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Decentralized control for composite systems with input saturation

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Abstract: The problem of decentralized stabilization for composite systems with input saturation is studied. *M*-matrix method is used to investigate the problem and a sufficient condition for the composite systems with input saturation to be stabilized by using decentralized linear state feedback control is derived. Moreover, a simpler sufficient condition for the symmetric composite systems with input saturation to be decentralized stabilized is also given.

Key words: decentralized control; composite systems; input saturation; stabilization; algebraic Riccati equation; *M*-matrix; symmetric composite systems

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具有输入饱和的组合系统的分散控制

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摘要: 研究了一类具有饱和输入的组合系统的分散可镇定问题. 运用 *M*-矩阵方法, 通过线性分散状态反馈得出具有饱和输入组合系统的可镇定的充分条件. 对于具有对称结构的组合系统, 给出了更为简单的镇定条件.

关键词: 分散控制; 组合系统; 饱和输入; 镇定; 代数 Riccati 方程; *M*-矩阵; 对称组合系统

1 Introduction

Input saturation is a common feature of control systems. Recently, linear systems with input saturation have been intensively studied. For example, the global asymptotical stabilization for such systems is studied by using nonlinear feedback laws in [1], and the semiglobal stabilization for the systems by using linear feedback laws is studied in [2]. However, little attention has been paid so far to the decentralized stabilization for composite systems with input saturation.

This paper will study the decentralized stabilization for composite systems with input saturation. By using the *M*-Matrix method, a sufficient condition for the system to be decentralized stabilized is derived using linear state feedback.

The second part of this paper considers a class of large-scale systems which were called symmetric composite systems by Lunze[3]. Symmetric composite systems are encountered in electric power systems, industrial manipulators, computer networks, etc. See [3 ~ 5] for other examples and references. Because of the special structure of symmetric composite systems, many analysis and design problems for them can be simplified[3 ~ 5]. This paper will show that the decentralized stabilization for symmetric composite systems with input saturation can also be simplified.

2 Preliminaries

First introduce the definition of saturation function.

Definition 1 A vector-valued function $\sigma:\mathbb{R}^m \to \mathbb{R}^m$ is called a saturation function if

$$\sigma(s) = [\mathrm{sat}_1(s_1), \mathrm{sat}_2(s_2), \cdots, \mathrm{sat}_m(s_m)]^{\mathrm{T}},$$

where

$$\mathrm{sat}_j(s_j) = \begin{cases} -s_{j0}, & \text{if } s_j < -s_{j0}, \\ s_j, & \text{if } |s_j| \leqslant s_{j0}, \\ -s_{j0}, & \text{if } s_j > s_{j0}, \end{cases}$$

$$(s_{j0} > 0, j = 1, 2, \cdots, m).$$

Remark 1 It can be easily seen that for a saturation function $\sigma:\mathbb{R}^m \to \mathbb{R}^m$, there is

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$$\left|\frac{1}{2}s_j - \text{sat}_j(s_j)\right| \leqslant \frac{1}{2}|s_j|,$$
$$\forall s_j \in \mathbb{R},\ \forall j = 1,2,\cdots,m.$$

Hence $\left\|\frac{1}{2}s - \sigma(s)\right\| \leqslant \frac{1}{2}\|s\|$, $\forall s \in \mathbb{R}^m$. where $\|\cdot\| = \|\cdot\|_2$.

We also need the definition and properties of M-matrix which were given in [6].

Lemma 1[6] Let $W = [w_{ij}] \in \mathbb{R}^{N\times N}$ be a real square matrix with nonpositive off-diagonal elements. Then the following conditions are mutually equivalent:

C1) The principal minors of W are all positive.

C2) The leading principal minors of W are all positive.

C3) There is a vector x (or y) whose elements are all positive such that the elements of Wx (or $W^{\mathrm{T}}y$) are all positive.

C4) W is nonsingular and the elements of W^{-1} are all non-negative.

C5) There exist N numbers $d_1, d_2, \cdots, d_N$ with $d_i > 0, i = 1,2,\cdots,N$, such that

$$d_i w_{ii} > \sum_{j=1,j\neq i}^{N} d_j |w_{ij}|, i = 1,2,\cdots,N.$$

C6) The real parts of the eigenvalues of W are all positive.

Definition 2[6] If a real square matrix with non-positive off-diagonal elements W satisfies one of the conditions in Lemma 1, then it is called an M-matrix.

Lemma 2[6] A real square matrix with non-positive off-diagonal elements W is an M-matrix if and only if there exists a diagonal matrix with positive diagonal elements D, such that $W^{\mathrm{T}}D + DW$ is a symmetric positive definite matrix.

3 Problem statement

Consider the system composed of N subsystems

$$\dot{x}_i = A_i x_i + \sum_{j=1,j\neq i}^{N} A_{ij}x_j + B_i\sigma_i(u_i), \qquad (1)$$
$$i = 1,2,\cdots,N,$$

where $x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}$ are vectors of the subsystem state, control input, respectively. $A_i \in \mathbb{R}^{n_i\times n_i}, B_i \in \mathbb{R}^{n_i\times m_i}$ and $\sigma_i: \mathbb{R}^{m_i} \to \mathbb{R}^{m_i} (i = 1,2,\cdots,N)$ are all saturation functions, $A_{ij} \in \mathbb{R}^{n_i\times n_j} (i \neq j)$ are the interconnective matrices.

This paper will study the following problem for system (1).

Decentralized stabilization problem: Find, if possible, a decentralized linear state feedback

$$u_i = K_i x_i,\ i = 1,2,\cdots,N, \qquad (2)$$

such that

S1) The ith closed-loop subsystem

$$\dot{x}_i = A_i x_i + B_i\sigma_i(K_i x_i), \qquad (3)$$

satisfies that the point $x_i = 0$ is uniformly asymptotically stable.

S2) The overall closed-loop system

$$\dot{x}_i = A_i x_i + \sum_{j=1,j\neq i}^{N} A_{ij}x_j + B_i\sigma_i(K_i x_i), \qquad (4)$$
$$i = 1,2,\cdots,N$$

satisfies that the point $x_i = 0$ is uniformly asymptotically stable.

In this paper, we will also consider the decentralized stabilization problem for the following special composite system

$$\dot{x}_i = A_1 x_i + \sum_{j=1,j\neq i}^{N} A_{12}x_j + B_1\sigma_i(u_i), \qquad (5)$$
$$i = 1,2,\cdots,N,$$

where $x_i \in \mathbb{R}^{n_1}, u_i \in \mathbb{R}^{m_1}, A_1, A_{12} \in \mathbb{R}^{n_1\times n_1}, B \in \mathbb{R}^{n_1\times m_1}$ and $\sigma_i: \mathbb{R}^{m_1} \to \mathbb{R}^{m_1} (i = 1,2,\cdots,N)$, are saturation functions. That is

$$n_1 = n_2 = \cdots = n_N,\ m_1 = m_2 = \cdots = m_N,$$
$$A_1 = A_2 = \cdots = A_N,\ B_1 = B_2 = \cdots = B_N$$

and $A_{ij} = A_{12}$ for all $i \neq j$ in (1).

Remark 2 We will refer to the system (5) as a symmetric composite system with input saturation. Symmetric composite systems are used widely in practice. For example, A multimachine power system consists of a large number of similar power generators feeding a network of loads, which are symmetrically interconnected. Symmetric composite systems without input saturation has been studied in [3 ~ 5].

4 Main results

In this paper, $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalues of the matrix, which are written in the brackets, respectively.

For the ith subsystem, choose positive definite matrices $Q_i \in \mathbb{R}^{n_i\times n_i}, R_i \in \mathbb{R}^{m_i\times m_i}$.

Let P_i be the unique positive definite solution of the algebraic Riccati equation.

$$A_i^{\mathrm{T}}P_i + P_iA_i - P_iB_iR_i^{-1}B_i^{\mathrm{T}}P_i + Q_i = 0.$$

Denote

$$r_i = \lambda_m(Q_i) - \| P_iB_i \| \; \| R_i^{-1}B_i^{\mathrm{T}}P_i \| .$$

The following theorem gives a sufficient condition under which the decentralized stabilization problem for system (1) can be solved.

Theorem 1 If there exist positive definite matrices $Q_i \in \mathbb{R}^{n_i \times n_i}, R_i \in \mathbb{R}^{m_i \times m_i} (i = 1,2,\cdots,N)$ such that the matrix $W = [w_{ij}]$ definited by

$$w_{ij} = \begin{cases} \dfrac{r_i}{2\lambda_M(P)}, & i = j, \\ -\| A_{ij} \|, & i \neq j \end{cases} \tag{6}$$

is an M-matrix. Then the decentralized linear state feedback

$$u_i = - R_i^{-1}B_i^{\mathrm{T}}P_ix_i \tag{7}$$

satisfies S1) and S2).

In the following, we will study the decentralized stabilization for symmetric composite systems with input saturation.

Because of the special structure of the system (5), the following simpler sufficient condition for it to be decentralized stabilized can be obtained.

Theorem 2 The decentralized stabilization problem for the system (5) is solvable if there exist two positive definite matrices $Q_1 \in \mathbb{R}^{n_1 \times n_1}$ and $R_1 \in \mathbb{R}^{m_1 \times m_1}$ such that

$$\frac{r_1}{2\lambda_M(P_1) \| A_{12} \|} > N - 1,$$

where $r_1 = \lambda_m(Q_1) - \| P_1B_1 \| \; \| R_1^{-1}B_1^{\mathrm{T}}P_1 \|$. In this case, the stabilizable decentralized state feedback control law is

$$u_i = - R_1^{-1}B_1^{\mathrm{T}}P_1x_i, \; i = 1,2,\cdots,N.$$

5 Conclusion

In this paper, we obtained a sufficient condition for the composite systems with input saturation to be stabilized by using decentralized linear state feedback control. Because of the special structure of symmetric composite systems, the condition becomes quite simple.

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