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Output feedback stabilization for stochastic time-delay nonlinear systems

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Abstract: A class of stochastic time-delay nonlinear systems was studied. A controller designed such that the closed-loop system was exponentially stable. It was shown that the stabilization via output feedback could be solved by a Lyapunov-based recursive design method.

Key words: stochastic time-delay nonlinear system; output feedback; backstepping; stabilization

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非线性随机时滞系统输出反馈镇定

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摘要: 研究了非线性随机时滞系统的镇定问题.运用 Backstepping 递推设计方法,得到了使得非线性随机时滞系统镇定的输出反馈控制器的设计算法.

关键词: 非线性随机时滞系统; 输出反馈; 递推设计; 镇定

1 Introduction

Stochastic modelling has played an important role in many branches of science and industry. An area of particular interest has been devoted to the automatic control of stochastic systems, with consequent emphasis placed on the analysis of stability in stochastic models. Time delay is commonly encountered and is often the resource of instability. It should be noted that the exponential stability of stochastic differential delay equations have been studied by many authors, e. g., Kushner[1] and Mao[2,3]. In this paper, we shall investigate an exponential stabilization problem for a class of stochastic time-delay nonlinear systems. Inspired by the recent work of stochastic free-delay nonlinear control[4,5], we show that the stabilization can be achieved for the stochastic time-delay nonlinear systems by employing a Lyapunov-based recursive controller design method. The output feedback control is considered. Our results extend existing stabilization results for stochastic systems without delay to control of stochastic time-delay nonlinear systems.

2 Preliminaries

Consider the stochastic time-delay nonlinear system

$$dx(t) = f(t,x(t),x(t-\tau))dt + g(t,x(t),x(t-\tau))d\omega \qquad (2.1)$$

on $t \geqslant 0$, where $x \in \mathbb{R}^n$ is the state with initial data $x(t) = \xi(t)$ for $-\tau \leqslant t \leqslant 0$, ω is an r-dimensional Brownian motion defined on a complete probability space (Ω, F, P) with a natural filtration $\{F_t\}_{t\geqslant 0}$ (i.e., $F_t = \sigma\{\omega(s): 0 \leqslant s \leqslant t\}$), $\tau > 0$ is the time lag. and $f: \mathbb{R}^n \to \mathbb{R}^n$ and $g: \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz continuous functions. Assume also that $f(t,0,0) = 0$ and $g(t,0,0) = 0$, so the equation admits a trivial solution $x(t,0) = 0$. Moreover, denoted by $L^2_{F_0}([-\tau,0],\mathbb{R}^n)$ there exists the family of $\mathbb{R}^n$- valued stochastic processes $\xi(s)$, $-\tau \leqslant s \leqslant 0$ such that $\xi(s)$ is F_0-measurable for every second and $\int_{-\tau}^{0} \mathrm{E}\,\|\xi(s)\|^p ds < \infty$ for $p > 0$.

Definition 1[6] The system (2.1) is said to be p-th

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moment exponentially stable if there exists a pair of constants $\lambda > 0$ and $\eta > 0$ such that for all ξ

$$E \| x(t,\xi) \|^p \leqslant \eta E \sup_{-\tau \leqslant \theta \leqslant 0} \| \xi(\theta) \|^p e^{-\lambda t}, \ t \geqslant 0, \tag{2.2}$$

where $p > 0$. When $p = 2$, it is usually called the exponential stability in mean square. Moreover, (2.2) implies

$$\limsup_{t \to \infty} \frac{1}{t} \log (E \| x(t,\xi) \|^p) \leqslant -\lambda \ \text{a.s.} \tag{2.3}$$

This left hand side term is called the p-th moment Lyapunov exponent of the solution.

Throughout this paper, we let

$$LV(x) = \frac{\partial V}{\partial x} f + \frac{1}{2} \mathrm{tr} \{ g^T \frac{\partial^2 V}{\partial x^2} g \}, \tag{2.4}$$

where tr $\{A\}$ is the trace of matrix A.

3 Output feedback control

In this section, we consider the following system:

$$\begin{cases} dx_i(t) = (x_{i+1}(t) + f_i(y(t)) + h_i(y(t-\tau)))dt + \\ \qquad g_i(y)^T d\omega, \ i = 1, \cdots, n-1, \\ dx_n(t) = (u(t) + f_n(y(t)) + h_n(y(t-\tau)))dt + g_n(y)^T d\omega, \\ y(t) = x_1(t), \end{cases} \tag{3.1}$$

where x_i, u and y represent the system states with initial data $x_i(t) = \xi_i(t)$ for $-\tau \leqslant t \leqslant 0$, control input and to-be-controlled output, respectively, ω and $\xi(t) = [\xi_1(t) \ \cdots \ \xi_n(t)]^T$ are the same as in the previous section. Given this system structure (3.1), we assume that the output y is measurable, and f_i, h_i and g_i with $f_i(0) = 0$, $h_i(0) = 0$ and $g_i(0) = 0$ are smooth nonlinear functions, so the equation admits a trivial solution $x(t,0) = 0$.

Since the states $x_2, \cdots, x_n$ are not measured, we first design an observer which would provide exponentially convergent estimates of the unmeasured states in the absence of noise. The observer is designed as

$$\begin{aligned} \dot{\hat{x}}_i &= \hat{x}_{i+1} + k_i(y - \hat{x}_1) + f_i(y(t)) + \\ &\quad h_i(y(t-\tau)), \ i = 1, \cdots, n, \end{aligned} \tag{3.2}$$

where $\dot{\hat{x}}_{n+1} = u$. The observation errors $\tilde{x} = x - \hat{x}$ satisfy

$$d\tilde{x}(t) = A\tilde{x}(t)dt + g(y(t))^T d\omega, \tag{3.3}$$

where $A = \begin{bmatrix} -k_1 & & \\ \vdots & & I \\ -k_n & 0 & \cdots & 0 \end{bmatrix}$, $g(y(t))^T = \begin{bmatrix} g_1(y(t))^T \\ \vdots \\ g_n(y(t))^T \end{bmatrix}$, and A is designed to be asymptotically stable. Now the entire system can be expressed as

$$\begin{cases} d\tilde{x}(t) = A\tilde{x}(t)dt + g(y(t))^T d\omega, \\ dy(t) = (\hat{x}_2(t) + \tilde{x}_2(t) + f_1(y(t)) + \\ \qquad h_1(y(t-\tau)))dt + g_1(y(t))^T d\omega, \\ d\hat{x}_2(t) = (\hat{x}_3 + k_2(y - \hat{x}_1) + f_2(y(t)) + \\ \qquad h_2(y(t-\tau)))dt, \\ \vdots \\ d\hat{x}_n = (u + k_n(y - \hat{x}_1) + f_n(y(t)) + h_n(y(t-\tau)))dt. \end{cases} \tag{3.4}$$

Our output feedback design will involve applying a backstepping procedure to the system $(y, \hat{x}_2, \cdots, \hat{x}_n)$, which also takes care of the feedback connection through the $\tilde{x}$ system. In the backstepping design, the error variables z_i are given by

$$\begin{aligned} z_1 &= y, \\ z_i &= \hat{x}_i - \alpha_{i-1}(\bar{\hat{x}}_{i-1}, y), \ i = 2, \cdots, n, \end{aligned} \tag{3.5}$$

where $\bar{\hat{x}}_i = [\hat{x}_2 \ \cdots \ \hat{x}_i]^T$. According to Ito's differentiation rule[7], we have

$$\begin{aligned} dz_1 &= (\hat{x}_2 + \tilde{x}_2 + f_1(y(t)) + h_1(y(t-\tau)))dt + \\ &\quad g_1(y(t))^T d\omega, \end{aligned} \tag{3.6a}$$

$$\begin{aligned} dz_i &= (\hat{x}_{i+1} + k_i\tilde{x}_1 + f_i + h_i - \\ &\quad \sum_{l=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_l}(\hat{x}_{l+1} + k_l\tilde{x}_l + f_l + h_l) - \\ &\quad \frac{\partial \alpha_{i-1}}{\partial y}(\hat{x}_2 + \tilde{x}_2 + f_1 + h_1) - \\ &\quad \frac{1}{2}\frac{\partial^2 \alpha_{i-1}}{\partial y^2} g_1(y)^T g_1(y))dt - \\ &\quad \frac{\partial \alpha_{i-1}}{\partial y} g_1(y)^T d\omega, \ i = 1, \cdots, n. \end{aligned} \tag{3.6b}$$

We start by an important preparatory comment. Since $f(0) = 0$, $h(0) = 0$ and $g(0) = 0$, $f(y(t))$, $h(y(t-\tau))$ and $g(y(t))$ can be expressed, respectively, as

$$\begin{cases} f(y(t)) = y(t)\bar{f}(y(t)), \\ h(y(t-\tau)) = y(t-\tau)\bar{h}(y(t-\tau)), \\ g(y(t))^T = y(t)\bar{g}(y(t))^T, \end{cases} \tag{3.7}$$

where

$$\bar{f}(y(t)) = \begin{bmatrix} \bar{f}_1(y(t)) \\ \vdots \\ \bar{f}_n(y(t)) \end{bmatrix},$$

$$\bar{h}(y(t-\tau)) = \begin{bmatrix} \bar{h}_1(y(t-\tau)) \\ \vdots \\ \bar{h}_n(y(t-\tau)) \end{bmatrix},$$

$$\bar{g}(y(t))^{\mathrm{T}} = \begin{bmatrix} \bar{g}_1(y(t))^{\mathrm{T}} \\ \vdots \\ \bar{g}_n(y(t))^{\mathrm{T}} \end{bmatrix}.$$

Now, we are ready to start the backstepping design procedure. We employ a Lyapunov function of a form

$$V(\tilde{x},y,z) = \frac{a}{2}(\tilde{x}^{\mathrm{T}}P\tilde{x})^2 + \frac{1}{4}y^4 + \frac{1}{4}\sum_{i=2}^{n} z_i^4 + \int_{t-\tau}^{t} q(y(\tau))\mathrm{d}\tau, \tag{3.8}$$

where P is a positive definite matrix which satisfies $A^{\mathrm{T}}P + PA = -I$, and $q(y)$ is a positive function yet to be determined.

Now we start the process of selecting the functions α_i to make LV negative definite. Along the trajectories of (3.3) and (3.6), we have

$$\begin{aligned} LV = & \\ & -a\,\tilde{x}^{\mathrm{T}}P\,\tilde{x}\,\|\,\tilde{x}\,\|^2 + \\ & 2a\mathrm{tr}\,\{g(y)(2Px\,\tilde{x}^{\mathrm{T}}P + \tilde{x}^{\mathrm{T}}P\,\tilde{x}P)g(y)^{\mathrm{T}}\} + \\ & y^3(\alpha_1 + z_2 + \tilde{x}_2 + f_1(y) + \\ & h_1(y(t-\tau))) + \frac{3}{2}y^2 g_1(y)^{\mathrm{T}} g_1(y) + \\ & \sum_{i=2}^{n} z_i^3(\alpha_i + z_{i+1} + k_i\tilde{x}_1 + f_i + h_i - \\ & \sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}(\hat{x}_{l+1} + k_l\tilde{x}_1 + f_l + h_l) - \\ & \frac{\partial\alpha_{i-1}}{\partial y}(\hat{x}_2 + \tilde{x}_2 + f_1 + h_1) - \\ & \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_1(y)^{\mathrm{T}}g_1(y)) + \\ & \frac{3}{2}\sum_{i=2}^{n} z_i^2(\frac{\partial\alpha_{i-1}}{\partial y})^2 g_1(y)^{\mathrm{T}}g_1(y) + \\ & q(y(t)) - q(y(t-\tau)) \leqslant \\ & -\Pi_0\,\|\,\tilde{x}\,\|^4 + y^3(\alpha_1 + \Pi_1) + \\ & \sum_{i=2}^{n-1} z_i^3(\alpha_i + \Pi_i) + z_n^3(u + \Pi_n) + q(y(t)) - \\ & q(y(t-\tau)) + (\frac{1}{4\xi_1^4} + \sum_{i=2}^{n}\frac{1}{4\gamma_i^4} + \sum_{i=2}^{n}\frac{1}{4\lambda_i^4} + \\ & \sum_{i=2}^{n}\sum_{l=2}^{i-1}\frac{1}{4\mu_{il}^4})y^4(t-\tau)\,\|\,\bar{h}(y(t-\tau))\,\|^4, \end{aligned}$$

where

$$-\Pi_0 = -a\,\lambda_{\min} + 3an\sqrt{n}\,\varepsilon_2^2\,\|\,P\,\|^4 + \frac{1}{4\varepsilon_1^4} + \frac{1}{4}\sum_{i=2}^{n}\frac{1}{\eta_i^4}, \tag{3.9}$$

$$\begin{aligned} \Pi_1 = & \frac{3}{4}\delta_1^{\frac{4}{3}}y + \frac{3}{4}\varepsilon_1^{\frac{4}{3}}y + \frac{3}{4}\xi_1^{\frac{4}{3}}y + \\ & \frac{3}{4}\sum_{i=2}^{n}\xi_i^2 y(\bar{g}_1(y)^{\mathrm{T}}\bar{g}_1(y))^2 + f_1(y) + \\ & \frac{3an\sqrt{n}}{\varepsilon_2^2}y\,\|\,\bar{g}(y)\,\|^4 + \frac{3}{2}y\bar{g}_1(y)^{\mathrm{T}}\bar{g}_1(y), \end{aligned} \tag{3.10}$$

$$\begin{aligned} \Pi_i = & \frac{3}{4}\delta_i^{\frac{4}{3}}z_i + \frac{1}{4\delta_{i-1}^4}z_i + k_i\tilde{x}_1 + f_i - \\ & \sum_{l=2}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l}(\hat{x}_{l+1} + k_l\tilde{x}_1 + f_l) - \frac{\partial\alpha_{i-1}}{\partial y}\hat{x}_2 - \\ & \frac{1}{2}\frac{\partial^2\alpha_{i-1}}{\partial y^2}g_1(y)^{\mathrm{T}}g_1(y) + \frac{3}{4}\eta_i^{\frac{4}{3}}(\frac{\partial\alpha_{i-1}}{\partial y})^{\frac{4}{3}}z_i + \\ & \frac{3}{4}\lambda_i^{\frac{4}{3}}z_i + \frac{3}{4}\gamma_i^{\frac{4}{3}}(\frac{\partial\alpha_{i-1}}{\partial y})^{\frac{4}{3}}z_i + \\ & \frac{3}{4}\frac{1}{\xi_i^2}(\frac{\partial\alpha_{i-1}}{\partial y})^4 z_i + z_i\sum_{l=2}^{i-1}\frac{3}{4}\mu_{il}^{\frac{4}{3}}(\frac{\partial\alpha_{i-1}}{\partial\hat{x}_l})^{\frac{4}{3}}, \end{aligned} \tag{3.11}$$

$$\begin{aligned} \Pi_n = & \frac{1}{4\delta_{n-1}^4}z_n + k_n\tilde{x}_1 + f_n - \\ & \sum_{l=2}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{x}_l}(\hat{x}_{l+1} + k_l\tilde{x}_1 + f_l) - \\ & \frac{\partial\alpha_{n-1}}{\partial y}\hat{x}_2 - \frac{1}{2}\frac{\partial^2\alpha_{n-1}}{\partial y^2}g_1(y)^{\mathrm{T}}g_1(y) + \\ & \frac{3}{4}\eta_n^{\frac{4}{3}}(\frac{\partial\alpha_{n-1}}{\partial y})^{\frac{4}{3}}z_n + \frac{3}{4}\lambda_n^{\frac{4}{3}}z_n + \\ & \frac{3}{4}\gamma_n^{\frac{4}{3}}(\frac{\partial\alpha_{n-1}}{\partial y})^{\frac{4}{3}}z_n + \frac{3}{4}\frac{1}{\xi_n^2}(\frac{\partial\alpha_{n-1}}{\partial y})^4 z_n + \\ & z_n\sum_{l=2}^{n-1}\frac{3}{4}\mu_{nl}^{\frac{4}{3}}(\frac{\partial\alpha_{n-1}}{\partial\hat{x}_l})^{\frac{4}{3}}, \end{aligned} \tag{3.12}$$

where $\lambda_{\min} > 0$ is the smallest eigenvalue of P, and ξ_1, $\varepsilon_i, \delta_i, \eta_i, \lambda_i, \mu_{il}$ and γ_i are positive constants to be chosen by the designer. The inequality comes from Young's inequality in Appendix B and the result of [4]. Due to the limit of the paper space, the details are omitted. At this point, we can see that all the terms can be cancelled by q, u and α_i. If we choose $a, \varepsilon_1, \varepsilon_2$ and η_i to satisfy

$$-\Pi_0 < 0, \tag{3.13}$$

and q, α_i and u as

$$q(y(t)) = \Big(\frac{1}{4\xi_1^4} + \sum_{i=2}^{n}\frac{1}{4\gamma_i^4} + \sum_{i=2}^{n}\frac{1}{4\lambda_i^4} + \sum_{i=2}^{n}\sum_{l=2}^{i-1}\frac{1}{4\mu_{il}^4}\Big)y^4(t)\,\|\bar{h}(y(t))\|^4, \tag{3.14}$$

$$\begin{cases}\alpha_1 = -c_1 y - \Pi_1 - \dfrac{1}{y^3(t)}q(y(t)),\\ \alpha_i = -c_i z_i - \Pi_i,\\ u = -c_n z_n - \Pi_n,\end{cases} \tag{3.15}$$

where $c_i > 0$, then the infinitesimal generator of the closed-loop systems (3.3), (3.6) and (3.15) is negative definite.

$$LV \leqslant -\Pi_0\|\tilde{x}\|^4 - \sum_{i=1}^{n}c_i z_i^4. \tag{3.16}$$

Assumption 1 $\|\bar{h}(y(t))\| \leqslant \bar{\sigma}_h$, where $\bar{\sigma}_h$ is a positive constant.

Along the trajectories of the system (3.4), by Ito's differentiation rule[7] we have

$$dV = LVdt + (2a\,\tilde{x}^{\mathrm{T}}P\tilde{x}\tilde{x}^{\mathrm{T}}Pg^{\mathrm{T}} + y^3 g_1^{\mathrm{T}})d\omega. \tag{3.17}$$

With (3.16) and (3.17), we have

$$\begin{aligned} d(e^{\varepsilon t}V) &= e^{\varepsilon t}(\varepsilon V dt + dV) \leqslant \\ & e^{\varepsilon t}\Big(\frac{a\varepsilon}{2}\lambda_{\max}^2\ \|\tilde{x}\|^4 + \frac{\varepsilon}{4}y^4 + \frac{\varepsilon}{4}\sum_{i=2}^{n}z_i^4 - \\ & \Pi_0\|\tilde{x}\|^4 - \sum_{i=1}^{n}c_i z_i^4 + \varepsilon\sigma_h\int_{t-\tau}^{t}y^4(\theta)d\theta\Big)dt + \\ & (2a\,\tilde{x}^{\mathrm{T}}P\tilde{x}\tilde{x}^{\mathrm{T}}Pg^{\mathrm{T}} + y^3 g_1^{\mathrm{T}})d\omega \leqslant \\ & e^{\varepsilon t}\Big(\sigma_a\varepsilon\sum_{i=0}^{n}z_i^4 - c\sum_{i=0}^{n}z_i^4 + \sigma_h\varepsilon\int_{t-\tau}^{t}\sum_{i=0}^{n}z_i^4(\theta)d\theta\Big)dt + \\ & (2a\tilde{x}^{P}\tilde{x}\tilde{x}^{\mathrm{T}}Pg^{\mathrm{T}} + y^3 g_1^{\mathrm{T}})d\omega, \end{aligned} \tag{3.18}$$

where $\sigma_h = \Big(\frac{1}{4\xi_1^4} + \sum_{i=2}^{n}\frac{1}{4\gamma_i^4} + \sum_{i=2}^{n}\frac{1}{4\lambda_i^4} + \sum_{i=2}^{n}\sum_{l=2}^{i-1}\frac{1}{4\mu_{il}^4}\Big)\bar{\sigma}_h^4$, $c = \min_i\{c_i, \Pi_0\}$, $z_0^4 = \|\tilde{x}\|^4$, $\sigma_a = \max\{\frac{a}{2}\lambda_{\max}^2, \frac{1}{4}\}$ and $\lambda_{\max} > 0$ the biggest eigenvalue of P.

For any given $T > 0$, integrating both sides of (3.18) from 0 to T and then taking expectation we obtain that

$$e^{\varepsilon t}\mathrm{E}\ V \leqslant \alpha + (\sigma_a\varepsilon - c)\mathrm{E}\int_0^T e^{\varepsilon t}\sum_{i=0}^{n}z_i^4(t)dt + \varepsilon\sigma_h\mathrm{E}\int_0^T e^{\varepsilon t}\int_{t-\tau}^{t}\sum_{i=0}^{n}z_i^4(\theta)d\theta dt, \tag{3.19}$$

where $\alpha = (\sigma_a + \tau\varepsilon\sigma_h)\sup_{-\tau\leqslant\theta\leqslant 0}\mathrm{E}\sum_{i=0}^{n}z_i^4(\theta)$.

Compute

$$\begin{aligned} &\int_0^T e^{\varepsilon t}\int_{t-\tau}^{t}\sum_{i=0}^{n}z_i^4(\theta)d\theta dt = \\ &\int_{-\tau}^{T-\tau}\Big(\int_{\theta\vee 0}^{(\theta+\tau)\wedge T}e^{\varepsilon t}dt\Big)\sum_{i=0}^{n}z_i^4(\theta)d\theta \leqslant \\ &\int_{-\tau}^{T}\Big(\int_{\theta}^{\theta+\tau}e^{\varepsilon t}dt\Big)\sum_{i=0}^{n}z_i^4(\theta)d\theta \leqslant \\ &\int_{-\tau}^{T}\tau e^{\varepsilon(\theta+\tau)}\sum_{i=0}^{n}z_i^4(\theta)d\theta \leqslant \\ &\tau e^{\varepsilon\tau}\int_0^T e^{\varepsilon t}\sum_{i=0}^{n}z_i^4(t)dt + \tau e^{\varepsilon\tau}\int_{-\tau}^{0}\sum_{i=0}^{n}z_i^4(t)dt. \end{aligned} \tag{3.20}$$

If $c = \sigma_a\varepsilon + \varepsilon\sigma_h\tau e^{\varepsilon\tau}$, substituting (3.20) into (3.19) we obtain that

$$e^{\varepsilon t}\mathrm{E}\ V \leqslant \alpha + \beta, \tag{3.21}$$

where $\beta = \varepsilon\sigma_h\tau^2 e^{\varepsilon t}\sup_{-\tau\leqslant\theta\leqslant 0}\mathrm{E}\sum_{i=0}^{n}z_i^4(\theta)$.

In consequence, since $T > 0$ is arbitrary, we have

$$\mathrm{E}\|z(t)\|_4^4 \leqslant \eta\sup_{-\tau\leqslant\theta\leqslant 0}\|z(\theta)\|_4^4 e^{-\varepsilon t},\ t \geqslant 0, \tag{3.22}$$

where $\|z\|_4 = (\sum_{i=0}^{n}z_i^4)^{\frac{1}{4}}$ and $\eta = \dfrac{\sigma_a + \varepsilon\tau\sigma_h + \varepsilon\sigma_h\tau^2 e^{\varepsilon\tau}}{\min\{\frac{a}{2}\lambda_{\min}^2, \frac{1}{4}\}}$.

With (3.22), we have the following stability result.

Theorem 1 Consider the stochastic time-delay nonlinear system (3.1) satisfying Assumption 1. If $a, \varepsilon_1, \varepsilon_2, \eta_i, c_i$ and ε are chosen such that

$$c = \sigma_a\varepsilon + \varepsilon\sigma_h\tau e^{\varepsilon\tau},$$

then the fourth-moment exponential stability is guaranteed via output feedback.

4 Conclusion

The problem of exponential stabilization was studied in this paper. It has been shown that the output feedback nonlinear control problems for a class of stochastic time-delay nonlinear systems can be solved by using a Lyapunov-based recursive design approach. Our results extend the existing stabilization of stochastic nonlinear systems without delay to the control of stochastic time-delay nonlinear systems. The proposed method can be extended to the control of stochastic multi-time-delay nonlinear systems.

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Appendix B

In this appendix, we use Young's inequality[8]

$$xy \leqslant \frac{\varepsilon^p}{p} | x |^p + \frac{1}{q\varepsilon^q} | y |^q,$$

where $\varepsilon > 0$, the constants $p > 1$ and $q > 1$ satisfy $(p - 1)(q - 1) = 1$, and $(x, y) \in \mathbb{R}^2$.

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