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Tuning of a modified Smith predictor for processes with time delay

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Abstract: A modified Smith predictor proposed in Majhi and Atherto (1999) is shown to be equivalent to a modified internal model control (IMC) structure, and then a three-stage design method is proposed for the Smith predictor. To achieve compromise between disturbance rejection and stability robustness, a robust control method is used to tune the feedback-loop controller. Design for some typical integrating and unstable processes with time delay shows that the proposed method can achieve good compromise between setpoint tracking, disturbance rejection and stability robustness.

Key words: Smith predictor; unstable and integrating processes; H_∞ control; robust tuning; robust stability and performance

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时滞过程改进型 Smith 预估器的整定

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摘要: 证明 Majhi 和 Atherton(1999)文所提出的改进型 Smith 预估器等价于一改进的内模控制结构(IMC),并对该结构提出一种三阶段设计方法.为获得扰动抑制和稳定鲁棒性的均衡,采用了鲁棒控制方法来整定反馈环控制器.针对某些典型的积分和不稳定时滞过程的设计表明所提方法能获得较好的扰动抑制和稳定鲁棒性的均衡.

关键词: Smith 预估器; 不稳定与积分过程; H_∞ 控制; 鲁棒整定; 鲁棒稳定及性能

1 Introduction

It is well-known that the Smith predictor is an effective control structure for processes with long time delay. However, due to internal stability issue, it cannot be applied to unstable processes. Many efforts have been made to extend the Smith predictor to unstable and integrating processes, e.g., [1, 2]. Astrom, Hang and Lim[3] proposed a modified Smith predictors for controlling processes with an integrator and long dead-time. But a number of tuning parameters is required. Matausek and Micic[4,5] proposed a deadtime compensator (DTC) structure which reduced the number of tuning parameters to three.

For unstable processes with time delay, another modified Smith predictor structure (Fig. 1) was proposed in [6], and a method based on the direct method was used to tune the three controllers. A common character of these structures is that the design of setpoint tracking and disturbance rejection can be done separately. It is also noted that the DTC structure in [5] is a special case of that in [6]. The latter can be applied to both integrating and unstable processes.

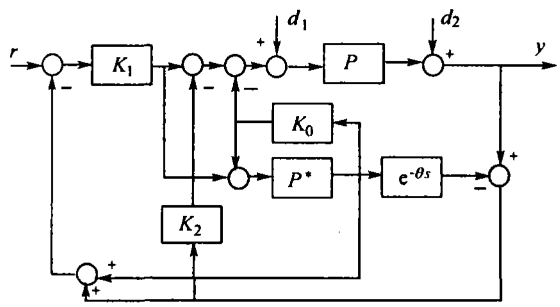

Fig. 1 Modified Smith predictor

In this paper we show that the modified Smith predictor proposed in [6] is equivalent to a modified internal model control (IMC) structure (Fig.2), so the setpoint tracking can be tuned by the well-known IMC tuning method[7]. To achieve compromise between disturbance

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rejection and stability robustness, we propose a method to tune the feedback-loop controller. By combining two tuning methods, good setpoint tracking, good disturbance rejection and good stability robustness can be achieved for the Smith predictor.

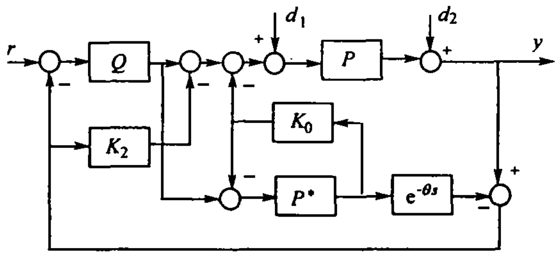


Fig. 2 Modified IMC structure

2 Modified Smith predictor

For the modified Smith predictor shown in Fig. 1, we have

$$y = \frac{PK_1(1 + P^* e^{-\theta s}K_2) r}{(1+P^* K_0+P^* K_1)(1+PK_2)+(P-P^* e^{-\theta s})K_1} + \frac{(1 + P^* K_0 + P^* K_1 - P^* e^{-\theta s}K_1)(Pd_1 + d_2)}{(1+P^* K_0+P^* K_1)(1+PK_2)+(P-P^* e^{-\theta s})K_1}. \tag{1}$$

If the model is perfect, i.e., $P = P^* e^{-\theta s}$, then

$$y = \frac{PK_1 r}{1+P^* K_0 + P^* K_1} + \left(1 - \frac{P^* e^{-\theta s}K_1}{1+P^* K_0 + P^* K_1}\right) \frac{Pd_1+d_2}{1+PK_2}. \tag{2}$$

Now consider input-output relation in the modified IMC structure shown in Fig. 2, where the symbols have the same meanings as those in Fig. 1, except now Q, K_0 and K_2 are controllers to be tuned. We have

$$y = \frac{PQ(1+P^* e^{-\theta s}K_2)r+(1+P^* K_0-P^* e^{-\theta s}Q)(Pd_1+d_2)}{(1+P^* K_0)(1+PK_2)+(P-P^* e^{-\theta s})Q}. \tag{3}$$

If the model is perfect, then

$$y = \frac{PQ}{1 + P^* K_0} r + \left(1 - \frac{P^* e^{-\theta s}Q}{1 + P^* K_0}\right) \frac{Pd_1 + d_2}{1 + PK_2}. \tag{4}$$

Let

$$G^* := \frac{P^*}{1 + P^* K_0},\ G := \frac{P}{1 + P^* K_0} = G^* e^{-\theta s}. \tag{5}$$

Obviously if

$$Q = \frac{K_1}{1 + G^* K_1}, \tag{6}$$

then the two structures are equivalent, and the input-output relation is simplified to

$$y = GQr + (1 - GQ) \frac{Pd_1 + d_2}{1 + PK_2}. \tag{7}$$

The equivalence is not surprising. For stable processes the relation has already been known, e.g., in the book of Morari and Zafiriou[7].

3 Controller tuning

By Eq. (7), the setpoint tracking is not related to K_2, so the setpoint tracking can be designed independently. By the analysis in the previous section, Q can be designed as an IMC controller for the stabilized model G. We note that an IMC controller inverts the invertible part of G^*, thus if K_0 does not introduce additional right-half-plane (RHP) zeros, its effect will be canceled by Q. So K_0 can be chosen arbitrarily as long as it stabilizes the delay-free part and does not introduce RHP zeros. If the process is (marginally) stable, then it can be set to zero. It is just for internal stability of the structure. The final performance of the system does not depend on it.

Once K_0 and Q are chosen, the other degree of freedom provided by K_2 can be used to improve the disturbance rejection of the closed loop system. Note that the transfer functions from $d_1(d_2)$ to y is

$$T_{yd_1} = (1 - GQ) \frac{P}{1 + PK_2}, \quad T_{yd_2} = (1 - GQ) \frac{1}{1 + PK_2}. \tag{8}$$

If P is unstable, clearly the response from d_1 to y will be unstable without K_2. To have good disturbance rejection, K_2 should stabilize the plant P and minimize the norms of T_{yd_1} and T_{yd_2}. Moreover, we require that the performance can be achieved robustly. Since model uncertainty usually occurs at high frequency and $1 - GQ \approx 1$ at high frequency (since it is the sensitivity function of an IMC structure without K_2), so robustness of the closed-loop system depends mainly on K_2. Now we need to consider the following problem:

$$\inf_{K_2} \| (1 + P_\Delta K_2)^{-1} \|_\infty, \tag{9}$$

where P_Δ is an uncertainty model of the plant.

Suppose the uncertainty is a multiplicative one, i.e.,

$$P_\Delta = P(1 + \Delta_S), \tag{10}$$

then

$$(1 + P_\Delta K_2)^{-1} = (1 + PK_2)^{-1} - (1 + PK_2)^{-1} P\Delta_S \cdot (1+K_2(1+PK_2)^{-1}P\Delta_S)^{-1}K_2(1+PK_2)^{-1}. \quad (11)$$

Define a new uncertainty structure

$$\Delta = \begin{bmatrix} \Delta_P & 0 \\ 0 & \Delta_S \end{bmatrix}, \quad (12)$$

where Δ_P is an imaginary block. By the main-loop theorem[8], robust performance defined in Eq.(9) is equivalent to

$$\inf_{K_2} \mu_\Delta(M), \quad (13)$$

where

$$M: = \begin{bmatrix} (1 + PK_2)^{-1} & (1 + PK_2)^{-1}P \\ K_2(1 + PK_2)^{-1} & K_2(1 + PK_2)^{-1}P \end{bmatrix}. \quad (14)$$

This is a μ-synthesis problem. It can be solved by *D-K* iteration. However, the procedure is complex. We can use a constant scale to compute the upper-bound of the structured singular value[8], i.e., to solve the following problem:

$$\inf_{K_2, \lambda_2} \left\| \begin{bmatrix} 1 & 0 \\ 0 & \lambda_2^{-1} \end{bmatrix} M \begin{bmatrix} 1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \right\|_\infty . \quad (15)$$

Here λ_2 can be thought of as a weighting between stability robustness and performance. Frequency-based scaling can also be used, but it will increase the order of the final controller. It can be shown that the problem in Eq.(15) is equivalent to a loop-shaping H_∞ design procedure[9] with $W_2 = 1$ and $W_1 = \lambda_2$.

In summary, we propose the following three-stage controller design procedure for the modified Smith predictor.

1) Delay-free part stabilizer. A controller K_0 is chosen to stabilize the delay-free part of the model, i.e., P^*. For stable and integrating processes K_0 can be chosen as zero.

2) Setpoint tracking. A controller Q is designed as an IMC controller for the stabilized model G. Then

$$K_1 = \frac{Q}{1 - G^* Q}.$$

3) Load disturbance rejection and robustness. A loop shaping H_∞ controller K_2 is designed to robustly stabilize the original delayed process P.

It is well-known that an IMC controller has one tuning parameter. It is easy to find out that in the proposed method we have essentially two tuning parameters: one for tuning the IMC controller, or K_1; and the other (λ_2) for tuning K_2. Detailed tuning procedure for some typical processes with time delay are illustrated in the next section.

4 Tuning for typical processes with time delay

4.1 An integrating process with time delay

For the integrating process

$$P(s) = \frac{k}{s(\tau s + 1)} e^{-\theta s}, \quad (16)$$

K_0 can be chosen as zero, and the IMC controller Q can be chosen as $Q = \frac{s(\tau s + 1)}{k(\lambda_1 s + 1)^2}$, where λ_1 is a tuning parameter. Then

$$K_1 = \frac{Q}{1 - G^* Q} = \frac{\tau s + 1}{k(\lambda_1^2 s + 2\lambda_1)}. \quad (17)$$

K_2 can be designed by solving H_∞ problem (15) with λ_2 chosen to make the infimum equal 2.5 for a compromise between robustness and disturbance attenuation. Using a curve fitting approach, we get the following tuning formula for K_2:

$$\begin{cases} K_2 = K_c(1 + T_c s), \\ K_c = \dfrac{1}{(1.33456\theta/\tau + 0.4455)k}, \\ T_c = (0.40136\theta + 0.5691\tau). \end{cases} \quad (18)$$

4.2 A process with an integrator and time delay

For the process

$$P(s) = \frac{k}{s} e^{-\theta s}, \quad (19)$$

K_0 can also be chosen as zero, and the IMC controller Q can be chosen as $Q = \frac{s}{k(\lambda_1 s + 1)}$. Then

$$K_1 = \frac{Q}{1 - G^* Q} = \frac{1}{k\lambda_1}. \quad (20)$$

For this process if we choose $\lambda_2 = 1.8/\tau$, we get the following tuning formula for K_2:

$$K_2 = \frac{0.7851}{\theta k} \frac{1 + 0.5\theta s}{1 + 0.09516\theta s}, \quad (21)$$

or we can approximate it with a PD controller with

$$K_2 = \frac{0.7851}{\theta k}(1 + 0.40496\theta s). \quad (22)$$

4.3 First-order unstable processes with time delay

For this process

$$P(s) = \frac{k}{\tau s - 1}e^{-\theta s}. \tag{23}$$

The delay-free part of the process is $P^* = \frac{k}{\tau s - 1}$. So

$$G^*(s) = \frac{k}{\tau s - 1 + kK_0}. \tag{24}$$

If K_0 is chosen to be larger than $1/k$, then G^* will be stable. For simplicity, we choose $K_0 = \frac{2}{k}$, which makes $G^*(s) = \frac{k}{\tau s + 1}$. Now an IMC controller Q can be chosen as $Q = \frac{\tau s + 1}{k(\lambda_1 s + 1)}$ and

$$K_1 = \frac{Q}{1 - G^* Q} = \frac{\tau}{k\lambda_1}(1 + \frac{1}{\tau s}). \tag{25}$$

Now we need to design K_2. It is known that the stabilizing proportional gain for a first-order delayed unstable process is bounded both below and above. We take the average value of the two bounds as the desired normalized loop gain (λ_2) and solve problem in Eq. (15) against the normalized delay. We can also approximate the optimal controller with a PD controller. Using a curve fitting approach, we get the following tuning formula for K_2 for a first-order unstable process with time delay:

$$K_2 = K_c(T_c s + 1), \tag{26}$$

where

$$K_c = \begin{cases} \frac{1}{k}\left(\frac{0.533\tau}{\theta} + 0.746\right), & \text{if } \theta/\tau \leq 0.7, \\ \frac{1}{k}\left(\frac{0.490\tau}{\theta} + 0.694\right), & \text{if } 0.7 < \theta/\tau \leq 1.5, \end{cases} \tag{27}$$

$$T_c = (0.426\theta/\tau - 0.014)\tau. \tag{28}$$

5 Examples

Example 1 Consider a process with transfer function

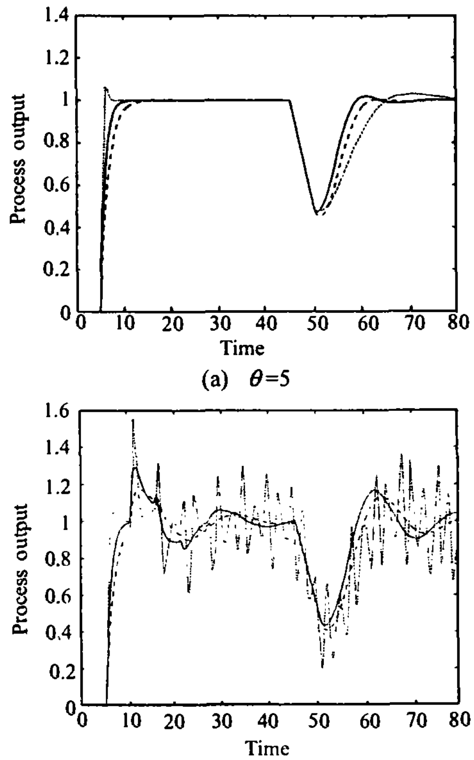
$$P(s) = \frac{1}{s}e^{-\theta s}, \tag{29}$$

where $\theta = 5$. The controller setting for the modified Smith predictor tuned by the proposed method ($\lambda_1 = 1$) is shown in Table 1. So are settings tuned in [6] and [1].

Table 1 Controller setting for Example 1

	K_0	K_1	K_2
Proposed	0	1	$0.157(1+2.205s)$
[5]	0	0.6	$0.1448\frac{1+2s}{1+0.2s}$
[6]	4.131	$0.1(1+\frac{1}{0.01s})$	0.105

The setpoint and load disturbance responses are shown in Fig.3(a). We see that the proposed controller setting has the best load response and the setpoint response is faster than that for [6] but slower than that for [1]. However, the setting in [1] is too aggressive as shown in Fig.3(b) where the delay has 10% uncertainty. The proposed setting has the best compromise between load disturbance rejection and stability robustness.



(a) θ=5

(b) θ increases by 10%

Fig. 3 Responses for Example 1

Example 2 Consider a process with transfer function

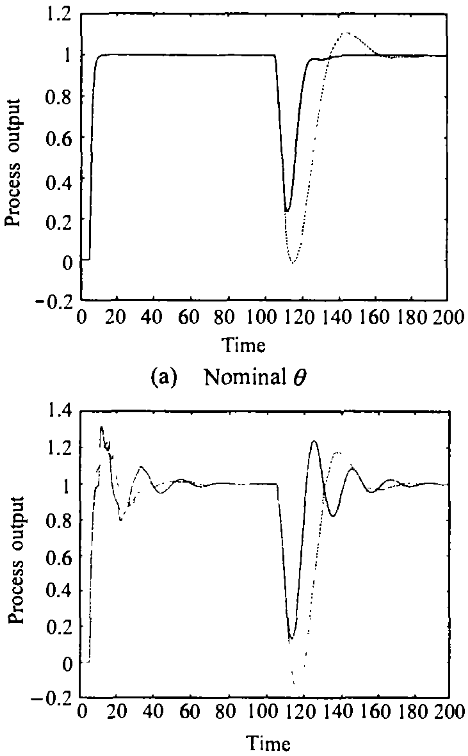
$$P(s) = \frac{1}{10s - 1}e^{-5s}. \tag{30}$$

The controller setting for the modified Smith predictor tuned by the proposed method ($\lambda_1 = 1$) is shown in Table 2. So is the setting tuned in [6].

Table 2 Controller settings for Example 2

	K_0	K_1	K_2
Proposed	2	$10(1+1/10s)$	$1.812(1+1.99s)$
[1]	13.468	$1+1/0.1s$	1.414

The setpoint and load disturbance responses for nominal delay and delay increased by 10% are shown in Fig.4. We see that the proposed controller setting has better load disturbance rejection than that by [6], since in the latter case K_2 is just a static gain.



(a) Nominal θ

(b) θ increases by 10%

Fig. 4 Responses for Example 2

6 Conclusions

A tuning method for a modified Smith predictor for integrating and unstable processes with time delay was proposed in this paper. Essentially two parameters are needed to tune the controller setting: one is responsible for the setpoint tracking, and the other for load disturbance rejection and stability robustness. The design for some typical processes with time delay shows that the proposed method can achieve good compromise between time domain performance and stability robustness.

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