm{T}} + \alpha M_1M_1^{\mathrm{T}} + Q_2^{-1}\Pi_2Q_2^{-1} + A_d\Pi_2^{-1}A_d^{\mathrm{T}} + Q_2^{-1}\Sigma_2Q_2^{-1}. \tag{24}$$

From (23), if we choose F by setting $S_{12} = 0$, then

$$F = A - GC + [D_1D_1^{\mathrm{T}} + \alpha M_1M_1^{\mathrm{T}} - G(D_2D_1^{\mathrm{T}} + \alpha M_2M_1^{\mathrm{T}})]Q_2 = A_1 - GC_1, \tag{25}$$

which is the same as (18) from (17). Substituting (25) into (22) and using (17), we get

$$S_{11} = A_1Q_1 + Q_1A_1^{\mathrm{T}} + \frac{1}{\gamma^2}Q_1L^{\mathrm{T}}LQ_1 + R_1 + Q_1\Pi_1Q_1 + A_d\Pi_1^{-1}A_d^{\mathrm{T}} - (Q_1C_1^{\mathrm{T}} + R_{12})(R_2 + C_d\Sigma_1^{-1}C_d^{\mathrm{T}})^{-1}(Q_1C_1^{\mathrm{T}} + R_{12})^{\mathrm{T}} + Q_1\Sigma_1Q_1 + [G - (Q_1C_1^{\mathrm{T}} + R_{12})(R_2 + C_d\Sigma_1^{-1}C_d^{\mathrm{T}})^{-1}][R_2 + C_d\Sigma_1^{-1}C_d^{\mathrm{T}}][G - (Q_1C_1^{\mathrm{T}} + R_{12})(R_2 + C_d\Sigma_1^{-1}C_d^{\mathrm{T}})^{-1}]^{\mathrm{T}}. \tag{26}$$

In order to ensure that (20) is satisfied as far as possible, G is chosen according to (19), so that the last term on the right-hand side of (26) vanishes, i.e.

$$S_{11} = A_1Q_1 + Q_1A_1^{\mathrm{T}} + \frac{1}{\gamma^2}Q_1L^{\mathrm{T}}LQ_1 + R_1 + Q_1\Pi_1Q_1 + A_d\Pi_1^{-1}A_d^{\mathrm{T}} + Q_1\Sigma_1Q_1 - (Q_1C_1^{\mathrm{T}} + R_{12})(R_2 + C_d\Sigma_1^{-1}C_d^{\mathrm{T}})^{-1}(Q_1C_1^{\mathrm{T}} + R_{12})^{\mathrm{T}}. \tag{27}$$

Moreover, (24) can be rewritten as

$$S_{22} = Q_2^{-1}[A^{\mathrm{T}}Q_2 + Q_2A + \frac{1}{\alpha}N^{\mathrm{T}}N + Q_2R_1Q_2 + \Pi_2 + Q_2A_d\Pi_2^{-1}A_d^{\mathrm{T}}Q_2 + \Sigma_2]Q_2^{-1}. \tag{28}$$

Recalling the conditions (15) and (16), we have that $S_{11} < 0$ and $S_{22} < 0$. This implies that $S < 0$. It follows from Theorem 2 that the augmented system (7) is asymptotically stable and $\| H(s) \|_\infty < \gamma$. Hence, there exists the asymptotically stable state estimator (4) with the choices (18) and (19) such that $\| H(s) \|_\infty < \gamma$. This completes the proof of Theorem 3.

Q.E.D.

Remark 1 From Theorem 3, it has been shown that the problem of robust H_∞ state estimation for systems with time delays and norm-bounded parameter uncertainties is characterized as a feasibility problem of finding a pair of positive definite matrices Q_1 and Q_2 in accordance with (15) and (16). (15) is coupled unilaterally with (16). Hence, we can always solve for Q_2 from (16) first and then Q_1 from (15). They can be solved using LMI technique[2] or modified algebraic Riccati equation approach[20].

Remark 2 Note that the state estimator (4) contains state-delay terms. If the time delays are unknown, the state estimator (4) is not adopted.

4 Robust H_∞ state estimation for unknown time-delays systems

Consider the system (1) with unknown time-delays d_1 and d_2. A state estimator for this system is constructed as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= F\hat{x}(t) + Gy(t), \\ \hat{z}(t) &= L\hat{x}(t), \end{aligned} \tag{29}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state, $\hat{z}(t) \in \mathbb{R}^m$ is an estimate for $z(t)$, and F and G are state estimator parameters to be determined.

Define the state estimation error $e(t)$ and the output estimation error $e_z(t)$ in (5) and (6). In terms of the state variable $e(t)$ and $x(t)$, the state equations describing the augmented system which is formed from (1) and (29) are as follows

$$\begin{cases} \dot{x}_e(t) = (A_e+\Delta A_e)x_e(t)+A_{de}x_e(t-d_1)+ \\ \qquad C_{de}x_e(t-d_2)+B_e w(t), \\ e_z(t) = C_e x_e(t), \end{cases} \tag{30}$$

where

$$\begin{cases} A_e = \begin{bmatrix} F & A-GC-F \\ 0 & A \end{bmatrix}, \\ \Delta A_e = \begin{bmatrix} M_1-GM_2 \\ M_1 \end{bmatrix}\Gamma[0 \quad N] =: M_e\Gamma N_e, \end{cases} \tag{31a}$$

$$A_{de} = \begin{bmatrix} 0 & A_d \\ 0 & A_d \end{bmatrix},\ C_{de} = \begin{bmatrix} 0 & -GC_d \\ 0 & 0 \end{bmatrix}, \tag{31b}$$
$$B_e = \begin{bmatrix} D_1-GD_2 \\ D_1 \end{bmatrix},\ C_e = [L \quad 0].$$

Theorem 4 Consider the system (1) with unknown time-delays d_1 and d_2, and suppose there exist symmetric positive definite matrices $Q_1>0, Q_2>0, \Pi_1>0, \Pi_2>0, \Sigma_1>0$ and $\Sigma_2>0$, and a positive constant $\alpha>0$ such that

$$A_1Q_1+Q_1A_1^T+\frac{1}{\gamma^2}Q_1L^TLQ_1+R_1+$$
$$Q_1\Pi_1Q_1+A_d\Pi_2^{-1}A_d^T+Q_1\Sigma_1Q_1-(Q_1C_1^T+$$
$$R_{12})(R_2+C_d\Sigma_2^{-1}C_d^T)^{-1}(Q_1C_1^T+R_{12})^T<0, \tag{32}$$

$$A^TQ_2+Q_2A+\frac{1}{\alpha}N^TN+Q_2R_1Q_2+$$
$$\Pi_2+Q_2A_d\Pi_2^{-1}A_d^TQ_2+\Sigma_2<0, \tag{33}$$

where

$$\begin{cases} A_1 = A+(A_d\Pi_2^{-1}A_d^T+R_1)Q_2, \\ C_1 = C+R_{12}^TQ_2,\ R_1 = D_1D_1^T+\alpha M_1M_1^T, \\ R_2 = D_2D_2^T+\alpha M_2M_2^T,\ R_{12} = D_1D_2^T+\alpha M_1M_2^T, \end{cases} \tag{34}$$

then for all admissible perturbation ΔA and ΔC, there exists the asymptotically stable state estimator (29) with

$$F = A_1-GC_1, \tag{35}$$

$$G = (Q_1C_1^T+R_{12})(R_2+C_d\Sigma_2^{-1}C_d^T)^{-1}, \tag{36}$$

such that $\|H(s)\|_\infty<\gamma$.

Proof The proof is similar to Theorem 3 and omitted.

Remark 3 In Theorem 4, the state estimator (29) does not contain the time delays, because time delays d_1 and d_2 are unknown. This causes more conservative in estimation process. Appropriately selecting the positive definite matrices Π_1, Π_2, Σ_1 and Σ_2 may reduce the conservativeness.

5 Numerical example

Consider continuous linear time-invariant system with time delays and parameter uncertainties described by (1) with

$$A = \begin{bmatrix} -3 & -2 \\ 1 & -5 \end{bmatrix},\ A_d = \begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},\ C_d = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.8 \end{bmatrix},\ D_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.9 \end{bmatrix},\ L = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$$
$$\Delta A = M_1\Gamma N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}\Gamma\begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix},$$
$$\Delta C = M_2\Gamma N = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}\Gamma\begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix},$$

where Γ is an uncertain matrix with $\Gamma\Gamma^T \leqslant I$. We wish to design an asymptotically stable state estimator such that $\|H(s)\|_\infty<\gamma$, where $\gamma=0.5$. First, selecting

$$\alpha = 0.1,$$
$$\Pi_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \Pi_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \Sigma_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

a) When d_1 and d_2 are known, then solving algebraic Riccati Ineqs. (16) and (15), we obtain

$$Q_2 = \begin{bmatrix} 1.9749 & -0.5374 \\ -0.5374 & 0.5212 \end{bmatrix},\ Q_1 = \begin{bmatrix} 0.0097 & 0.0015 \\ 0.0015 & 0.0120 \end{bmatrix}.$$

From (18), (19) and (4), the state estimator is given by

$$\dot{\hat{x}}(t) =$$
$$\begin{bmatrix} -3.7623 & -1.9908 \\ 0.9464 & -6.3401 \end{bmatrix}\hat{x}(t)+\begin{bmatrix} 0.2 & 0 \\ 0 & 0 \end{bmatrix}\hat{x}(t-d_1)+$$
$$\begin{bmatrix} 0.7863 & -0.0069 \\ -0.0109 & 0.7036 \end{bmatrix}\left[y(t)-\begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}\hat{x}(t-d_2)\right],$$
$$\hat{z}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\hat{x}(t).$$

b) When d_1 and d_2 are unknown, then solving algebraic Riccati Ineqs. (33) and (32), we obtain

$$Q_2 = \begin{bmatrix} 1.9749 & -0.5374 \\ -0.5374 & 0.5212 \end{bmatrix},\ Q_1 = \begin{bmatrix} 0.1248 & -0.0029 \\ -0.0029 & 0.0502 \end{bmatrix}.$$

From (35), (36) and (29), the state estimator is given by

$$\dot{\hat{x}}(t)=\begin{bmatrix}-2.7690 & -2.2036\\ 0.7809 & -5.2726\end{bmatrix}\hat{x}(t)+\begin{bmatrix}0.6566 & -0.0167\\ -0.0167 & 0.2541\end{bmatrix}y(t),$$

$$\hat{z}(t)=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\hat{x}(t).$$

Therefore, through this numerical example, we have shown how to design the state estimators for the systems with known time delays and unknown time delays. Although the state estimators designed by the proposed approach may be more conservative, it is demonstrated that they can guarantee the prescribed H_∞ performance.

6 Conclusion

The problem of robust H_∞ state estimation for linear systems with time delays and parameter perturbations has been investigated in this paper. For known state delay and measurement delay systems, the state estimator with time delays has been constructed to guarantee the prescribed H_∞ performance in terms of two algebraic Riccati inequalities. For unknown state delay and measurement delay systems, the state estimator without time-delay has been constructed to guarantee the prescribed H_∞ performance in terms of two algebraic Riccati inequalities. They can be solved via Riccati equation approach or LMI technique that is similar to the one used in the solution of H_∞ estimation of a corresponding process without time delay. The approach proposed in this paper can be easily extended to the case of discrete-time systems.

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