

多时变状态和控制时滞系统的绝对稳定性

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摘要: 研究了具有多时变状态和控制时滞的 Lurie 控制系统基于线性矩阵不等式(LMI)的绝对稳定性条件. 首先, 构造了关于 Lyapunov 泛函中正定矩阵和积分项系数等自由参数的 LMI, 获得了系统时滞无关绝对稳定条件. 进一步引入自由权矩阵来表示牛顿-莱布尼兹公式中各项的相互关系, 得到了系统绝对稳定的时滞相关条件. 最后, 通过一个实例阐述了本文方法的有效性和相比已有结果的优越性.

关键词: Lurie 控制系统; 绝对稳定; 线性矩阵不等式; 时变时滞

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Absolute stability for Lurie control systems with time-varying delays in both state and control input

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Abstract: Based on the linear matrix inequality (LMI), some absolute stability conditions for Lurie control systems with time-varying delays in both state and control input are derived. First, the delay-independent absolute stability conditions were obtained, which were described by LMI for the free parameters such as the positive definite matrix and the coefficients of the integral terms in the Lyapunov functional. Moreover, new delay-dependent absolute stability criteria were presented, in which some free weighting matrices that express the relationships between the terms in the Leibniz-Newton formula were introduced. Finally, a numerical example was provided to demonstrate the effectiveness of the proposed method.

Key words: Lurie control system; absolute stability; linear matrix inequality (LMI); time-varying delays

1 引言(Introduction)

近年来,对于具有滞后型非线性反馈的 Lurie 控制系统,文献[1~3]讨论了其绝对稳定性的时滞无关条件,其中,文献[1,2]利用矩阵范数得到的充分条件通常是比较保守的,文献[3]利用线性矩阵不等式(LMI),所得结果具有可解性,降低了文献[1,2]的保守性,但其结果也只适用于有限扇形角的情形,且在非线性条件的处理上,它采用的是一个简单的向量不等式,导致较大的保守性.另外,基于文献[4]的 Park 的不等式,文献[3]还给出了具有滞后型非线性反馈的 Lurie 控制系统用 LMI 描述的绝对稳定的时滞相关条件,如文献[5]所述,其利用牛顿-莱布尼兹公式将 Lyapunov 泛函的导数中某些 $x(t - \tau)$ 用 $x(t) - \int_{t-\tau}^t \dot{x}(s)ds$ 代替,而另外的 $x(t - \tau)$ 却

保留下来,也就是说,他们采用的是固定的权矩阵,是导致时滞相关条件保守性的一个重要原因.

本文对一般的具有多时变状态和控制时滞的 Lurie 控制系统,采用 S-过程处理非线性反馈项并获得绝对稳定的时滞无关条件,减少了向量不等式带来的保守性,并且所得结果不仅适用于有限扇形角,而且适用于无穷扇形角.同时利用文献[5]提出的自由权矩阵方法来考虑 $x(t - \tau)$ 和 $x(t) - \int_{t-\tau}^t \dot{x}(s)ds$ 相互关系,使得表示它们相互关系的最优权矩阵可以通过 LMI 的解获得,通过其解来构造 Lyapunov 泛函,减少了 Lyapunov 泛函中自由参数选择的随意性,克服了已有方法的保守性.

2 系统描述(Problem statement)

考虑具有多时变状态和控制时滞的 Lurie 控制

系统

$$\begin{cases} \dot{x}(t) = Ax(t) + \sum_{i=1}^l B_i x(t - h_i(t)) + \\ \sum_{i=0}^l D_i f(\sigma(t - h_i(t))), \\ \sigma(t) = C^T x(t), x(t) = \varphi(t), t \in [-h, 0], \end{cases} \quad (1)$$

这里

$x(t) \in \mathbb{R}^n, A, B_i (i = 1, 2, \dots, l) \in \mathbb{R}^{n \times n},$
 $D_i (i = 0, 1, \dots, l) \in \mathbb{R}^{n \times m},$
 $C = (c_1, c_2, \dots, c_m) \in \mathbb{R}^{n \times m},$
 $c_j \in \mathbb{R}^n (j = 1, 2, \dots, m),$
 $\sigma(t) = (\sigma_1(t), \sigma_2(t), \dots, \sigma_m(t))^T,$
 $f(\sigma(t)) = (f_1(\sigma_1(t)), f_2(\sigma_2(t)), \dots, f_m(\sigma_m(t)))^T.$
 时滞 $h_i(t) (i = 0, 1, 2, \dots, l)$ 是时变连续函数且满足

$$\begin{aligned} h_0(t) &= 0, 0 \leq h_i(t) \leq \bar{h}_i, \\ 0 \leq \dot{h}_i(t) &\leq \mu_i < 1, i = 1, 2, \dots, l, \end{aligned} \quad (2)$$

其中, $\bar{h}_i, \mu_i (i = 1, 2, \dots, l)$ 是常数, 并且 $h = \max \{\bar{h}_i (i = 1, 2, \dots, l)\}.$ 每一个非线性反馈项具有无穷扇形角约束, 即

$$\begin{aligned} f_j(\cdot) &\in K_j[0, \infty] = \\ \{f_j(\sigma_j) \mid f_j(0) &= 0, \\ \sigma_j f_j(\sigma_j) > 0 (\sigma_j \neq 0)\}, j &= 1, 2, \dots, m, \end{aligned} \quad (3)$$

或具有有限扇形角约束, 即

$$\begin{aligned} f_j(\cdot) &\in K_j[0, k_j] = \\ \{f_j(\sigma_j) \mid f_j(0) &= 0, \\ 0 < \sigma_j f_j(\sigma_j) \leq k_j \sigma_j^2 (\sigma_j \neq 0)\}, j &= 1, 2, \dots, m. \end{aligned} \quad (4)$$

3 绝对稳定性的时滞无关条件(Delay-independent absolute stability conditions)

考虑具有多时变状态和控制时滞的控制系统(1)在无穷扇形角(3)和(4)下的绝对稳定性, 有

定理 1 对给定的 $0 \leq \mu_i < 1 (i = 1, 2, \dots, l),$ 如果 LMI

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} + CS_0 & \Phi_{14} \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} & \Phi_{24} + \Psi_{24} \\ \Phi_{13}^T + S_0 C^T & \Phi_{23}^T & \Phi_{33} & \Phi_{34} \\ \Phi_{14}^T & \Phi_{24}^T + \Psi_{24}^T & \Phi_{34}^T & \Phi_{44} \end{bmatrix} < 0, \quad (5)$$

存在关于

$P > 0, Q_i > 0, R_i > 0 (i = 1, 2, \dots, l),$
 $S_i = \text{diag}\{s_{i1}, s_{i2}, \dots, s_{im}\} \geq 0 (i = 0, 1, 2, \dots, l),$
 $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\} \geq 0$
 的可行解, 则系统(1)在无穷扇形角(3)下是绝对稳定的. 其中

$$\begin{aligned} \Phi_{11} &= A^T P + PA + \sum_{i=1}^l Q_i, \\ \Phi_{12} &= [PB_1 \quad PB_2 \quad \dots \quad PB_l], \\ \Phi_{13} &= A^T CA + PD_0, \\ \Phi_{14} &= [PD_1 \quad PD_2 \quad \dots \quad PD_l], \\ \Phi_{22} &= \text{diag}\{-(1-\mu_1)Q_1, -(1-\mu_2)Q_2, \dots, -(1-\mu_l)Q_l\}, \\ \Phi_{23} &= [\Lambda C^T B_1 \quad \Lambda C^T B_2 \quad \dots \quad \Lambda C^T B_l]^T, \\ \Phi_{24} &= [0 \quad 0 \quad \dots \quad 0], \Phi_{33} = \Lambda C^T D_0 + D_0^T CA + \sum_{i=1}^l R_i, \\ \Phi_{34} &= [\Lambda C^T D_1 \quad \Lambda C^T D_2 \quad \dots \quad \Lambda C^T D_l], \\ \Phi_{44} &= \text{diag}\{-(1-\mu_1)R_1, -(1-\mu_2)R_2, \dots, -(1-\mu_l)R_l\}, \\ \Psi_{24} &= \text{diag}\{CS_1, CS_2, \dots, CS_l\}. \end{aligned}$$

定理 2 对于给定的 $0 \leq \mu_i < 1 (i = 1, 2, \dots, l),$ 如果如下 LMI

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} + CKS_0 & \Phi_{14} \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} & \Phi_{24} + \Omega_{24} \\ \Phi_{13}^T + S_0 KC^T & \Phi_{23}^T & \Phi_{33} - 2S_0 & \Phi_{34} \\ \Phi_{14}^T & \Phi_{24}^T + \Omega_{24}^T & \Phi_{34}^T & \Phi_{44} + \Omega_{44} \end{bmatrix} < 0, \quad (6)$$

存在关于

$P > 0, Q_i > 0, R_i > 0 (i = 1, 2, \dots, l),$
 $S_i = \text{diag}\{s_{i1}, s_{i2}, \dots, s_{im}\} \geq 0 (i = 0, 1, 2, \dots, l),$
 $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_m\} \geq 0$
 的可行解, 则系统(1)在有限扇形角(4)下是绝对稳定的. 其中

$$\begin{aligned} \Omega_{24} &= \text{diag}[CKS_1, CKS_2, \dots, CKS_l], \\ \Omega_{44} &= \text{diag}\{-2S_1, -2S_2, \dots, -2S_l\}, \\ K &= \text{diag}\{k_1, k_2, \dots, k_m\}, \end{aligned}$$

$\Phi_{ij} (i = 1, \dots, 4; i \leq j \leq 4)$ 定义于式(5).

4 绝对稳定性的时滞相关条件(Delay-dependent absolute stability conditions)

考虑具有多时变状态和控制时滞的控制系统(1)在无穷扇形角(3)和(4)下当时滞满足条件(2)时的绝对稳定性, 设

$$\begin{aligned} \Gamma &= (\Gamma_{ij})_{4 \times 4}, \Gamma_{ij} = \Phi_{ij} + \Xi_{ij} + \Theta_{ij}, \\ i &= 1, \dots, 4, i \leq j \leq 4, \end{aligned}$$

Φ_{ij} 定义于式(5),

$$\Xi_{11} = \sum_{j=1}^l (M_{0j} + M_{0j}^T + \bar{h}_j X_{0j}), \Xi_{12} = \left[\sum_{j=1}^l M_{1j}^T - M_{01} \quad \sum_{j=1}^l M_{2j}^T - M_{02} \quad \cdots \quad \sum_{j=1}^l M_{lj}^T - M_{0l} \right],$$

$$\Xi_{13} = \sum_{j=1}^l N_{0j}^T, \Xi_{14} = \left[\sum_{j=1}^l N_{1j}^T \quad \sum_{j=1}^l N_{2j}^T \quad \cdots \quad \sum_{j=1}^l N_{lj}^T \right],$$

$$\Xi_{22} = \begin{bmatrix} \sum_{j=1}^l (\bar{h}_j X_{1j}) - M_{11} - M_{11}^T & -M_{12} - M_{21}^T & \cdots & -M_{1l} - M_{l1}^T \\ -M_{12}^T - M_{21} & \sum_{j=1}^l (\bar{h}_j X_{2j}) - M_{22} - M_{22}^T & \cdots & -M_{2l} - M_{l2}^T \\ \vdots & \vdots & \ddots & \vdots \\ -M_{1l}^T - M_{l1} & -M_{2l}^T - M_{l2} & \cdots & \sum_{j=1}^l (\bar{h}_j X_{lj}) - M_{ll} - M_{ll}^T \end{bmatrix}$$

$$\Xi_{23} = [-N_{01} \quad -N_{02} \quad \cdots \quad -N_{0l}]^T, \Xi_{24} = \begin{bmatrix} -N_{11}^T & -N_{21}^T & \cdots & -N_{l1}^T \\ -N_{12}^T & -N_{22}^T & \cdots & -N_{l2}^T \\ \vdots & \vdots & \ddots & \vdots \\ -N_{1l}^T & -N_{2l}^T & \cdots & -N_{ll}^T \end{bmatrix},$$

$$\Xi_{33} = \sum_{j=1}^l \bar{h}_j Y_{0j}, \Xi_{34} = [0 \quad 0 \quad \cdots \quad 0], \Xi_{44} = \text{diag} \left(\sum_{j=1}^l \bar{h}_j Y_{1j}, \sum_{j=1}^l \bar{h}_j Y_{2j}, \cdots, \sum_{j=1}^l \bar{h}_j Y_{lj} \right),$$

$$\Theta = (\Theta_{ij})_{4 \times 4} = \begin{bmatrix} [A^T H A] & [A^T H B_1] & \cdots & [A^T H B_l] & [A^T H D_0] & [A^T H D_1] & \cdots & [A^T H D_l] \\ [B_1^T H A] & [B_1^T H B_1] & \cdots & [B_1^T H B_l] & [B_1^T H D_0] & [B_1^T H D_1] & \cdots & [B_1^T H D_l] \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ [B_l^T H A] & [B_l^T H B_1] & \cdots & [B_l^T H B_l] & [B_l^T H D_0] & [B_l^T H D_1] & \cdots & [B_l^T H D_l] \\ [D_0^T H A] & [D_0^T H B_1] & \cdots & [D_0^T H B_l] & [D_0^T H D_0] & [D_0^T H D_1] & \cdots & [D_0^T H D_l] \\ [D_1^T H A] & [D_1^T H B_1] & \cdots & [D_1^T H B_l] & [D_1^T H D_0] & [D_1^T H D_1] & \cdots & [D_1^T H D_l] \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ [D_l^T H A] & [D_l^T H B_1] & \cdots & [D_l^T H B_l] & [D_l^T H D_0] & [D_l^T H D_1] & \cdots & [D_l^T H D_l] \end{bmatrix},$$

$$H = \sum_{j=1}^l \bar{h}_j W_j, \Pi_j = \begin{bmatrix} X_{0j} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & M_{0j} \\ 0 & X_{1j} & \cdots & 0 & 0 & 0 & \cdots & 0 & M_{1j} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & X_{lj} & 0 & 0 & \cdots & 0 & M_{lj} \\ 0 & 0 & \cdots & 0 & Y_{0j} & 0 & \cdots & 0 & N_{0j} \\ 0 & 0 & \cdots & 0 & 0 & Y_{1j} & \cdots & 0 & N_{1j} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & Y_{lj} & N_{lj} \\ M_{0j}^T & M_{1j}^T & \cdots & M_{lj}^T & N_{0j}^T & N_{1j}^T & \cdots & N_{lj}^T & W_j \end{bmatrix}, j = 1, 2, \dots, l.$$

定理 3 对给定 $\bar{h}_j \geq 0, 0 \leq \mu_j < 1 (j = 1, 2, \dots, l)$, 0 存在关于 l), 如果一组 LMIs, $\Pi_j \geq 0 (j = 1, 2, \dots, l)$ 和 $\bar{\Gamma} < P > 0, Q_i > 0, R_i > 0 (i = 1, 2, \dots, l)$,

$S_i = \text{diag} \{s_{i1}, s_{i2}, \dots, s_{im}\} \geq 0$ ($i = 0, 1, 2, \dots, l$),
 $\Lambda = \text{diag} \{\lambda_1, \lambda_2, \dots, \lambda_m\} \geq 0$, $X_{ij} \geq 0$, $Y_{ij} \geq 0$,
 M_{ij}, N_{ij} ($i = 0, 1, \dots, l; j = 1, 2, \dots, l$), $W_j \geq 0$ ($j = 1, 2, \dots, l$) 的可行解, 则系统(1)在无穷扇形角(3)下是绝对稳定的. 其中,

$$\bar{\Gamma} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} + CS_0 & \Gamma_{14} \\ \Gamma_{12}^T & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} + \Psi_{24} \\ \Gamma_{13}^T + S_0 C^T & \Gamma_{23}^T & \Gamma_{33} & \Gamma_{34} \\ \Gamma_{14}^T & \Gamma_{24}^T + \Psi_{24}^T & \Gamma_{34}^T & \Gamma_{44} \end{bmatrix},$$

Ψ_{24} 定义于式(5).

定理 4 对给定 $\bar{h}_j \geq 0, 0 \leq \mu_j < 1$ ($j = 1, 2, \dots, l$), 如果一组 LMIs, $\Pi_j \geq 0$ ($j = 1, 2, \dots, l$) 和 $\bar{\Gamma} < 0$ 存在关于

$P > 0, Q_i > 0, R_i > 0$ ($i = 1, 2, \dots, l$),
 $S_i = \text{diag} \{s_{i1}, s_{i2}, \dots, s_{im}\} \geq 0$ ($i = 0, 1, 2, \dots, l$),
 $\Lambda = \text{diag} \{\lambda_1, \lambda_2, \dots, \lambda_m\} \geq 0$, $X_{ij} \geq 0$, $Y_{ij} \geq 0$,
 M_{ij}, N_{ij} ($i = 0, 1, \dots, l; j = 1, 2, \dots, l$),
 $W_j \geq 0$ ($j = 1, 2, \dots, l$) 的可行解, 则系统(1)在有限扇形角(4)下是绝对稳定的. 其中,

$$\bar{\Gamma} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} + CKS_0 & \Gamma_{14} \\ \Gamma_{12}^T & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} + \Omega_{24} \\ \Gamma_{13}^T + S_0 KC^T & \Gamma_{23}^T & \Gamma_{33} - 2S_0 & \Gamma_{34} \\ \Gamma_{14}^T & \Gamma_{24}^T + \Omega_{24}^T & \Gamma_{34}^T & \Gamma_{44} + \Omega_{44} \end{bmatrix},$$

Ω_{24} 和 Ω_{44} 定义于式(6).

注 1 定理 3 和定理 4 的时滞相关条件事实上包含了定理 1 和定理 2 的时滞无关条件. 因为设定自由参数矩阵 $W_j = 0$ ($j = 1, 2, \dots, l$), $X_{ij} = 0, Y_{ij} = 0, M_{ij} = 0, N_{ij} = 0$, ($i = 0, 1, \dots, l; j = 1, 2, \dots, l$), 定理 3 和定理 4 就分别变成了定理 1 和定理 2, 也就是说, 如果系统(1)由定理 1 或定理 2 判定是时滞无关稳定的, 那么对于任意时滞, 定理 3 或定理 4 的条件肯定满足, 只要设定上述自由参数矩阵为 0, 其他参数矩阵用定理 1 或定理 2 的解即可判定系统的绝对稳定性.

5 实例 (Example)

考虑系统(1), 假设

$$n = 2, m = 1, l = 1,$$

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 0 \end{bmatrix},$$

$$D_0 = [0 \ 0]^T, D_1 = [0.5 \ 0.5]^T, C = [1 \ -1]^T.$$

若 $k_1 = 0.5, \mu_1 = 0$, 利用 MATLAB 解定理 2 的 LMI (6) 可知, 系统时滞无关稳定. 同时, 对于任意给定的时滞, 时滞相关定理 4 给定的一组 LMIs 也有解, 这就验证了注 1 的说明. 本文的结果已经大大优于文

献[3]([3]获得时滞界为 0.69). 更进一步, 下表还列出了利用定理 2 和定理 4 计算的不同扇形角和不同时滞导数的界所对应的最大时滞界限.

表 1 扇形角与保证系统绝对稳定的时滞界的关系
 Table 1 Relationship between the sector and the upper bound of delay guaranteeing absolute stability

$f_1(\cdot)$	$K_1[0, 0.5]$	$K_1[0, 3]$	$K_1[0, 10]$
$\mu_1 = 0$	任意时滞	任意时滞	0.71
$\mu_1 = 0.5$	任意时滞	2.84	0.52
$\mu_1 = 0.9$	2.66	1.38	0.48

6 结论 (Conclusion)

本文对于具有多时变状态和控制时滞以及无穷扇形角或有限扇形角的 Lurie 控制系统, 基于 S-过程和 LMI, 分别获得了系统时滞无关和时滞相关绝对稳定的准则, 克服了已有结果的保守性, 并说明了时滞相关条件和时滞无关条件的相互关系. 最后给出实例说明本文方法的有效性以及扇形角大小、时滞界和时滞导数上限的关系.

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