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Adaptive robust iterative learning control for uncertain robotic systems

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Abstract: The uncertain model of the robotic system was decomposed into repetitive and non-repetitive parts, and the normal model of the system was taken into account. By using Lyapunov method, an adaptive robust iterative learning control scheme was presented for the robotic system with both structured and unstructured uncertainties, and the overall stability of the system in the iteration domain was established. In the scheme the bound parameter estimates and the iterative learning control input were adjusted in the iteration domain. The validity of the scheme is illustrated through a simulation example.

Key words: iterative learning control; robust control; adaptive control; robot systems
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不确定性机器人系统自适应鲁棒迭代学习控制

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摘要: 利用 Lyapunov 方法,提出了一种不确定性机器人系统的自适应鲁棒迭代学习控制策略,整个系统在迭代域里是全局渐近稳定的.所考虑的机器人系统同时包含了结构和非结构不确定性.在设计时,系统的不确定性被分解成可重复性和非重复性两部分,并考虑了系统的标称模型.在所提出的控制策略中,自适应策略用来估算做法确定性的界,界的修正与迭代学习控制量一样的迭代域得以实现的.计算机仿真表明本文提出的控制策略是有效的.

关键词: 迭代学习控制; 鲁棒控制; 自适应控制; 机器人系统

1 Introduction

The iterative learning control (ILC) methods deal with systems that perform the same tasks repetitively over a finite time interval. The basic idea of the ILC is that the information obtained from the previous trial is used to improve the control input for the next trial. The control input in each trial is adjusted by using the tracking error signals obtained from previous trial. As the iteration continues the control system eventually learns the task and follows the desired trajectory with little or no error. Depending on whether the system parameters are estimated or not, the ILC schemes may be classified into two categories: nonadaptive and adaptive. In the nonadaptive case, the current input profile is computed simply by adding the scaled system error and/or its time derivative to the previous input profile[1~3]. In the adaptive case, however, uncertain parameters are estimated and used to identify the system dynamics, which is in turn used to generate the control input profile[4~8].

There have been substantial research effort in the adaptive ILC area. Among the reported results, Park et al[4] proposed an adaptive iterative learning controller (AILC) for uncertain robotic systems by using the fact that they are linearly parameterizable. Seo et al[5] extended the result to a class of general nonlinear systems and proposed an intelligent learning control scheme. In the above control schemes[4,5], the parameter estimate and the learning control input were updated in the iteration domain. On the other hand, French & Roger[6] and J. Y. Chol & J. S. Lee[7] gave another AILC in which the parameter estimate is updated in the time domain.

However, in these controllers[3~6], the normal mod-

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els are not taken into account, and the uncertain parameters are not treated differently. Xu & Viswanathan[8] decomposed a class of MIMO nonlinear dynamical systems into periodic and nonperiodic parts, without taking the normal model into account.

In this paper, by using Lyapunov method an adaptive robust iterative learning control scheme is presented for the robotic system with both structured and unstructured uncertainties. In this control scheme, the whole control variable consists of three parts, i.e., the iterative learning control input, the normal model based computed torque control input, and the robust control input. The bound parameter estimates for the robust input and the learning control input were updated in the iteration domain.

In the subsequent discussion, the following definition is used. For any matrix A, the induced matrix norm $\| A \|$ is defined as $\| A \| = [\lambda_{\max}(A^{\mathrm{T}}A)]^{\frac{1}{2}}$, where $\lambda_{\max}$ is the largest eigenvalue.

2 Problem formulation

Consider an uncertain robot system with n rigid bodies:

$$[M_0(q)+\Delta M(q)]\ddot{q}+[C_0(q,\dot{q})+\Delta C(q,\dot{q})]\dot{q}+ [G_0(q)+\Delta G(q)]+d+f=u, \quad (1)$$

where $M_0(q)\in\mathbb{R}^{n\times n}$, $C_0(q,\dot{q})\in\mathbb{R}^{n\times n}$ and $G_0(q)\in\mathbb{R}^n$ are the known normal inertia matrix, centripetal plus Coriollis force matrix and gravitational vector, and $\Delta M(q),\Delta C(q,\dot{q}),\Delta G(q)$ are the corresponding uncertain parts, respectively; u is the control input, d and f are the unknown disturbances which are assumed to be bounded, and $q,\dot{q},\ddot{q}$ are the generalized joint position, velocity, and acceleration, respectively. Furthermore, the disturbance d is assumed to be repetitive and f non-repetitive. For notational convenience, in Eq. (1) and other expressions all over the paper, the time argument t is omitted.

Assumption A1 The uncertain models $\Delta M(q)$, $\Delta C(q,\dot{q}),\Delta G(q)$ are Lipschitz continuous. In other words, the following inequalities hold:

$$\| \Delta M(q)-\Delta M(q^d) \| \leqslant l_M \| q-q^d \| = l_M \| e \|,$$

$$\| \Delta C(q,\dot{q}) - \Delta C(q^d,\dot{q}^d) \| \leqslant l_C[\| q-q^d \| + \| \dot{q}-\dot{q}^d \|] = l_C[\| e \| + \| \dot{e} \|],$$

$$\| \Delta G(q) - \Delta G(q^d) \| \leqslant l_G \| q-q^d \| = l_G \| e \|,$$

where $e=q-q^d$ and l_M, l_C, l_G are the unknown finite Lipschitz constants. $q^d,\dot{q}^d,\ddot{q}^d$ represent the desired joint position, velocity, and acceleration, respectively.

Assumption A2 Like most other iterative learning control schemes, the initial error is assumed to satisfy the equalities: $e(0)=0$ and $\dot{e}(0)=0$.

3 Adaptive robust iterative learning control

For system (1), the presented adaptive robust iterative learning control scheme is as follows:

$$u_j = u_j^c + u_j^l + u_j^r. \quad (2)$$

Where u_j denotes the control input at the j-th iteration, u_j^c is the computed torque control input, u_j^r the robust control input, and u_j^l the learning control input. The computed torque control input u_j^c is generated by

$$u_j^c = M_0(q_j)[\ddot{q}^d-(k+\alpha I)\dot{e}_j-\alpha k e_j]+ C_0(q_j,\dot{q}_j)\dot{q}_j+G_0(q_j), \quad (3)$$

where $e_j=q_j-q^d$, $q_j,\dot{q}_j,\ddot{q}_j$ denote the actual joint position, velocity, and acceleration at the j-th iteration, respectively; k is a positive feedback matrix and α is a positive parameter.

Define $\sigma_j=\dot{e}_j+ke_j$. Substituting Eqs. (2) and (3) into Eq. (1) yields

$$M_0(q_j)[\dot{\sigma}_j+\alpha\sigma_j] = u_j^l+u_j^r-\{\Delta M(q_j)\ddot{q}_j+\Delta C(q_j,\dot{q}_j)\dot{q}_j+\Delta G(q_j)+d+f\}. \quad (4)$$

Let $\Delta M,\Delta C,\Delta G$ denote $\Delta M(q_j),\Delta C(q_j,\dot{q}_j)$, $\Delta G(q_j)$, and $\Delta M_d,\Delta C_d,\Delta G_d$ denote $\Delta M(q^d)$, $\Delta C(q^d,\dot{q}^d),\Delta G(q^d,\dot{q}^d)$ respectively. We can get

$$\Delta M(q_j)\ddot{q}_j+\Delta C(q_j,\dot{q}_j)\dot{q}_j+\Delta G(q_j)+d+f= \Delta M\dot{\sigma}_j+\mathscr{R}+\bar{h}. \quad (5)$$

Where

$$\mathscr{R}=\Delta M_d\ddot{q}^d+\Delta C_d\dot{q}^d+\Delta G_d+d,$$

$$\bar{h}=(\Delta M-\Delta M_d)\ddot{q}^d-\Delta Mk\dot{e}+(\Delta C-\Delta C_d)\dot{q}^d+\Delta C\dot{e}+(\Delta G-\Delta G_d)+f.$$

Substituting Eq. (5) into Eq. (4) yields

$$(M_0+\Delta M)\dot{\sigma}_j+\alpha M_0\sigma_j = u_j^l+u_j^r-\mathscr{R}-\bar{h}. \quad (6)$$

Obviously, the uncertain part $\mathscr{R}$ is repetitive and $\bar{h}$ is non-repetitive. From Assumption A1, the following inequality holds:

$$\| \bar{h} \| \leqslant [l_M\| \ddot{q}^d \| + l_C\| \dot{q}^d \| + l_G]\cdot\| e \| + [k\| \Delta M \| +$$

$$l_C \| \dot{q}^d \| + \| \Delta C \|] \cdot \| \dot{e} \| + \| f \| = \Gamma E. \quad (7)$$

Where

$$\begin{cases} \Gamma = [l_1 \quad l_2 \quad l_3], \\ l_1 = l_M \| \ddot{q}^d \| + l_C \| \dot{q}^d \| + l_G, \\ l_2 = k \| \Delta M \| + l_C \| \dot{q}^d \| + \| \Delta C \|, \\ l_3 = \| f \|, \\ E = [\| e \|, \| \dot{e} \|, 1]^{\mathrm{T}}. \end{cases} \quad (8)$$

Then, the main aim is to find out the iterative learning algorithm and to determine the robust control input and its bound parameter estimates. This would be discussed in the next section.

4 Main results

The mains results of this paper are described as following theorem and corollaries.

If the system satisfied Assumptions A1 and A2 described in Section 1, then the learning algorithm, adaptive update law and the system's convergence are given as Theorem 1.

Theorem 1 If the iterative learning algorithm and robust control input are given as

$$u_j^l = u_{j-1}^l - \beta\sigma_j, \; u_1^l = 0, \quad (9)$$

$$u_j^r = -(\frac{1}{2}\beta^{-1}P + C_0 + \hat{\xi}_j)\sigma_j - \hat{\Gamma}_j E \operatorname{sgn}(\sigma_j), \quad (10)$$

where β is a positive constant, $\hat{\xi}$ and $\hat{\Gamma}$ are the j-th estimates of parameters ξ^* ($\xi^* = \| \Delta C \|$) and Γ, respectively, and if the adaptive updating laws of $\hat{\xi}_j$ and $\hat{\Gamma}_j$ are given as

$$\hat{\xi}_j = \hat{\xi}_{j-1} + \beta \| \sigma_j \|^2, \; \hat{\xi}_1 = 0, \quad (11)$$

$$\hat{\Gamma}_j = \hat{\Gamma}_{j-1} + \beta \| \sigma_j \| \eta^{\mathrm{T}}, \; \hat{\Gamma}_1 = [0,0,0], \quad (12)$$

where the parameter $\eta = E = [\| e \|, \| \dot{e} \|, 1]^{\mathrm{T}}$, then system (1) is asymptotically stabilized under the control scheme (2).

Proof Choose a Lyapunov function

$$V_j = \int_0^T (\| \mathscr{R} - u_j^l \|^2 + \sigma_j^{\mathrm{T}} P \sigma_j + \| \hat{\xi}_j - \xi^* \|^2 + \| \hat{\Gamma}_j - \Gamma \|^2) \mathrm{d}t, \quad (13)$$

where P is a symmetry positive matrix. Then

$$\begin{aligned} V_j - V_{j-1} = & \\ & \int_0^T (u_j^l - u_{j-1}^l)^{\mathrm{T}} (u_j^l + u_{j-1}^l - 2\mathscr{R}) \mathrm{d}t + \\ & \int_0^T (\sigma_j^{\mathrm{T}} P \sigma_j - \sigma_{j-1}^{\mathrm{T}} P \sigma_{j-1}) \mathrm{d}t + \\ & \int_0^T (\hat{\xi}_j - \hat{\xi}_{j-1})^{\mathrm{T}} (\hat{\xi}_j + \hat{\xi}_{j-1} - 2\xi^*) \mathrm{d}t + \\ & \int_0^T (\hat{\Gamma}_j - \hat{\Gamma}_{j-1})(\hat{\Gamma}_j + \hat{\Gamma}_{j-1} - 2\Gamma)^{\mathrm{T}} \mathrm{d}t. \end{aligned}$$

From (9), (11) and (12) we have

$$\begin{aligned} V_j - V_{j-1} = & \\ & \int_0^T (-\beta\sigma_j)^{\mathrm{T}} (\beta\sigma_j + 2(u_j^l - \mathscr{R})) \mathrm{d}t + \\ & \int_0^T (\sigma_j^{\mathrm{T}} P \sigma_j - \sigma_{j-1}^{\mathrm{T}} P \sigma_{j-1}) \mathrm{d}t + \\ & \int_0^T (\beta \| \sigma_j \|^2)^{\mathrm{T}} (-\beta \| \sigma_j \|^2 + 2(\hat{\xi}_j - \xi^*)) \mathrm{d}t + \\ & \int_0^T (\beta \| \sigma_j \| \eta^{\mathrm{T}})(-\beta \| \sigma_j \| \eta^{\mathrm{T}} + 2(\hat{\Gamma}_j - \Gamma))^{\mathrm{T}} \mathrm{d}t. \end{aligned} \quad (14)$$

The equation (6) can be converted into

$$\begin{aligned} & u_j^l - \mathscr{R} = \\ & (M_0 + \Delta M)\dot{\sigma}_j + \alpha M_0 \sigma_j - (u_j^r - \bar{h}) = \\ & M\dot{\sigma}_j + \alpha M_0 \sigma_j - (u_j^r - \bar{h}). \end{aligned} \quad (15)$$

Substituting (15) into (14), the following inequality is derived easily.

$$\begin{aligned} V_j - V_{j-1} \leqslant & \\ & \int_0^T (-2\beta\sigma_j^{\mathrm{T}} M \dot{\sigma}_j - 2\alpha\beta\sigma_j^{\mathrm{T}} M_0 \sigma_j) \mathrm{d}t + \\ & \int_0^T (\sigma_j^{\mathrm{T}} P \sigma_j \mathrm{d}t + \int_0^T 2\beta\sigma_j^{\mathrm{T}} (u_j^r - \bar{h}) \mathrm{d}t + \\ & \int_0^T 2\beta \| \sigma_j \|^2 (\hat{\xi}_j - \xi^*) \mathrm{d}t + \int_0^T 2\beta \| \sigma_j \| (\hat{\Gamma}_j - \Gamma) \eta \mathrm{d}t - \\ & \int_0^T \beta^2 \sigma_j^{\mathrm{T}} \sigma_j \mathrm{d}t - \int_0^T \sigma_{j-1}^{\mathrm{T}} P \sigma_{j-1} \mathrm{d}t. \end{aligned}$$

Since

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}} M \dot{\sigma}_j \mathrm{d}t = \beta\sigma_j^{\mathrm{T}} M \sigma_j \mid_0^T - \int_0^T \beta\sigma_j^{\mathrm{T}} \dot{M} \sigma_j \mathrm{d}t,$$

we have

$$\begin{aligned} V_j - V_{j-1} \leqslant & \\ & \int_0^T (\beta\sigma_j^{\mathrm{T}} \dot{M} \sigma_j - 2\alpha\beta\sigma_j^{\mathrm{T}} M_0 \sigma_j) \mathrm{d}t - \\ & \beta\sigma_j^{\mathrm{T}} M \sigma_j \mid_0^T + \int_0^T \sigma_j^{\mathrm{T}} P \sigma_j \mathrm{d}t + \int_0^T 2\beta\sigma_j^{\mathrm{T}} (u_j^r - \bar{h}) \mathrm{d}t + \\ & \int_0^T 2\beta \| \sigma_j \|^2 (\hat{\xi}_j - \xi^*) \mathrm{d}t + \int_0^T 2\beta \| \sigma_j \| (\hat{\Gamma}_j - \Gamma) \eta \mathrm{d}t - \\ & \int_0^T \beta^2 \sigma_j^{\mathrm{T}} \sigma_j \mathrm{d}t - \int_0^T \sigma_{j-1}^{\mathrm{T}} P \sigma_{j-1} \mathrm{d}t = \end{aligned}$$

$$\int_0^T \sigma_j^{\mathrm{T}}(P + \beta(\dot{M} - 2C) - 2\alpha\beta M_0)\sigma_j \mathrm{d}t +$$

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}(C_0 + \Delta C)\sigma_j \mathrm{d}t +$$

$$\beta(\sigma_j^{\mathrm{T}}(0)M(0)\sigma_j(0) - \sigma_j^{\mathrm{T}}(T)M(T)\sigma_j(T)) +$$

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}(u_j^r - \bar{h})\mathrm{d}t + \int_0^T 2\beta \| \sigma_j \|^2(\hat{\xi}_j - \xi^*)\mathrm{d}t +$$

$$\int_0^T 2\beta \| \sigma_j \| (\hat{\Gamma}_j - \Gamma)\eta \mathrm{d}t - \int_0^T \beta^2 \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t -$$

$$\int_0^T \sigma_{j-1}^{\mathrm{T}} P\sigma_{j-1}\mathrm{d}t,$$

where $M(0) = M(q(0)), M(T) = M(q(T))$.

It is well known that $\dot{M} - 2C$ is a skew-symmetric matrix and the inertia M is positive. From Assumption A2, we get $\sigma_j(0) = 0$. Thus, we have

$$V_j - V_{j-i} \leqslant$$

$$\int_0^T \sigma^{\mathrm{T}} P\sigma_j \mathrm{d}t + \int_0^T 2\beta\sigma_j^{\mathrm{T}}(C_0 + \Delta C)\sigma_j \mathrm{d}t +$$

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}(u_j^r - \bar{h})\mathrm{d}t + \int_0^T 2\beta \| \sigma_j \|^2(\hat{\xi}_j - \xi^*)\mathrm{d}t +$$

$$\int_0^T 2\beta \| \sigma_j \| (\hat{\Gamma}_j - \Gamma)\eta \mathrm{d}t - \int_0^T \beta^2 \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t - \int_0^T \sigma_{j-1}^{\mathrm{T}} P\sigma_{j-1}\mathrm{d}t.$$

From (10) we get

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}(u_j^r - \bar{h})\mathrm{d}t =$$

$$\int_0^T - 2\beta\sigma_j^{\mathrm{T}}(\frac{1}{2}\beta^{-1}P + C_0 + \hat{\xi}_j)\sigma_j \mathrm{d}t -$$

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}\hat{\Gamma}_j E \mathrm{sgn}\ (\delta_j)\mathrm{d}t - \int_0^T 2\beta\sigma_j^{\mathrm{T}}\bar{h}\mathrm{d}t.$$

Substituting this equation into the above inequality yields

$$V_j - V_{j-1} \leqslant$$

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}(\Delta C)\sigma_j \mathrm{d}t - \int_0^T 2\beta \hat{\xi}_j \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t +$$

$$\int_0^T 2\beta \| \sigma_j \|^2(\hat{\xi}_j - \xi^*)\mathrm{d}t - \int_0^T 2\beta\sigma_j^{\mathrm{T}}\hat{\Gamma}_j E \mathrm{sgn}\ (\delta_j)\mathrm{d}t -$$

$$\int_0^T 2\beta\sigma_j^{\mathrm{T}}\bar{h}\mathrm{d}t + \int_0^T 2\beta \| \sigma_j \| (\hat{\Gamma}_j - \Gamma)\eta \mathrm{d}t -$$

$$\int_0^T \beta^2 \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t - \int_0^T \sigma_{j-1}^{\mathrm{T}} P\sigma_{j-1}\mathrm{d}t.$$

Since all the desired and estimative bound parameters are positive, $\xi^* = \| \Delta C \|$, and $\sigma_j^{\mathrm{T}}\mathrm{sgn}\ (\sigma_j) \geqslant \| \sigma_j \|$ and $\sigma_j^{\mathrm{T}}\sigma_j = \| \sigma_j \|^2$ for σ_j is a column vector, such that

$$\int_0^T \sigma_j^{\mathrm{T}}(\Delta C)\sigma_j \mathrm{d}t \leqslant \int_0^T \| \sigma_j \|^2 \xi^* \mathrm{d}t,$$

$$\int_0^T \hat{\xi}_j \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t = \int_0^T \hat{\xi}_j \| \sigma_j \|^2 \mathrm{d}t,$$

$$\int_0^T \sigma_j^{\mathrm{T}}\hat{\Gamma}_j E \mathrm{sgn}\ (\delta_j)\mathrm{d}t \geqslant \int_0^T \| \sigma_j \| \hat{\Gamma}_j E \mathrm{d}t.$$

So the following inequality can be obtained easily

$$V_j - V_{j-1} \leqslant$$

$$- \int_0^T 2\beta \| \sigma_j \| \hat{\Gamma}_j E \mathrm{d}t - \int_0^T 2\beta\sigma_j^{\mathrm{T}}\bar{h}\mathrm{d}t +$$

$$\int_0^T 2\beta \| \sigma_j \| (\hat{\Gamma}_j - \Gamma)\eta \mathrm{d}t -$$

$$\int_0^T \beta^2 \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t - \int_0^T \sigma_{j-1}^{\mathrm{T}} P\sigma_{j-1}\mathrm{d}t.$$

For the parameter $\eta = E = [\ \| e \|\ ,\ \| \dot{e} \|\ ,1]^{\mathrm{T}}$, besides expression (7), the following inequality holds

$$V_j - V_{j-1} \leqslant -\int_0^T \beta^2 \sigma_j^{\mathrm{T}}\sigma_j \mathrm{d}t - \int_0^T \sigma_{j-1}^{\mathrm{T}} P\sigma_{j-1}\mathrm{d}t \leqslant 0.$$

When and only when $\sigma_{j-1} = 0, \sigma_j = 0$ for all $t \in [0, T]$, the equality $V_j - V_{j-1} = 0$ holds. The proof of the theorem is completed.

Since Assumption 2 is too strict and difficult to achieve in practice. Following corollaries can remove this constrain.

Corollary 1 If Assumption 2 is not found, and the condition

$$\lim_{j \to \infty} \sigma_j(0) = 0$$

is satisfied, then the system (1) is also asymptotically stabilized under the control scheme (2), in which the robust control and iterative learning input and the adaptive update law, as in the theorem, are given as (9) ~ (12).

Proof For $\lim_{j \to \infty} \sigma_j(0) = 0$, we have

$$\lim_{j \to \infty} \beta\sigma_j^{\mathrm{T}}(0)M(0)\sigma_j(0) = 0.$$

So Corollary 1 can be provided easily, and the proof of the corollary is the same as the proof of the theorem.

In practice, the condition given above is usually unrealizable. And when the norm of the initial error $\| \sigma_j(0) \| \leqslant \delta, j = 1,2,\cdots$, where δ is a small enough positive constant. Then the convergence of the system can be achieved as in Corollary 2.

Corollary 2 If the following condition about initial error

$$\| \sigma_j(0) \| \leqslant \delta,\ j = 1,2,\cdots$$

is satisfied, then the system is stable and following inequality is found.

$$\lim_{j\to\infty}\int^{T}\sigma_j^{\mathrm T}\sigma_j \leqslant \beta^{-1}\lambda\delta^2, \tag{16}$$

where λ is the maximum eigenvalue of matrix $M(0)$, and β is the learning parameter.

Proof Like the proof of the theorem given above, when

$$\| \sigma_j(0) \| \leqslant \delta,\ j = 1,2,\cdots,$$

we can get

$$V_j - V_{j-1} \leqslant$$
$$-\int_0^T\beta^2\sigma_j^{\mathrm T}\sigma_j\mathrm{d}t-\int_0^T\sigma_{j-1}^{\mathrm T}P\sigma_{j-1}\mathrm{d}t+\beta\sigma_j^{\mathrm T}(0)M(0)\sigma_j(0)\leqslant$$
$$-\int_0^T\beta^2\sigma_j^{\mathrm T}\sigma_j\mathrm{d}t + \beta\lambda\delta^2.$$

If

$$\int_0^T\sigma_j^{\mathrm T}\sigma_j > \beta^{-1}\lambda\delta^2,$$

we have

$$V_j - V_{j-1} \leqslant -\int_0^T\beta^2\sigma_j^{\mathrm T}\sigma_j\mathrm{d}t + \beta\lambda\delta^2 < 0.$$

So the inequality (16) is satisfied, and Corollary 2 is provided.

5 Simulation

To illustrate the effectiveness of the presented adaptive robust iterative learning control scheme, a two-link planar robot is designed to follow a continuous path. The robot system is shown as Fig.1. The vectors q_1, q_2 denote the angles of the two links. We assume that the links are homogeneous rigid bodies with the mass m_1, m_2 and the length l_1, l_2, respectively. The model of this system with disturbance is given in Appendix.

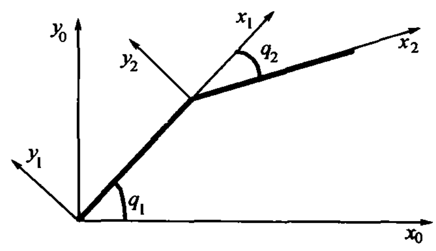

Fig. 1 Two-link robot configuration

In this simulation example, we select

$$l_1 = 1.0\ \mathrm{m},\ l_2 = 0.5\ \mathrm{m},$$

$$m_1 = 10.0\ \mathrm{kg},\ m_2 = \begin{cases}4.5\ \mathrm{kg},\ 0\ \mathrm{s} \leqslant t < 2.5\ \mathrm{s},\\ 5.5\ \mathrm{kg},\ 2.5\ \mathrm{s} \leqslant t \leqslant 5\ \mathrm{s},\end{cases}$$

The desired trajectories are required as

$$q^d = [q_1^d, q_2^d]^{\mathrm T} = [\sin(3t), \cos(3t)]^{\mathrm T},\ t \in [0,5],$$

and the repetitive and non-repetitive disturbances are assumed as

$$d = [d_1, d_2]^{\mathrm T} = [2-t/10,\ 2-t/10]^{\mathrm T},\ t \in [0,5],$$
$$f = [f_1, f_2]^{\mathrm T} = [2\mathrm{rand}\ (1), 2\mathrm{rand}\ (1)]^{\mathrm T},$$

where rand $(1) \in (-1,1)$ is a random function. In the simulation, the learning gain and other parameters were chosen as

$$\beta = 0,5,\ \alpha = 2,$$
$$P = \begin{bmatrix}20 & 0\\ 0 & 30\end{bmatrix},\ k = \begin{bmatrix}3 & 0\\ 0 & 5\end{bmatrix}.$$

The simulation results for the first and fifth iteration are shown in Fig.2 and Fig.3, respectively. It is proved that the control scheme presented in this paper is very effective. In addition, since the normal model of the uncertain system has been taken into account, the system can achieve good performance at a few times of iterations.

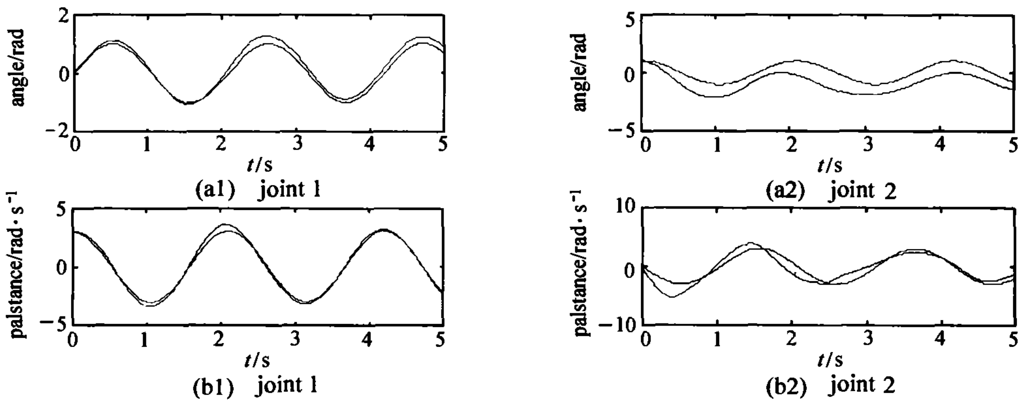

Fig. 2 Responses at the first iteration (solid: desired dashed: actual)

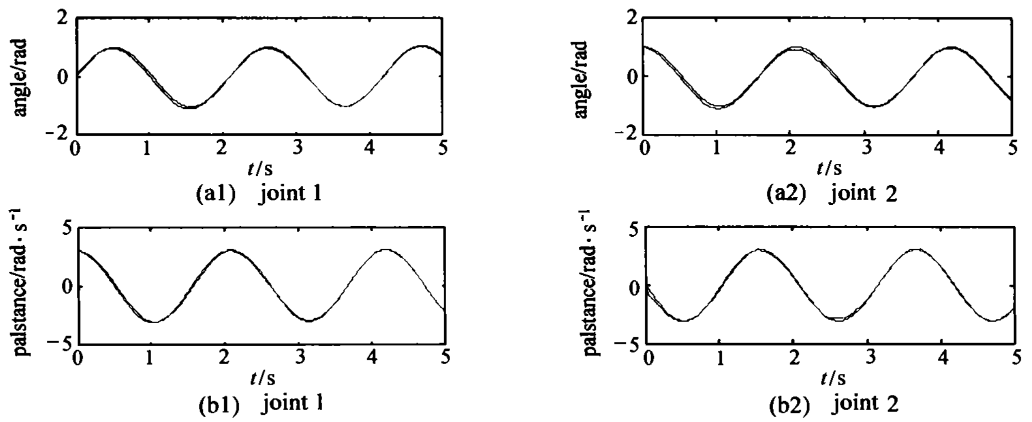

Fig. 3 Responses at the fifth iteration(solid: desired dashed: actual)

6 Conclusion

By using the Lyapunov method, an adaptive robust iterative learning control scheme is presented for the robotic system with both structured and unstructured uncertainties. Its distinct feature against other iterative learning schemes is that the uncertain model of the robotic system is decomposed into repetitive and non-repetitive parts, the normal model is took into account, and the bound parameter estimates and the iterative learning control input were adjusted in the iteration domain. The simulation results show the control scheme is very effective.

References:

[1] ARIMOTO S, KAWAMURA S, MIYASKI F. Bettering operation of robots by learning [J]. *J of Robotic Systems*, 1984, 1(2): 123 - 140.

[2] JANG T, CHOI C H, AHN H S. Iterative learning control in feedback systems [J]. *Automatica*, 1995, 31(2): 243 - 248.

[3] LEE J H, LEE K S, KIM W C. Model-based iterative learning control with a quadratic criterion for time-varying linear systems [J]. *Automatica*, 2000, 36(4): 641 - 657.

[4] PARK K H, KUC T Y, LEE J S. Adaptive learning control of uncertain robotic systems [J]. *Int J Control*, 1996, 65(5): 725 - 744.

[5] SEO W G, PARK K H, LEE J S. Intelligent learning control for a class of nonlinear dynamics system [J]. *IEE Proc — Control Theory and Applications*, 1999, 146(2): 165 - 170.

[6] FRENCH M, ROGERS E. Nonlinear iterative learning by an adaptive Lyapunov technique [A]. *Proc of the 37th IEEE Conference on Decision and Control* [C]. Tampa, FL: Piscataway, 1998: 175 - 180.

[7] CHOI J Y, LEE J S. Adaptive iterative learning control of uncertain robotic systems [J]. *IEE Proc—Control Theory and Applications*, 2000, 147(2): 217 - 223.

[8] XU Jianxin, VISWNATHAN B. Adaptive robust iterative learning control with dead zone scheme [J]. *Automatica*, 2000, 36(1): 91 - 99.

Appendix

The dynamic model of the two-links robot system shown as Fig. 1 can be written as

$$M(q)\ddot{q} + D(q,\dot{q})\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + G(q) + d + f = u.$$

Where d and f are repetitive and non-repetitive disturbances respectively;

$$M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{bmatrix},$$

$$m_{11} = \frac{1}{3}m_1 l_1^2 + m_2 l_1^2 + \frac{1}{3}m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2),$$

$$m_{21} = m_{12} = \frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos(q_2),$$

$$m_{22} = \frac{1}{3}m_2 l_2^2,$$

$$D(q,\dot{q}) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22}\end{bmatrix},$$

$$d_{11} = -m_2 l_1 l_2 \sin(q_2)\dot{q}_2,$$

$$d_{21} = d_{12} = -\frac{1}{2}m_2 l_1 l_2 \sin(q_2)\dot{q}_2,$$

$$d_{22} = 0,$$

and

$$G(q) = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix},$$

where

$$g_1 = \frac{1}{2}m_1 g l_1 \cos(q_1) + \frac{1}{2}m_2 g l_2 \cos(q_1 + q_2) + m_2 g l_l \cos(q_1),$$

$$g_2 = \frac{1}{2}m_2 g l_2 \cos(q_1 + q_2).$$

In above equaltiy, g is the acceleration of gravity.

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