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# Synthesis of Petri nets controller for discrete event systems based on finite capacity places - Part 2

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**Abstract**: The basic idea behind FCP (finite capacity places) method has been introduced in Part 1. In this sequel, the synthesis algorithm of Petri net controller for DES to enforce the general case of the constraint is presented and the maximal permissiveness of FCP method is proved. In addition, the reported example of AGV (automated guided vehicles) coordination system is used to illustrate the advantages and characteristics of FCP method.

**Key words**: discrete event systems; controller synthesis; Petri nets; finite capacity places

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# 基于有限容量库所的离散事件系统的 Petri 网控制器综合——第 2 部分

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**摘要**: FCP 方法的基本思想已在第一部分作了介绍.第二部分给出了离散事件系统在最一般情况下约束的 Petri 网控制器的设计方法,并证明 FCP 方法是最大容许控制的.此外,已有文献里的自动导航车辆协调系统的例子将被用来说明 FCP 方法所具有的优点和特点.

**关键词**: 离散事件系统; 控制器综合; Petri 网; 有限容量库所

## 1 Introduction

In [1], several algorithms of synthesizing PN controller to enforce their corresponding special form of linear inequality constraint have been introduced. In this sequel, the general algorithms are developed, in which the algorithms in [1] are included. The maximal permissiveness of the FCP method is also proved. The advantages and the characteristics of the method are illustrated by the AGV coordination system example.

In this paper the notation and terminology of [1] will be used freely.

## 2 The general case

In [1], the plant we treated is an ordinary Petri nets and the coefficients of the constrained places are Boolean variable. In this section, we will investigate the controller synthesis method when the plant is a generalized PN and the coefficients of the constrained places may be any integers. Besides, the assumption that the constrained places have no common input and/or output places is also removed. The principal idea we utilized in the procedure of the controller design is still the technique of complementary place.

### 2.1 'Less than or equal to' constraint

This case is expressed in the same form as Ineq.(1) in [1], which is rewritten in the following:

$$\sum_{i=1}^{n} l_i \mu_i \leqslant b, \qquad (1)$$

where the coefficient $l_i$ may be any integer and the constrained places may have common input and/or output transitions.

Given the constraint (1), the desired controller can be synthesized by using following algorithm.

**Algorithm 3**

1) Evaluate the related sets $C_p, {}^{\circ}C_p, C_p{}^{\circ}, CC_t, {}^{\circ}C_{\text{pure-}t}$

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and $C^{\circ}{}_{\text{pure-}t}$.

2) For each $t \in {}^{\circ}C_{\text{pure-}t}$, draw an arc between the controller place $p_c$ and the transition $t$, the weight function $w$ of the arc satisfies

$$w = |\omega| \tag{2}$$

and

$$\omega = \sum_{i=1}^{n} w(t,p_i) \times l_i, \tag{3}$$

where $p_i \in t^{\circ}$, $p_i \in C_p$ and $|\omega|$ denotes the absolute value of $\omega$. If $\omega > 0 (\omega < 0)$, let $p_c$ be the input (output) place of $t$ and if $\omega = 0$, there is no arc between $p_c$ and $t$ at all.

3) For each $t \in C^{\circ}{}_{\text{pure-}t}$, draw an arc between the controller place $p_c$ and the transition $t$, the weight function $w$ of the arc satisfies

$$w = |\omega| \tag{4}$$

and

$$\omega = \sum_{i=1}^{n} w(p_i,t) \times l_i, \tag{5}$$

where $p_i \in {}^{\circ}t$ and $p_i \in C_p$. If $\omega > 0$ $(\omega < 0)$, let $p_c$ be the output (input) place of $t$ and if $\omega = 0$, there is no arc between $p_c$ and $t$ at all.

4) For each $t \in CC_t$, draw an arc between the controller place $p_c$ and the transition $t$, the weight function $w$ of the arc satisfies

$$w = |\omega| \tag{6}$$

and

$$\omega = \sum_{i=1}^{n} w(p_i,t) \times l_i - \sum_{j=1}^{n} w(t,p_j) \times l_j, \tag{7}$$

where $p_i \in {}^{\circ}t, p_j \in t^{\circ}, p_i$ and $p_j \in C_p$. If $\omega > 0$ $(\omega < 0)$, let $p_c$ be the output (input) place of $t$ and if $\omega = 0$, there is no arc between $p_c$ and $t$ at all.

5) According to the following equation, calculate the initial marking of $p_c$, that is,

$$M_0(p_c) = b - \sum_{i=1}^{n} l_i \mu_{i0}. \tag{8}$$

**Remark 1** Algorithm 1 of [1] is the special cases of Algorithm 3. Algorithm 1 is designed for the case when the plant is an ordinary plant and the coefficient $l_i$ is a Boolean variable, thus $\omega$ is equal to 1 both in Step 2 and Step 3 in Algorithm 3. Besides, $\omega$ in Step 4 equals 0 since we assumed the constrained places had no common input or output transitions in Algorithm 1.

## 2.2 'Greater than or equal to' constraint

This case is expressed in the same form as Ineq.(12) in [1], which is rewritten in the following:

$$\sum_{i=1}^{n} l_i \mu_i \geqslant b, \tag{9}$$

where the coefficient $l_i$ may be any integer.

Given the constraint (9), the desired controller can be synthesized by using the following algorithm.

**Algorithm 4**

1) Evaluate the related sets $C_p, {}^{\circ}C_p, C_p{}^{\circ}, CC_t, {}^{\circ}C_{\text{pure-}t}$ and $C^{\circ}{}_{\text{pure-}t}$.

2) For each $t \in {}^{\circ}C_{\text{pure-}t}$, draw an arc between the controller place $p_c$ and the transition $t$, the weight function $w$ of the arc satisfies

$$w = |\omega| \tag{10}$$

and

$$\omega = \sum_{i=1}^{n} w(t,p_i) \times l_i, \tag{11}$$

where $p_i \in t^{\circ}$ and $p_i \in C_p$. If $\omega > 0$ $(\omega < 0)$, let $p_c$ be the output (input) place of $t$ and if $\omega = 0$, there is no arc between $p_c$ and $t$ at all.

3) For each $t \in C^{\circ}{}_{\text{pure-}t}$, draw an arc between the controller place $p_c$ and the transition $t$, the weight function $w$ of the arc satisfies

$$w = |\omega| \tag{12}$$

and

$$\omega = \sum_{i=1}^{n} w(p_i,t) \times l_i, \tag{13}$$

where $p_i \in {}^{\circ}t$ and $p_i \in C_p$. If $\omega > 0$ $(\omega < 0)$, let $p_c$ be the input (output) place of $t$ and if $\omega = 0$, there is no arc between $p_c$ and $t$ at all.

4) For each $t \in CC_t$, draw an arc between the controller place $p_c$ and the transition $t$, the weight function $w$ of the arc satisfies

$$w = |\omega| \tag{14}$$

and

$$\omega = \sum_{i=1}^{n} w(p_i,t) \times l_i - \sum_{j=1}^{n} w(p_j,t) \times l_j, \tag{15}$$

where $p_i \in {}^{\circ}t, p_j \in t^{\circ}, p_i$ and $p_j \in C_p$. If $\omega > 0$ $(\omega < 0)$, let $p_c$ be the input (output) place of $t$ and if $\omega = 0$, there is no arc between $p_c$ and $t$ at all.

5) According to the following equation, calculate the initial marking of $p_c$, that is,

$$M_0(p_c) = \sum_{i=1}^{n} l_i \mu_{i0} - b. \tag{16}$$

**Remark 2** Algorithm 4 can be reduced to Algorithm 2 of [1] when the specified conditions are satisfied.

## 3 Maximal permissiveness

In this section, we will prove that the synthesis method is the maximally permissive control policy for the linear inequality. The results obtained by Yamalidou et al. [2] will be used in the following proof.

For simplicity, we only consider the constraint (1) in [1] and the corresponding Algorithm 1 presented in [1]. The notations such as $D_p$, $D_c$ and $D$ used here have the same meanings as that in [2], i.e., $D_p$, $D_c$ and $D$ represent the incidence matrices of the plant, the controller net and the controlled net, respectively. $D_p$ is an $n \times m$ matrix, with $n$ being the number of places and $m$ the number of transitions of the plant. $D_c$ is an $m$-dimensional row vector and

$$D = \begin{bmatrix} D_p \\ D_c \end{bmatrix}.$$

We assume that there are $s\,(s \leqslant n)$ entries in the constrained place set $C_p$ and these constrained places are denoted by $p_v\,(1 \leqslant v \leqslant s)$. It is always possible because we can denote the plant net again to satisfy the above assumption if its places are not denoted like this. We also assume that the common constrained transition set $CC_t$, pure output constrained transition set $C^{\circ}{}_{\text{pure-}t}$ and pure input constrained transition set ${}^{\circ}C_{\text{pure-}t}$ have $i$, $j$ and $k$ entries, respectively. These transitions are denoted by $t_w\,(1 \leqslant w \leqslant i)$, $t_x\,(i+1 \leqslant x \leqslant i+j)$ and $t_y\,(i+j+1 \leqslant y \leqslant i+j+k)$, respectively. Because the constrained place $p_v$ may have a couple of pure output and pure input constrained transitions, we also assume the place $p_v$ have $r_v\,(1 \leqslant r_v \leqslant j)$ and $h_v\,(1 \leqslant h_v \leqslant k)$ pure output and pure input constrained transitions, respectively, where $r_v$ and $h_v$ satisfy the following equations, respectively

$$\sum_{v=1}^{s} r_v = j \tag{17}$$

and

$$\sum_{v=1}^{s} h_v = k. \tag{18}$$

With the above assumptions, the incidence matrix $D_p$ of the plant is then has form of (19), where $l = n - s$.

Before we start to explain the matrix $D_p$, we denote it with partitioned matrix, that is,

$$D_p = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix},$$

$$D_1 = [D_{11} \quad D_{12} \quad D_{13} \quad D_{14}],$$

where $D_{11}$, $D_{12}$, $D_{13}$ and $D_{14}$ are $s \times i$, $s \times j$, $s \times k$ and $s \times (m - i - j - k)$ matrices, respectively, and $D_2$ is an $(n - s) \times m$ matrix. $D_{11}$ represents the connection between the constrained places and the common constrained transitions. Recall the assumption in Section 3.1 of [1] that there are no common input and common output transitions of constrained places, so the entries in every column are all zero except for two having the value of 1 and $-1$. Note that the two non-zero entries may be any arbitrary entries in the column. Again as the assumption, the only non-zero entry in the columns of $D_{12}$ and $D_{13}$ is also equal to $-1$ or 1. Note that the non-zero entries are placed together in each row due to the intentional index of the related transitions. It is easy to know that $D_{14}$ is a zero-matrix since there is no arc between the constrained places and the transitions $t_z\,(i+j+k+1 \leqslant z \leqslant m)$. The entries denoted with $*$ in $D_2$ have no relation with our problem.

$$D_p = \begin{array}{c} \begin{array}{ccccccc} \overbrace{\hspace{3em}}^{i} & \overbrace{\overbrace{\hspace{5em}}^{r_1}\hspace{4em}\overbrace{\hspace{3em}}^{r_s}}^{j} & \overbrace{\overbrace{\hspace{3em}}^{h_1}\hspace{4em}\overbrace{\hspace{3em}}^{h_s}}^{k} & \hspace{5em} \end{array} \\ \underbrace{\left[\begin{array}{ccccccccccccccccccc} 1 & 1 & -1 & \cdots & -1 & & 0 & \cdots & 0 & 1 & \cdots & 1 & & 0 & \cdots & 0 & 0 & \cdots & 0 \\ -1 & 0 & 0 & \cdots & 0 & & \vdots & & & 0 & \cdots & 0 & & \vdots & & & & & \\ 0 \cdots & \vdots & \vdots & & & \cdots & & & \vdots & \vdots & & & \cdots & & & \vdots & \vdots & & \vdots \\ \vdots & 0 & & & \vdots & & 0 & \cdots & 0 & & & \vdots & & 0 & \cdots & 0 & & & \\ 0 & -1 & 0 & \cdots & 0 & & -1 & \cdots & -1 & 0 & \cdots & 0 & & 1 & \cdots & 1 & 0 & \cdots & 0 \\ * & & & & & & & & \cdots & & & & & & & & & & * \\ \vdots & & & & & & & & & & & & & & & & & & \vdots \\ * & & & & & & & & \cdots & & & & & & & & & & * \end{array}\right]}_{m} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \\ \\ \end{array}\right\} s \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} l \end{array} \end{array}. \tag{19}$$

According to the Algorithm 1 in [1], we have the matrix

$$D_c = [\underbrace{0 \cdots 0}_{i} \ \underbrace{1 \cdots 1}_{j} \ \underbrace{-1 \cdots -1}_{k} \ \underbrace{0 \cdots 0}_{m-i-j-k}].$$

Note that $D_c = -[\underbrace{1 \cdots 1}_{s} \ \underbrace{0 \cdots 0}_{n-s}] \times D_p$. Then, we point out that the column vector

$$x^{\mathrm{T}} = [\underbrace{1 \cdots 1}_{s} \ \underbrace{0 \cdots 0}_{n-s} \ 1]$$

is the place invariant of the controlled system because the following equation holds

$$x^{\mathrm{T}} \times D = x^{\mathrm{T}} \times \begin{bmatrix} D_p \\ D_c \end{bmatrix} = 0,$$

where 0 is an $m$-dimensional zero-vector. Note that the support $P(x)$ of the place invariant $x$ is

$$P(x) = \{p = p_i \mid 1 \leqslant i \leqslant s, i \in \mathbb{N}\} \cup \{p_c\}, \tag{20}$$

i.e., the entries of the $P(x)$ are the constrained places and the controller place.

It was said in [2] that the place invariants of the uncontrolled plant are also invariants of the controlled plant for any Petri net control scheme that only adds places and arcs in order to control the plant. For FCP method, assume $x_p$ is a place invariant of the uncontrolled plant, then

$$x_p^{\mathrm{T}} \times D_p = 0$$

and

$$[x_p^{\mathrm{T}} \quad 0] \times D = [x_p^{\mathrm{T}} \quad 0] \times \begin{bmatrix} D_p \\ D_c \end{bmatrix} = 0$$

hold, i.e., $x_p$ is also a place invariant of the controlled plant. The place invariant numbers of the uncontrolled and controlled plant are $n$-rank $D_p$ and $n+1$-rank $D$, i.e., $n+1$-rank $D_p$, respectively. Thus the above vector $x$ is the only place invariant of the controlled systems as a result of the control law based on the concept of finite capacity places. Finally, since there are no new or unexpected invariants forced on the controlled system as a result of the control algorithm, which is the same as P-invariant[2], we can draw a conclusion that the control method presented in this paper is maximally permissive.

**Remark 3** The above proof procedure on Algorithm 1 in [1] can be applied to the other algorithms. What needs to be modified is the incidence matrix of the controlled system $D$ that is composed of $D_c$ and $D_p$.

## 4 Examples

In this section, the example of AGV coordination system is used to illustrate the FCP method. The example originally appeared in [3] and has been studied intensively in the area of DES [2, 4, 5]. We consider the example here so that the characteristics and the advantages of FCP of various methods can be better illustrated. For more examples, the reader may refer to [6].

The AGV example is about a flexible manufacturing cell, in which there are three workstations, two part-receiving stations, and one completed parts station. In this example, five AGV's transport material between pairs of stations pass through zones shared by other AGV's. The shared zones are shown as shaded regions in the PN model of Fig. 1[3]. The flow of tokens that model the part and the vehicles represents the evolution of the actual state of the system. Our aim is to design a maximally permissive controller to ensure that at the most one vehicle is permitted in each zone at any time.

The control objective can be written as the following constraints

$$\sum_{i \in Z_1} \mu_i \leqslant 1, \tag{21a}$$

$$\sum_{i \in Z_2} \mu_i \leqslant 1, \tag{21b}$$

$$\sum_{i \in Z_3} \mu_i \leqslant 1, \tag{21c}$$

$$\sum_{i \in Z_4} \mu_i \leqslant 1, \tag{21d}$$

where $Z_j (j = 1,2,3,4)$ is the set of indices of places which make up zone $j$.

For constraint (21a), it has the form of inequality (1) in [1]. So the Algorithm 1 can be used to design the controller. The constrained places are $p_1, p_2, p_3$ and $p_4$, that is, $C_p = \{p_1, p_2, p_3, p_4\}$. The corresponding input and output constrained transition sets are: ${}^{\circ}C_p = \{t_2, t_3, t_6, t_8\}$ and $C_p{}^{\circ} = \{t_1, t_4, t_5, t_7\}$, respectively, and two equations ${}^{\circ}C_{\text{pure-}t} = {}^{\circ}C_p$ and $C^{\circ}{}_{\text{pure-}t} = C_p{}^{\circ}$ hold since $CC_t = \phi$. Then, we draw the arcs as shown in Fig. 1 between the transitions in ${}^{\circ}C_{\text{pure-}t}$ and $C^{\circ}{}_{\text{pure-}t}$ and the controller place $C_1$. We know that the initial marking of $C_1$ is 1 according to equation (7) in [1]:

$$b - \sum_{i=1}^{n} l_i \mu_i = b - (\mu_1 + \mu_2 + \mu_3 + \mu_4) = 1.$$

The other controller places $C_i(i = 2,3,4)$ associated to the corresponding constraints in (21) can be synthesized with the same algorithm and the detailed procedure is omitted. The entire controlled PN constituted with the original plant and the additional controller net is illustrated in Fig. 1.

The controlled net obtained using FCP method is identical to that of Yamalidou et al based on $P$-invariant[2]. But it seems that FCP method is simpler than $P$-invariant method. We think that FCP method has advantage over the one of $P$-invariant because the incidence matrix and the operation on it, which play an important role in the calculation of $P$-invariant, can be avoided by FCP method. The advantage is obvious when the uncontrolled system is large and complex and thus the incidence matrix has high dimension and a large number of entries. Consider the AGV example again. The PN illustrated in Fig. 2 has 64 places and 53 transitions and consequently its incidence matrix is a 64 × 53-dimension matrix and has more than 3000 entries. The computation of the matrix itself as well as the operation on it is hard and prone to make mistake.

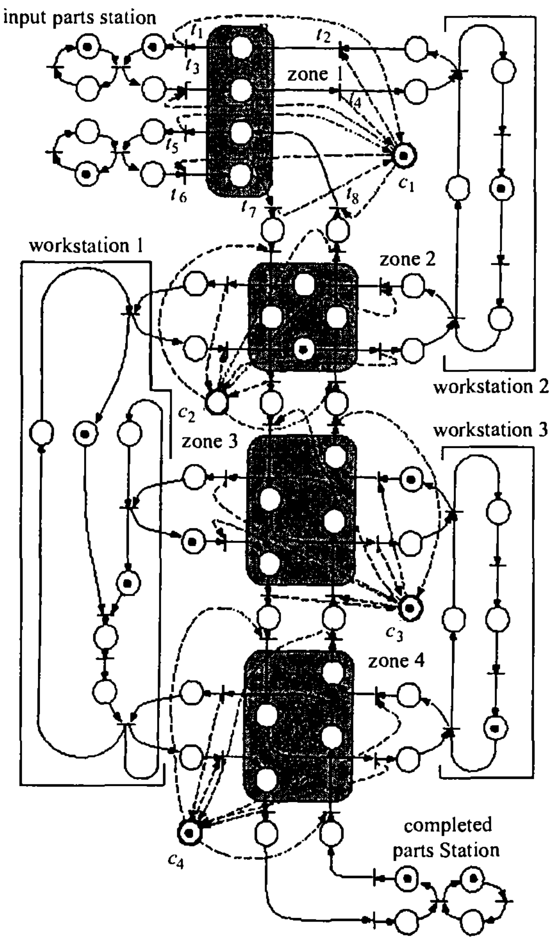


Fig. 1 The controlled PN of AGV system

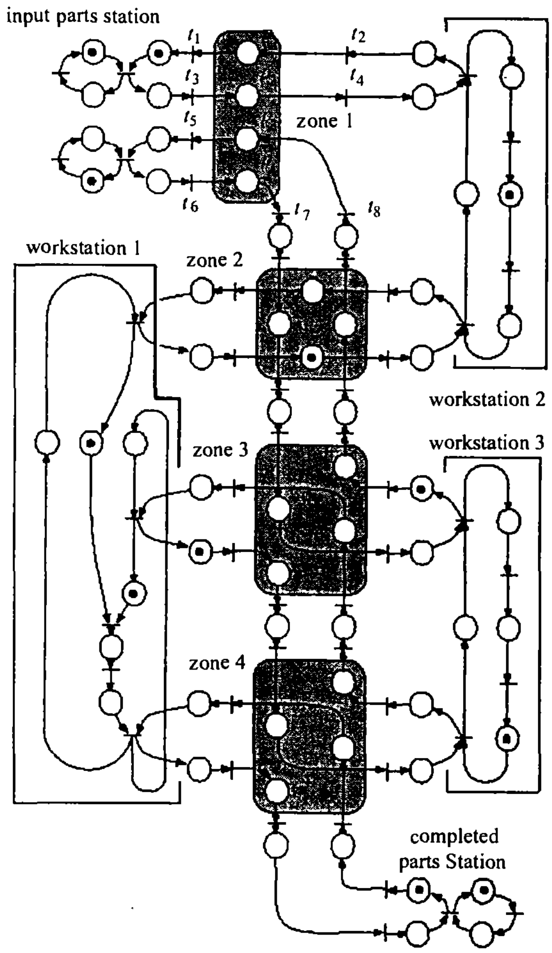


Fig. 2 The PN graph of AGV example

One may argue that $P$-invariant method only involves one incidence matrix while the FCP method needs to evaluate several place and transition sets. But the evaluation of these sets is rather easy and the entries involved in the sets are far fewer than that in the incidence matrix though there are several sets needed to be evaluated. The constrained place set $C_p$ is already known and we define such a set just for the sake of convenience of description. The sets of ${}^{\circ}C_p$, $C_p{}^{\circ}$ and $CC_t$, can be obtained via observing the nodes (places and transitions) connection of the net, and ${}^{\circ}C_{\text{pure-}t}$ and $C^{\circ}{}_{\text{pure-}t}$ are evaluated through a simple operation of set subtraction. Furthermore, few nodes are involved in the set evaluation. For the AGV example, the control objective is written as four similar linear inequality constraints. Every constraint has four constrained places and these places have four input and four output transitions, respectively, i.e., each of the sets of $C_p$, $^{\circ}C_p$ and $C_p{}^{\circ}$ has four entries. The equations $^{\circ}C_{\text{pure-}t} = {}^{\circ}C_p$, and $C^{\circ}_{\text{pure-}t} = C_p{}^{\circ}$ hold in this example since the set $CC_t$ is null. So there are only 20 entries (4 places and 16 transitions) for one constraint and 80 for all the constraints involved in the evaluation process. Note that we have to evaluate more than 3000 entries in order to compute the incidence matrix by using $P$-invariant method. We note that the larger and the more complex the plant is, the higher dimension the incidence matrix has, and the more obvious the advantage of FCP method is.

## 5 Conclusions

This paper has discussed the FCP method of synthesizing PN controller for DES with general case of linear inequality defined on the place marking. It has been proved in this paper that the control law determined by FCP method is maximally permissive. The example shows that the result obtained via FCP method is the same as the one obtained by using $P$-invariant method. However, FCP method has again a considerable improvement in overcoming the obstacle of computational complexity due to the avoidance of incidence matrix and the operation on it. Still, it is not easy to obtain the desired controller by exploiting $P$-invariant method when the plant is a large system, whose incidence matrix has a high dimension. This is illustrated sufficiently by the AGV example.

The presented synthesis method possesses all the characteristics of the method based on $P$-invariant. First, the design of the controller need no state enumeration, so the computational complexity of controller synthesis can be greatly reduced. Second, the control logic of this approach is included as part of the controlled net, that is, the controller net and the plant form a closed loop. Third, the method can be applied to the DES modeled by any generalized and untimed Petri nets.

FCP method is capable of dealing with nets with uncontrollable transitions in some cases. We note that there is no necessity to modify the FCP method itself when it is applied to the system in which some transitions are uncontrollable or unobservable. What is needed to do is seek an equivalent constraint set, e.g., cat and mouth problem[6]. FCP method is also applicable to the constraints involving firing vector elements only if the constraints can be transformed into the linear making inequality constraint. It can be expected that in the future research, efforts will be made to find an efficient method of constraint transformation.

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