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Kind of optimal investment strategy for dynamic alliance in the fuzzy sense

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Abstract: A kind of effective fuzzy investment decision making model for mutually complementary enterprises in a DA (dynamic alliance) or VE(virtual enterprises) is submitted, which computes total gross profits and coefficients of profits and simultaneously considers enterprises' relativity and investment incentive measure. An algorithm to the practical problem based on fuzzy optimizing technique is then analyzed stress in the fuzzy sense, the method can be used for the DA to obtain the maximum total gross profits from competitive and cooperative integrated market. Some key steps for implementating the decision making scheme are proposed too. Then, some examples illustrate the utility of the suggested method. It provides useful reference model for enterprises in a DA or VE to realize scientific optimal investment decision making strategy.

Key words: dynamic alliance (DA); optimal investment strategy; gross profits; fuzzy optimizing technique; mutually complementary enterprises

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动态联盟的一种模糊最优投资策略

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摘要: 针对动态联盟或虚拟企业中两个互补型合作企业,提出了一种模糊投资决策模型,该模型同时考虑了企业的相关性和鼓励投资的激励措施;应用模糊优化技术对该模型进行求解,确定了获得最大总利润的最优投资策略;并用计算实例证明了该方法的有效性.为虚拟企业进行科学的投资决策,提供了有益的借鉴.

关键词: 动态联盟; 最优投资策略; 利润; 模糊优化技术; 互补型企业

1 Introduction

Faced with the new integrated market economy competitive environment, enterprises must adjust themselves to new situation. Dynamic alliance (DA) is a kind of effective form for enterprises' competition and cooperation[1]. DA needs some scheme to make proper decision for its project investment[2]. For this purpose, a kind of investment decision making model in the crisp case is proposed. In order to tackle practical instances, the parameters in the model be fuzzified; an optimal investment strategy is then dealt with stress; and finally, some examples are given to show the validity of the suggested method.

2 Decision making model of investment

We are now discussing the DA's best investments I_1, I_2 of two mutually complementary enterprises X_1, X_2. In a monopoly market, for example, two enterprises X_1, X_2 are in the same industry consisting a DA. X_1, X_2 share the same resources and market, and they cooperate and compete with each other[3]. Hence if the investment of X_1 is too high, then the coefficient of profits of X_2 will decrease and vice verse; meanwhile an incentive measure is introduced to encourage the enterprises in the alliance invest as much as possible. Therefore, suppose the investment of X_1, X_2 are I_1, I_2, and the coefficients of profits are x_1, x_2, then we have:

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$$\begin{cases} x_1 = a_1 + a_2 I_1 - a_3 I_2, \\ x_2 = b_1 - b_2 I_1 + b_3 I_2, \\ 0 \leqslant I_1 \leqslant b_1/b_2,\ 0 \leqslant I_2 \leqslant a_1/a_3, \end{cases} \quad (1)$$

where $a_j > 0, b_j > 0, j = 1,2,3$; and a_j, b_j are known.

In order to ensure the success of the alliance, assume that I_1, I_2 are restricted at a certain scope:

$$\begin{cases} 0.5 - \varepsilon \leqslant I_1/(I_1 + I_2) \leqslant 0.5 + \varepsilon, \\ 0.5 - \varepsilon \leqslant I_2/(I_1 + I_2) \leqslant 0.5 + \varepsilon, \\ 0 < \varepsilon < 0.5, \end{cases} \quad (2)$$

where ε is known. Then the total gross profits are:

$$P(I_1, I_2) = x_1 I_1 + x_2 I_2 =$$
$$a_1 I_1 + a_2 I_1^2 + b_1 I_2 + b_3 I_2^2 - (a_3 + b_2) I_1 I_2,$$
$$0 \leqslant I_1 \leqslant b_1/b_2,\ 0 \leqslant I_2 \leqslant a_1/a_3. \quad (3)$$

But in a prefect competitive market, for the same investment I_1, I_2, it is not necessary to have the same coefficients of profits x_1 and x_2. We shall consider fuzzy coefficient instead[4,5]. Let $\tilde{x}_1, \tilde{x}_2$ denote fuzzy profit coefficients, the fuzzy total gross profits be $\tilde{P} = \tilde{x}_1 I_1 + \tilde{x}_2 I_1$, the problem can be described as:

$$\max \tilde{P} = \max (\tilde{x}_1 I_1 + \tilde{x}_2 I_2), \quad (4)$$

$$\text{s.t.} \begin{cases} \tilde{x}_1 = \tilde{A}_1 + \tilde{A}_2 I_1 - \tilde{A}_3 I_2, \\ \tilde{x}_2 = \tilde{B}_1 - \tilde{B}_2 I_1 + \tilde{B}_3 I_2, \end{cases} \quad (5)$$

where $\tilde{A}_j, \tilde{B}_j, j = 1,2,3$; are triangular fuzzy numbers:

$$\begin{cases} \tilde{A}_j = (a_j - \delta_{j1}, a_j, a_j + \delta_{j2}), \\ \tilde{B}_j = (b_j - \delta_{j3}, b_j, b_j + \delta_{j4}), \end{cases} j = 1,2,3, \quad (6)$$

where $0 < \delta_{j1} < a_j, \delta_{j2} > 0, 0 < \delta_{j3} < b_j, \delta_{j4} > 0, j = 1,2,3$, are known.

3 The optimal investment strategy of DA

3.1 Fuzzy operation property

From [6], let $\tilde{U}, \tilde{V}$ tbe the two fuzzy numbers denoted by $\tilde{U} = (u_1, u_2, u_3), \tilde{V} = (v_1, v_2, v_3)$. Their centroids are:

$$\begin{cases} \int_{-\infty}^{\infty} u\mu_{\tilde{U}}(u)\mathrm{d}u \Big/ \int_{-\infty}^{\infty} \mu_{\tilde{U}}(u)\mathrm{d}u = \frac{1}{3}(u_1 + u_2 + u_3), \\ \int_{-\infty}^{\infty} v\mu_{\tilde{V}}(u)\mathrm{d}v \Big/ \int_{-\infty}^{\infty} \mu_{\tilde{V}}(v)\mathrm{d}v = \frac{1}{3}(v_1 + v_2 + v_3). \end{cases} \quad (7)$$

Property 1 Let $\tilde{U} = (u_1, u_2, u_3), \tilde{V} = (v_1, v_2, v_3)$ be two triangular fuzzy numbers, $k_j \in (-\infty, \infty)$, $j = 1,2,3$. Suppose k_1, k_2 do not equal 0 simultaneously, then the fuzzy number $\tilde{y} = k_1 \tilde{U} + k_2 \tilde{V} + k_3$ is

$$\tilde{y} = \begin{cases} (k_1 u_1 + k_2 v_1 + k_3, k_1 u_2 + k_2 v_2 + k_3, k_1 u_3 + k_2 v_3 + k_3),\ \forall k_1 > 0, k_2 \geqslant 0; k_1 \geqslant 0, k_2 > 0, \\ (k_1 u_1 + k_2 v_3 + k_3, k_1 u_2 + k_2 v_2 + k_3, k_1 u_3 + k_2 v_1 + k_3),\ \forall k_1 > 0, k_2 \leqslant 0; k_1 \geqslant 0, k_2 < 0, \\ (k_1 u_3 + k_2 v_1 + k_3, k_1 u_2 + k_2 v_2 + k_3, k_1 u_1 + k_2 v_3 + k_3),\ \forall k_1 < 0, k_2 \geqslant 0; k_1 \leqslant 0, k_2 > 0, \\ (k_1 u_3 + k_2 v_3 + k_3, k_1 u_2 + k_2 v_2 + k_3, k_1 u_1 + k_2 v_1 + k_3),\ \forall k_1 < 0, k_2 \leqslant 0; k_1 \leqslant 0, k_2 < 0. \end{cases} \quad (8)$$

3.2 Optimal investment strategy

For the problem in section 2, let

$$\begin{cases} s_j(\delta_{j1}, \delta_{j2}) = a_j + 1/3(\delta_{j2} - \delta_{j1}) = \\ 2/3 a_j + 1/3 \delta_{j2} + 1/3(a_j - \delta_{j1}) > 0, \\ t_j(\delta_{j3}, \delta_{j4}) = b_j + 1/3(\delta_{j4} - \delta_{j3}) = \\ 2/3 b_j + 1/3 \delta_{j4} + 1/3(b_j - \delta_{j3}) > 0. \end{cases} \quad (9)$$

The centroids of $\tilde{A}_j, \tilde{B}_j$ are

$$M_{\tilde{A}_j} = a_j + 1/3(\delta_{j2} - \delta_{j1}) = s_j(\delta_{j1}, \delta_{j2}),\ j = 1,2,3,$$
$$M_{\tilde{B}_j} = b_j + 1/3(\delta_{j4} - \delta_{j3}) = t_j(\delta_{j3}, \delta_{j4}),\ j = 1,2,3.$$

If there are no changes in x_1, x_2, then $\delta_{j2} = \delta_{j1} = 0$, $\delta_{j4} = \delta_{j3} = 0, j = 1,2,3$, i.e. $M_{\tilde{A}_j} = a_j, M_{\tilde{B}_j} = b_j, j = 1,2,3$. By property 1 and (5),(6), since $I_j \geqslant 0, j = 1,2$, therefore

$$\tilde{x}_1 = [a_1 - \delta_{11} + (a_2 - \delta_{21})I_1 - (a_3 + \delta_{32})I_2,\ a_1 + a_2 I_1 - a_3 I_2,\ a_1 + \delta_{12} + (a_2 + \delta_{22})I_1 - (a_3 - \delta_{31})I_2],$$
$$\tilde{x}_2 = [b_1 - \delta_{13} - (b_2 + \delta_{24})I_1 + (b_3 - \delta_{33})I_2,\ b_1 - b_2 I_1 + b_3 I_2,\ b_1 + \delta_{14} - (b_2 - \delta_{23})I_1 + (b_3 + \delta_{34})I_2].$$

Let

$$\begin{aligned} g_1 &= t_1(\delta_{13}, \delta_{14})/t_2(\delta_{23}, \delta_{24}), \\ g_2 &= s_1(\delta_{11}, \delta_{12})/s_3(\delta_{31}, \delta_{32}). \end{aligned} \quad (10)$$

By (9), we have $g_j > 0, j = 1,2$. In the crisp case, $\delta_{ij} = 0, \delta_{i,j+2} = 0, i = 1,2,3; j = 1,2$; then by (10), we have $g_1 = b_1/b_2, g_2 = a_1/a_3$. In the crisp case, we have $0 \leqslant I_j \leqslant g_j, j = 1,2$, accordingly.

The centroids of $\tilde{x}_1, \tilde{x}_2$ are

$$M_{\tilde{x}_1}(I_1, I_2) = s_1(\delta_{11}, \delta_{12}) + s_2(\delta_{21}, \delta_{22})I_1 - s_3(\delta_{31}, \delta_{32})I_2,$$

$M_{\tilde{x}_2}(I_1, I_2) =$

$t_1(\delta_{13}, \delta_{14}) - t_2(\delta_{23}, \delta_{24}) I_1 + t_3(\delta_{33}, \delta_{34}) I_2$

respectively. These are the estimates of coefficients of profits x_1, x_2 in fuzzy sense. Here is a special case:

$$M_{\tilde{x}_1}(I_1, I_2) = a_1 + a_2 I_1 - a_3 I_2,$$
$$\forall \delta_{j2} = \delta_{j1},\ j = 1,2,3;$$
$$M_{\tilde{x}_2}(I_1, I_2) = b_1 - b_2 I_1 + b_3 I_2,$$
$$\forall \delta_{j4} = \delta_{j3},\ j = 1,2,3.$$

This coincides with crisp case. If $\delta_{ij} = 0, i = 1,2,3; j = 1,2,3,4$; then $\tilde{a}_j \equiv a_j, \tilde{b}_j \equiv b_j$. It is just a crisp case. The fuzzy total gross profits are

$$\tilde{P} = \tilde{x}_1 I_1 + \tilde{x}_2 I_2 =$$
$$\tilde{A}_1 I_1 + \tilde{A}_2 I_1^2 + \tilde{B}_1 I_2 + \tilde{B}_3 I_2^2 - (\tilde{A}_3 + \tilde{B}_2) I_1 I_2.$$

Let

$$P_1 = (a_1 - \delta_{11}) I_1 + (a_2 - \delta_{21}) I_1^2 + (b_1 - \delta_{13}) I_2 + (b_3 - \delta_{33}) I_2^2 - (a_3 + \delta_{32} + b_2 + \delta_{24}) I_1 I_2,$$
$$P_2 = a_1 I_1 + a_2 I_1^2 + b_1 I_2 + b_3 I_2^2 - (a_3 + b_2) I_1 I_2,$$
$$P_3 = (a_1 + \delta_{12}) I_1 + (a_2 + \delta_{22}) I_1^2 + (b_1 + \delta_{24}) I_2 + (b_3 + \delta_{34}) I_2^2 - (a_3 - \delta_{31} + b_2 - \delta_{23}) I_1 I_2.$$

By Property 1 and the fact that $I_j \geqslant 0, j = 1,2$; we have $\tilde{P} = (P_1, P_2, P_3)$. For $0 \leqslant I_j \leqslant g_j, j = 1,2$, the centroid of $\tilde{P}$ is

$$M_{\tilde{P}}(I_1, I_2) = 1/3(P_1 + P_2 + P_3) = s_1(\delta_{11}, \delta_{12}) I_1 + s_2(\delta_{21}, \delta_{22}) I_1^2 + t_1(\delta_{13}, \delta_{14}) I_2 + t_3(\delta_{33}, \delta_{34}) I_2^2 - [s_3(\delta_{31}, \delta_{32}) + t_2(\delta_{23}, \delta_{24})] I_1 I_2. \quad (11)$$

This is the estimate of P in the fuzzy sense.

It is obvious that

$$M_{\tilde{P}}(I_1, I_2) = M_{\tilde{x}_1}(I_1, I_2) I_1 + M_{\tilde{x}_2}(I_1, I_2) I_2,$$

when

$$\delta_{j2} = \delta_{j1}, \delta_{j4} = \delta_{j3},\ j = 1,2,3;$$

(11) becomes

$$M_{\tilde{P}}(I_1, I_2) = a_1 I_1 + a_2 I_1^2 + b_1 I_2 + b_3 I_2^2 - (a_3 + b_2) I_1 I_2,$$

which coincides with (3), i.e. $M_{\tilde{P}}(I_1, I_2) = P(I_1, I_2)$. When $\delta_{ij} = 0, i = 1,2,3, j = 1,2,3,4$, the fuzzy sense turns out to be a crisp case.

3.2.1 In the fuzzy sense

Since the crisp case is a special case in the fuzzy sense, we shall first work on the problem in fuzzy sense.

Objective function $M_{\tilde{P}}(I_1, I_2)$ is a continuous function for I_1, I_2 in the closed region $0 \leqslant I_j \leqslant g_j, j = 1,2$. The absolute maximum value does exist either at the point in the open region $0 < I_1 < g_1, 0 < I_2 < g_2$, where both the partial derivatives exist, equal to zero and have solutions which meet the requirements for the relative maximum region $\{(I_1, I_2) \mid 0 < I_1 < g_1, 0 < I_2 < g_2\}$ or on the boundary of $0 \leqslant I_j \leqslant g_j, j = 1,2$. But in conjunction with (2), the restrict conditions should become

$$\begin{cases} (0.5 - \varepsilon)/(0.5 + \varepsilon) g_2 \leqslant I_1 \leqslant \\ \min(g_1, (0.5 + \varepsilon)/(0.5 - \varepsilon) g_2), \\ (0.5 - \varepsilon)/(0.5 + \varepsilon) g_1 \leqslant I_2 \leqslant \\ \min(g_2, (0.5 + \varepsilon)/(0.5 - \varepsilon) g_1), \end{cases} \quad (12)$$

where ε is know.

1) In the open region.

$$\frac{\partial}{\partial I_1}(M_{\tilde{P}}(I_1, I_2)) = s_1(\delta_{11}, \delta_{12}) + 2 s_2(\delta_{21}, \delta_{22}) I_1 - [s_3(\delta_{31}, \delta_{32}) + t_2(\delta_{23}, \delta_{24})] I_2 = 0, \quad (13)$$

$$\frac{\partial}{\partial I_2}(M_{\tilde{P}}(I_1, I_2)) = t_1(\delta_{13}, \delta_{14}) - [s_3(\delta_{31}, \delta_{32}) + t_2(\delta_{23}, \delta_{24})] I_1 + 2 t_3(\delta_{33}, \delta_{34}) I_2 = 0. \quad (14)$$

Let

$$W = 4 s_2(\delta_{21}, \delta_{22}) t_3(\delta_{33}, \delta_{34}) - [s_3(\delta_{31}, \delta_{32}) + t_2(\delta_{23}, \delta_{24})]^2,$$

then

$$\begin{vmatrix} \frac{\partial^2}{\partial I_1^2} M_{\tilde{P}}(I_1, I_2) & \frac{\partial^2}{\partial I_1 \partial I_2} M_{\tilde{P}}(I_1, I_2) \\ \frac{\partial^2}{\partial I_1 \partial I_2} M_{\tilde{P}}(I_1, I_2) & \frac{\partial^2}{\partial I_2^2} M_{\tilde{P}}(I_1, I_2) \end{vmatrix} = W,$$

if $W > 0$, $M_{\tilde{P}}(I_1, I_2)$ it gets minimum at some points in open region; if $W \leqslant 0$, $M_{\tilde{P}}(I_1, I_2)$ it has no extremum in the open region. Therefore $M_{\tilde{P}}(I_1, I_2)$ reaches maximum on the boundary of (12) in both cases.

2) On the boundary.

It is easy to know there are no maximum on the lower boundary of $\lim I_i = 0^+$, so the maximum occurs on the upper boundary of

$$I_1 = \min(g_1, (0.5 + \varepsilon)/(0.5 - \varepsilon) g_2) = \bar{I}_1 = C_1,$$

or

$$I_2 = \min(g_2, (0.5 + \varepsilon)/(0.5 - \varepsilon) g_1) = \bar{I}_2 = C_2.$$

On the boundary $\bar{I}_2 = C_2$ (constant), I_1 vary, then

$$(0.5-\varepsilon)/(0.5+\varepsilon)g_2 \leqslant I_1 \leqslant \min(g_1,(0.5+\varepsilon)/(0.5-\varepsilon)g_2),$$

from (13),

$$\frac{\mathrm{d}}{\mathrm{d}I_1}M_{\tilde{P}}(I_1,I_2) = s_1(\delta_{11},\delta_{12}) + 2s_2(\delta_{21},\delta_{22})I_1 - [s_3(\delta_{31},\delta_{32}) + t_2(\delta_{23},\delta_{24})]C_2 = 0,$$

$$\frac{\mathrm{d}^2}{\mathrm{d}I_1^2}M_{\tilde{P}}(I_1,I_2) = 2s_2(\delta_{21},\delta_{22}) > 0,$$

therefore, when $I_1 = [-s_1(\delta_{11},\delta_{12}) + (s_3(\delta_{31},\delta_{32}) + t_2(\delta_{23},\delta_{24}))C_2]/2s_2(\delta_{21},\delta_{22}) = E$,

$$\max M_{\tilde{P}}(I_1,I_2) = M_{\tilde{P}}(I_{(1)1},I_{(1)2}) = M_{\tilde{P}}(I_{(1)1},C_2) = \begin{cases} M_{\tilde{P}}(I_{1(2)},C_2), & \forall E < I_{1(1)}, \\ \max(M_{\tilde{P}}(I_{1(1)},C_2), M_{\tilde{P}}(I_{1(2)},C_2)), & \forall I_{1(1)} \leqslant E \leqslant I_{1(2)}, \\ M_{\tilde{P}}(I_{1(1)},C_2), & \forall E > I_{1(2)}, \end{cases} \tag{15}$$

where

$$I_{1(1)} = (0.5-\varepsilon)/(0.5+\varepsilon)g_2,$$
$$I_{1(2)} = \min(g_1,(0.5+\varepsilon)/(0.5-\varepsilon)g_2),$$
$$I_{(1)2} = C_2 = \min(g_2,(0.5+\varepsilon)/(0.5-\varepsilon)g_1),$$
$$E = [-s_1(\delta_{11},\delta_{12}) + (s_3(\delta_{31},\delta_{32}) + t_2(\delta_{23},\delta_{24}))C_2]/2s_2(\delta_{21},\delta_{22}).$$

The same principle, on boundary $\bar{I}_1 = C_1$, the result is:

$$\max M_{\tilde{P}}(I_1,I_2) = M_{\tilde{P}}(I_{(2)1},I_{(2)2}) = M_{\tilde{P}}(C_1,I_{(2)2}) = \begin{cases} M_{\tilde{P}}(C_1,I_{2(2)}), & \forall F < I_{2(1)}, \\ \max(M_{\tilde{P}}(C_1,I_{2(1)}), M_{\tilde{P}}(C_1,I_{2(2)})), & \forall I_{2(1)} \leqslant F \leqslant I_{2(2)}, \\ M_{\tilde{P}}(C_1,I_{2(1)}), & \forall F > I_{2(2)}, \end{cases} \tag{16}$$

where

$$I_{2(1)} = (0.5-\varepsilon)/(0.5+\varepsilon)g_1,$$
$$I_{2(2)} = \min(g_2,(0.5+\varepsilon)/(0.5-\varepsilon)g_1),$$
$$I_{(2)1} = C_1 = \min(g_1,(0.5+\varepsilon)/(0.5-\varepsilon)g_2),$$
$$F = [-t_1(\delta_{13},\delta_{14}) + (s_3(\delta_{31},\delta_{32}) + t_2(\delta_{23},\delta_{24}))C_1]/2t_3(\delta_{33},\delta_{34}).$$

From (15) and (16), a theorem can be obtained.

Theorem 1 (In the fuzzy sense) If I_1, I_2 are the investment of two mutually complementary enterprises in an alliance X_1, X_2, respectively, the coefficients of profits of X_1, X_2 are

$$\tilde{x}_1 = \tilde{A}_1 + \tilde{A}_2 I_1 - \tilde{A}_3 I_2,$$
$$\tilde{x}_2 = \tilde{B}_1 - \tilde{B}_2 I_1 + \tilde{B}_3 I_2,$$
$$0 \leqslant I_1 \leqslant g_1,\ 0 \leqslant I_2 \leqslant g_2.$$
$$\text{s.t.}\ (0.5-\varepsilon)/(0.5+\varepsilon)g_2 \leqslant I_1 \leqslant \min(g_1,(0.5+\varepsilon)/(0.5-\varepsilon)g_2),$$
$$(0.5-\varepsilon)/(0.5+\varepsilon)g_1 \leqslant I_2 \leqslant \min(g_2,(0.5+\varepsilon)/(0.5-\varepsilon)g_1),$$

where $\tilde{A}_j, \tilde{B}_j, j = 1,2,3$; in according with (6), let $s_j(\delta_{j1},\delta_{j2}), t_j(\delta_{j3},\delta_{j4}), j = 1,2,3$; defined by (9),

$$g_1 = t_1(\delta_{13},\delta_{14})/t_2(\delta_{23},\delta_{24}),$$
$$g_2 = s_1(\delta_{11},\delta_{12})/s_3(\delta_{31},\delta_{32}),$$

then

$$\max M_{\tilde{P}}(I_1,I_2) = \max_{j=1,2} M_{\tilde{P}}(I_{(j)1},I_{(j)2}) \equiv M_{\tilde{P}}(I_1^*,I_2^*).$$

3.2.2 In crisp sense

It is easy to extend the Theorem 1 in the fuzzy sense to crisp case; the latter is a special case of the former.

Theorem 2 (In crisp sense) The same result as fuzzy sense can be used directly. i.e.

$$\max P(I_1,I_2) = \max_{j=1,2} P(I_{(j)1}^0,I_{(j)2}^0) \equiv P(I_1^{**},I_2^{**}).$$

4 Example

4.1 In the crisp case

Example 1 If x_1, x_2 are the coefficients of profits of two mutually complementary enterprises X_1, X_2 in an alliance, respectively. The expressions is:

$$x_1 = 0.3 + 0.05I_1 - 0.02I_2,$$
$$x_2 = 0.42 - 0.03I_1 + 0.06I_2,$$
$$\varepsilon = 0.3.$$

i.e.

$$a_1 = 0.3, a_2 = 0.05, a_3 = 0.02;$$
$$b_1 = 0.42, b_2 = 0.03, b_3 = 0.06;$$
$$g_1 = b_1/b_2 = 14, g_2 = a_1/a_3 = 15.$$

By (12), $3.75 \leqslant I_1 \leqslant 14, 3.5 \leqslant I_2 \leqslant 15$, By (15), $E = b_2/2a_2 \times g_2 = 4.5, 3.75 \leqslant E \leqslant 14$, so

$$\max P(I_1,I_2) = P(I_{(1)1}^0,I_{(1)2}^0) = \max(P(3.75,15),P(14,15)) = 23.3.$$

By (16), $F = a_3/2b_3 \times g_1 = 2.333 < 3.5$, so

$$\max P(I_1,I_2) = P(I_{(2)1}^0,I_{(2)2}^0) = P(14,15) = 23.3.$$

Therefore,

$\max P(I_1, I_2) =$

$\max (P(I_{(1)1}^0, I_{(1)2}^0),\ P(I_{(2)1}^0, I_{(2)2}^0)) = 23.3.$

4.2 In the fuzzy sense

Example 2 The other conditions are similar to Example 1, let

$$\delta_{12} - \delta_{11} = 0.02,\ \delta_{22} - \delta_{21} = -0.01,$$
$$\delta_{32} - \delta_{31} = 0.02;\ \delta_{14} - \delta_{13} = -0.02,$$
$$\delta_{24} - \delta_{23} = 0.03,\ \delta_{34} - \delta_{33} = -0.03;$$
$$0 < \delta_{j1} < a_j,\ 0 < \delta_{j2},$$
$$0 < \delta_{j3} < b_j,\ 0 < \delta_{j4}, j = 1,2,3.$$

By Theorem 1, from (9),

$$s_1(\delta_{11}, \delta_{12}) = 0.30667,$$
$$s_2(\delta_{21}, \delta_{22}) = 0.04667,$$
$$s_3(\delta_{31}, \delta_{32}) = 0.02667;$$
$$t_1(\delta_{13}, \delta_{14}) = 0.41333,$$
$$t_2(\delta_{23}, \delta_{24}) = 0.04,$$
$$t_3(\delta_{33}, \delta_{34}) = 0.05.$$

From (10), $g_1 = 10.333$, $g_2 = 11.499$. By (12),

$$2.875 \leqslant I_1 \leqslant 10.333,$$
$$2.583 \leqslant I_2 \leqslant 11.499.$$

By (15), $E = 4.928$, $2.875 < E < 10.333$, so

$\max M_{\tilde{P}}(I_1, I_2) =$

$M_{\tilde{P}}(I_{(1)1}, I_{(1)2}) = M_{\tilde{P}}(I_{(1)1}, C_2) =$

$\max (M_{\tilde{P}}(I_{(1)1}, C_2), M_{\tilde{P}}(I_{(1)2}, C_2)) = 11.551.$

By (16), $F = 2.756$, $2.583 < F < 11.499$, so

$\max M_{\tilde{P}}(I_1, I_2) =$

$M_{\tilde{P}}(I_{(2)1}, I_{(2)2}) = M_{\tilde{P}}(C_1, I_{(2)2}) =$

$\max (M_{\tilde{P}}(C_1, I_{(2)1}), M_{\tilde{P}}(C_1, I_{(2)2})) = 11.551.$

Therefore

$\max M_{\tilde{P}}(I_1, I_2) =$

$\max (M_{\tilde{P}}(I_{(1)1}, I_{(1)2}), (M_{\tilde{P}}(I_{(2)1}), I_{(2)2})) = 11.551.$

It is obvious that the situation in fuzzy sense is quite different from the crisp case, so investment strategy should be adjusted according to different situations.

5 Conclusions

1) The main achievements made in this paper are as follows: A kind of investment decision making model for enterprises of a DA is proposed; a kind of optimal investment strategy for DA to obtain maximum profits utilizing fuzzy optimizing technique based on the given model in the fuzzy sense is provided; and the utility of the described method is illustrated.

2) The following subjects should be paid more attention: The robustness and sensitivity analysis of the parameters or structure in the model; the consideration of constrains to each coefficient of profits; and the case more than two enterprises in a DA.

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