Article ID: 1000 - 8152(2003)05 - 0737 - 04

Identification and chaotifying control of a class of system without mathematical model

REN Hai-peng, LIU Ding
(School of Automation and Information Engineering, Xi'an University of Technology, Shanxi Xi'an 710048, China)

Abstract: Fuzzy neural network (FNN) was proposed to identify the dynamics of a class of non-chaotic system without mathematical model. The result of identification was then used in inverse system method, by which chaotifying control of the system could be implemented. This method was independent of the exact mathematic model of the system to be controlled. It was testified that error of control caused by the identification error was less than the identification error by properly designed control parameters. Simulation results for continuous and discrete systems show the effectiveness of the method.

Key words: system identification; chaotifying control; fuzzy neural networks; inverse system method
CLC number: TP273, TP13 **Document code**: A

一类模型未知系统的辨识和混沌化控制

任海鹏, 刘 丁
(西安理工大学 自动化与信息工程学院, 陕西 西安,710048)

摘要: 对于一类模型未知的非混沌系统采用模糊神经网络辨识其动力学特性,将得到的模糊神经网络辨识模型应用于逆系统方法中,实现了一类模型未知非混沌系统的混沌化控制.该方法不依赖于被控对象的数学模型,就可以进行有效控制.研究了模糊神经网络辨识误差对控制精度的影响,证明了适当设计参数可以使由辨识误差引起的控制误差小于辨识误差.针对连续和离散两类系统的仿真研究证明了该方法的有效性.

关键词: 系统辨识; 混沌化控制; 模糊神经网络; 逆系统方法

1 Introduction

In the last four decades, chaotic dynamics have been intensively studied, and the traditional trend of analyzing and understanding chaos has evolved to controlling and utilizing chaos[1]. On the one hand, chaos is weakened or completely suppressed[2~4] when it is harmful; on the other hand, non-chaotic system is chaotified[5~8], which means enhancing existing chaos or creating chaos purposefully when it is useful, because of its great potential in non-traditional applications in the field of mechanical, electronics, optical and especially the telecommunication system.

In this paper, a class of non-chaotic system without mathematical model is identified using FNN and the result of identification is used in inverse system control method, by which the non-chaotic system is forced to track a chaotic reference input, namely chaotification.

2 Identification of non-chaotic system using FNN

Consider a class of continuous non-chaotic system Σ as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \quad\vdots \\ \dot{x}_n = f(x_1, x_2, \cdots, x_n) + u, \\ y = x_1. \end{cases} \tag{1}$$

Where y is the output of the system, u is the input of the system. $x \in \mathbb{R}^n$ is the state variable, $f(\cdot)$ is a linear or nonlinear function of x, which dominates the dynamic activity of the system. If $f(\cdot)$ can be identified the dynamics is determined.

From (1), we have

$$y^{(n)} - u(t) = f(y, y^{(1)}, \cdots, y^{(n-1)}). \tag{2}$$

Received date: 2002 - 03 - 04; Revised date: 2003 - 02 - 17.

The input and output of the system can be used to identify the function $f(\cdot)$. FNN is intuitively considered because of its function approximation ability and convergence speed. The training process can be shown by Fig.1. The details about the FNN and its training can be found in [4].

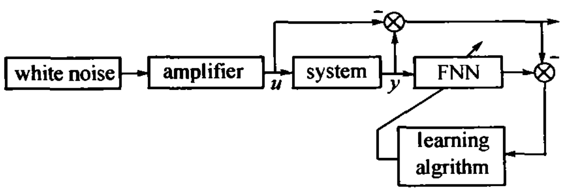

Fig. 1 Training process scheme

The discrete counterpart of (1) can be described as follows:

$$\begin{cases} x_1(k+1) = x_2(k), \\ x_2(k+1) = x_3(k), \\ \vdots \\ x_n(k+1) = \phi(x_1(k), x_2(k), \cdots, x_n(k)) + u(k), \\ y(k) = x_1(k), \end{cases} \tag{3}$$

The function $\phi(\cdot)$ can also be identified using the same process described above.

3 Chaotifying control via inverse system method

The inverse system of (1) can be described as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \vdots \\ \dot{x}_n = y^{(n)}, \\ u = y^{(n)} - f(x_1, x_2, \cdots, x_n). \end{cases} \tag{4}$$

The originally desired system[9] is

$$\begin{cases} \phi + a_1 y^{(n-1)} + \cdots + a_{n-1} y^{(1)} + a_n y = r, \\ \phi = y^{(n)}. \end{cases} \tag{5}$$

It is difficult to obtain desired aim by using Eq. (5) as the desired system. So the modified desired system is shown as follows

$$\begin{cases} \phi + a_1 y^{(n-1)} + \cdots + a_{n-1} y^{(1)} + a_n y = \\ r^{(n)} + a_1 r^{(n-1)} + \cdots + a_{n-1} r^{(1)} + a_n r, \\ \phi = y^{(n)}. \end{cases} \tag{6}$$

And assuming: 1) $r(t)$ and its derivatives up to $r^{(n)}(t)$ are bounded for all $t \geqslant 0$, and the n-th derivative $r^{(n)}(t)$ is at least a piecewise continuous function of t; 2) $r(t), r^{(1)} \cdots, r^{(n)}(t)$ are available online.

From Eqs. (4) and (6), we have

$$u = r^{(n)} + a_1 r^{(n-1)} + \cdots + a_{n-1} r^{(1)} + a_n r - a_1 y^{(n-1)} - \cdots - a_{n-1} y^{(1)} - a_n y - f(x_1, x_2, \cdots, x_n) \tag{7}$$

Definition 1 $e = r - y$ is the tracking error.

According to Definition 1 we have

$$e^{(n)} + a_1 e^{(n-1)} + \cdots + a_{n-1} e^{(1)} + a_n e = 0. \tag{8}$$

According to the pole placement rule, one can choose proper $a_i (i = 1, \cdots, n)$ to make the system (8) asymptotically stable, so that the output y can track the reference input r.

For discrete system (3), the inverse system controller can be derived from the similar procedure. It is given as follows

$$u(k) = r(k+n) + d_1 r(k+n-1) + \cdots + d_n r(k) - d_1 y(k+n-1) - \cdots - d_n y(k) - \phi(\cdot). \tag{9}$$

From Eqs.(7) and (9), if the function $f(\cdot)$ or $\phi(\cdot)$ can be identified from the input and output of the system, the control is free of the system model.

4 Theorem about tracking error

Definition 2 The identification model of $f(\cdot)$ is

$$y^{(n)} - u = \hat{f}(\cdot). \tag{10}$$

Definition 3 The identification error is

$$\Delta = \hat{f}(\cdot) - f(\cdot). \tag{11}$$

Definition 4 η_1 is the tracking error caused by Δ.

Theorem 1 For the given system (1), the inverse system controller Eq.(7) is used and the function $f(\cdot)$ in Eq. (7) is replaced by $\hat{f}(\cdot)$. If only the proper controller parameters $a_i (i = 1, 2, \cdots, n)$ are selected, the inequality $\eta_1 \leqslant \Delta$ can be ensured.

Proof Replacing $f(x_n)$ in Eq. (7) with $\hat{f}(x_n)$, we have

$$u = r^{(n)} + a_1 r^{(n-1)} + \cdots + a_{n-1} r^{(1)} + a_n r - a_1 y^{(n-1)} - \cdots - a_{n-1} y^{(1)} - a_n y - \hat{f}(\cdot). \tag{12}$$

Substituting Eq.(11) into Eq.(12), we have

$$u = r^{(n)} + a_1 r^{(n-1)} + \cdots + a_{n-1} r^{(1)} + a_n r - a_1 y^{(n-1)} - \cdots - a_{n-1} y^{(1)} - a_n y - \Delta - f(\cdot). \tag{13}$$

According to Definition 1, we have

$$e^{(n)} + a_1 e^{(n-1)} + \cdots + a_{n-1} e^{(1)} + a_n e = \Delta. \tag{14}$$

Since the system (8) is asymptotically stable, the system (14) has finite input and output.

Δ can be regarded as a series of pulse inputs with different amplitudes. So at the time t_i, the Δ can be represented by $\Delta(t_i)\delta(t - t_i)$, where $\Delta(t_i)$ is the identification model error at time t_i. Then the zero-state response of system (14) at t_i is

$$e(t - t_i) = \Delta(t_i)\sum_{i=1}^{n} A_i e^{p_i(t-t_i)}. \tag{15}$$

Where $A_i = 1/\prod_{\substack{k=1\\k\neq i}}^{n}(s - p_k)\Big|_{s=p_i}$, $p_i(i = 1,\cdots,n)$ are the poles of Eq. (14), which can be determined by the controller parameters $a_i(i = 1,2,\cdots,n)$. If the distance between two poles among all poles is big enough and the poles are negative enough, A_i is small enough and $e(t - t_i)$ attenuates quickly enough to warrant that the error caused by FNN model error is less than the FNN model error. In other words, $\eta_1 \leqslant \Delta$.

Therefore the clockwork control precision can be ensured if the precision of the identification can be ensured.

Remark 1 The requirement of the inequality $\eta_1 \leqslant \Delta$ is coincided with the requirement that the instantaneous state of e attenuates very quickly. So we can design $a_i(i = 1,2,\cdots,n)$ to make these poles be negative and far away from each other.

Remark 2 The error caused by FNN model is different from the total control error, the total control error or the tracking error is larger than that caused by the FNN model error.

Remark 3 There are some limitations in the parameter design. If the distance between any poles is too big, it may cause the control variable u in Eq.(7) to be too large to realize for a real controller. Therefore the parameters should be selected finely to balance the conflict described above.

The counterpart theorem of Theorem 1 for discrete system can be developed and testified using the method similar to what is found in the context of Literature [4].

For both the continuous and the discrete systems, if the reference input $r(t)$ or $r(k)$ is chaotic signal, the inverse system method can be used to control non-chaotic system to have chaotic output, namely chaotification.

5 Simulation examples

Example 1 The continuous case

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -x_1 - 2x_2 - x_3 + u, \\ y = x_1. \end{cases} \tag{16}$$

When $u = 0$, the system (16) is non-chaotic.

The reference system

$$\begin{cases} \dot{z}_1 = z_2, \\ \dot{z}_2 = z_3, \\ \dot{z}_3 = -b_1 z_3 - z_2 - z_1(t-\tau) - b_2(z_1(t-\tau))^3, \\ v = z_1. \end{cases} \tag{17}$$

When $b_1 = 0.5, b_2 = -\frac{1}{3}, \tau = 1$, the output v is chaotic.

Choose $a_1 = 21, a_2 = 129, a_3 = 315$ in Eq.(7). The forced output of the system (16) and the tracking error are shown in Fig.2(a) and Fig.2(b) respectively.

From Fig.2, it can be seen that the output of non-chaotic system (16) tracks the chaotic input and become chaotic.

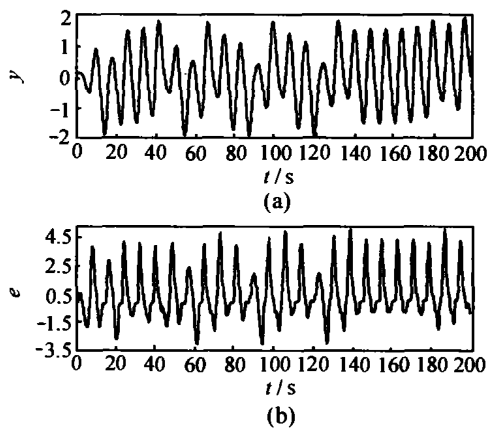

Fig. 2 Output of system (16) and tracking error

Example 2 For discrete case

$$\begin{cases} x_1(k+1) = x_2(k), \\ x_2(k+1) = -x_1(k) - x_2(k) + u(k), \\ y(k) = x_1(k). \end{cases} \tag{18}$$

The chaotic reference system is

$$\begin{cases} y_1(k+1) = -k\sin(y_2(k)) + y_1(k), \\ y_2(k+1) = y_2(k) + y_1(k+1), \\ w(k) = y_1(k). \end{cases} \tag{19}$$

When $k = 1.16$, the discrete system (19) is chaotic. Set $d_1 = 0.64$ and $d_2 = 1.5$ in Eq.(9). The forced output of the system (18) and the control error are shown in Fig.3(a) and Fig.3(b) respectively.

From Fig.3, it can be seen that the method is also effective for discrete systems.

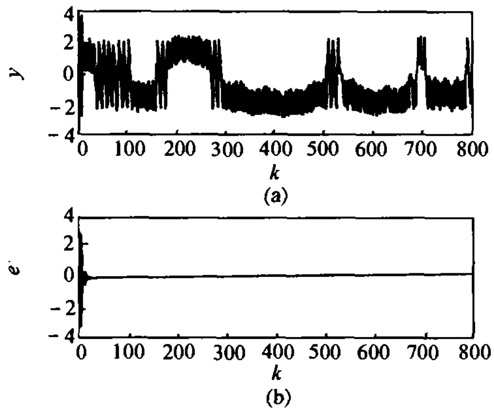

Fig. 3 Output and control error of discrete system (18)

6 Conclusions

In this paper, a class of non-chaotic system without mathematical model is identified by using FNN, the identification result is used in the inverse system method for chaotifying control of the system. The method is independent of the analytical model because of the use of FNN. Simulation results show that in order to obtain less tracking error the parameters of the controller system are large; as a result the control variable may be somewhat large. In its application, a balance should be kept between the tracking error and controller parameters. Moreover, this method has strong application potential in secure communication. The control variable in this method can be used as a key to chaos communication. Because this key signal must be used to control the system (1) to obtain the real chaos modulating signal at receiving end, thiskey signal is more secure.

References:

[1] CHEN Guanrong. What does chaos have to do with systems and control engineering [J]. *J of System Science and Complex*, 2001, 14(1): 32 - 39.

[2] OTT E, GREBOGI C, YORKE J A. Controlling chaos [J]. *Physics Review Letter*, 1990, 64(11): 1196 - 1199.

[3] CHEN Guanrong, DONG Xiaoning. On feedback control of chaotic dynamic systems [J]. *Int J of Bifurcation and Chaos*, 1992, 2(3): 407 - 411.

[4] REN Haipeng, LIU Ding. Inverse system control of chaos using fuzzy-neural network [J]. *Acta Physics Sinica*, 2002, 51(5): 982 - 986.

[5] CHEN Guanrong. Chaos: control and anticontrol [J]. *IEEE Circuits and Systems Society Newsletter*, 1998, 9(1): 1 - 5.

[6] CHEN Guanrong, LAI Dejian. Feedback anticontrol of the discrete chaos [J]. *Int J of Bifurcation and Chaos*, 1998, 8(7): 1585 - 1590.

[7] WANG Ciaofan, CHEN Guanrong. Yet another algorithm for chaotifying control of discrete chaos [J]. *Control Theory & Applications*, 2000, 17(3): 336 - 340.

[8] CHEN D S, WANG H O, CHEN Guanrong. Anti-control of Hopf bifurcations [J]. *IEEE Trans on Circuits and System- Ⅰ*, 2001, 48 (6): 661 - 671.

[9] LI Chenwen, FENG Yuankun. *Invert System Approach of Multivariable Nonlinear Control* [M]. Beijing: Tsinghua University Press, 1991.
(李春文,冯元琨.多变量非线性系统的逆系统方法[M].北京:清华大学出版社,1991.)

作者简介:

任海鹏 (1975 —),男,现为西安理工大学博士研究生,主要研究方向为混沌控制和应用,智能方法及应用,E-mail: renhaipeng@xaut.edu.cn;

刘 丁 (1957 —),男,教授,博士生导师,现为西安理工大学副校长,从事工业自动化,智能控制理论与应用等方面的研究.发表论文70余篇,获国家及省部级科技进步奖4项.