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时滞切换系统动态反馈H_∞控制

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摘要: 对于由若干线性时滞子系统构成的切换系统, 考虑了动态反馈控制与切换策略的设计, 以实现H_∞性能的优化. 利用其连续性不受切换行为影响的Lyapunov-Krasovskii泛函构造方式, 并结合闭环子系统的适当变换, 导出了与时滞相关的控制器及切换策略的存在性判据.通过参数代换与矩阵相似变换, 将此判据等价地转化为线性矩阵不等式, 从而解得泛函与控制器参数. 仿真结果验证了方法的有效性.

关键词: 切换系统; 时滞; 动态反馈控制; 线性矩阵不等式

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Dynamic feedback H-infinity control of switched systems with time-delay

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Abstract: For the switched system composed of linear subsystems with time-delay, the synthesis of dynamic feedback control and switching strategy is concerned with the optimization of H-infinity performance. Combining with the proper transformation of closed subsystem, an improved construction of Lyapunov-Krasovskii functional preserving the continuity free from switching actions is employed, and the delay-dependent criterion is established for the existence of controllers and switching strategy. Based on the technique of parameter transformation and matrix transformation, the proposed criterion is then equivalently reformulated in term of linear matrix inequality (LMI) from which the parameters of the functional and controllers are explicitly presented. Finally, a numerical example is given to illustrate the proposed method.

Key words: switched systems; time-delay; dynamic feedback control; linear matrix inequality

1 引言(Introduction)

切换与时滞是广泛存在的自然现象, 理论上通过建立切换系统与时滞系统模型以研究这些现象对系统动力学行为及控制性能的影响, 及如何利用或消除这种影响. 切换系统是由若干控制对象在切换信号驱动下构成的非连续时变系统, 分析切换的作用与影响时, 一般存在两种观点: 其一, 切换信号随时间演化, 即具有自治的特征, 可能导致复杂的动力学行为, 对于线性切换系统稳定性的基本问题[1]已经发展出相应的研究方法, 并在一定程度上得到解答[2~4]; 其二, 切换信号为系统状态所驱动, 即具有反馈控制的特征. 本文基于这种思想设计切换策略, 使得通过动态反馈实现闭环控制的线性时滞子系统在其驱动下形成一个有机整体, 并实现H_∞控制性能的优化.

在时域中分析时滞系统的稳定性与控制性能, 结果的保守性与参数的可解性依赖于Lyapunov函数或Lyapunov-Krasovskii泛函的选取. 对于具有时不变系数矩阵的线性时滞系统, 最一般的泛函构造形式导致偏微分方程边值问题[5,6], 不同的构造方式其保守性也不尽相同[7]. 在设计问题中, 泛函的选取在尽可能克服保守性的同时, 还要给出便于求解的控制器设计方法. 为此, 通常在泛函的构造中包含了系统矩阵或状态导数[8,9], 而系统的非连续变化导致这种泛函构造方式将失去连续性这一基本前提. 本文利用其连续性不受切换行为影响的Lyapunov-Krasovskii泛函构造方式, 并结合闭环子系统的适当变换, 导出了与时滞相关的控制器及切换策略的存在性判据; 通过参数代换与矩阵相似变换技巧, 将其等价地转化为线性矩阵不等式, 从而解得泛函与控制器参数.

文中基本符号说明如下: $C_{n,2r} := C([-2r,0],\mathbb{R}^n)$, $r>0$表示由 $[-2r,0]$ 映入 $\mathbb{R}^n$的具有一致范数的连续函数构成的Banach空间, $x_t \in C_{n,2r}, t \geqslant t_0$含义为 $x_t(\theta) := (t+\theta), \theta \in [-2r,0]$; D^+为实值连续函数的右导数算子.

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2 问题描述 (Problem statement)

考虑由下述若干线性子系统

$$\begin{cases} \dot{x}(t)=A_ix(t)+A_{i1}x(t-\tau)+B_{iw}w(t)+B_iu(t), \\ z(t) = E_ix(t) + F_iu(t),\ t \geqslant 0,\ i = 1,\cdots,N, \end{cases} \tag{1}$$

在右连续切换信号$s:\mathbb{R}_+ \rightarrow \{1,\cdots,N\}$驱动下构成的时滞切换系统. $0 < \tau \leqslant r$为时滞常数, 其上界$r > 0$精确已知; $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $w \in \mathbb{R}^p$, $z \in \mathbb{R}^q$分别为系统状态变量, 控制输入, 干扰输入, 及受控输出; $\{A_i, A_{i1}, B_{iw}, B_i, E_i, F_i\}_{i=1}^N$为适当维数矩阵.

本文目的是设计子系统全维动态控制器:

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}_i\hat{x}(t) + \hat{A}_{i1}\hat{x}(t-r) + \hat{B}_ix(t), \\ u(t) = \hat{C}_i\hat{x}(t) + \hat{D}_ix(t),\ i = 1,\cdots,N \end{cases} \tag{2}$$

及由状态变量$\xi := [x' \quad \hat{x}']'$所驱动的反馈切换策略:

$$s(t) = \pi(s(t^-), \xi(t)),\ t \geqslant 0, \tag{3}$$

使得对于给定干扰抑止水平$\gamma > 0$, 闭环子系统(1,2)在切换控制(3)作用下实现H_∞次优性能: 若$w(t) \equiv 0$, $t \geqslant 0$, 整个系统指数稳定; 若$\xi(\theta) \equiv 0$, $\theta \leqslant 0$, 由干扰输入到受控输出的$L_2 - L_2$增益满足$\int_0^\infty |z(t)|^2 \mathrm{d}t < \gamma^2 \int_0^\infty |w(t)|^2 \mathrm{d}t$, $w(\cdot) \in L_2^p[0,\infty)\backslash\{0\}$.

注 1 相对于静态反馈, 动态反馈控制具有较强的极点配置能力; 若从计算的角度分析控制性能的可实现性, 则在相似的约束条件下, 后者具有较高维数的解空间, 因此较为灵活.

本文将在相空间$C_{n,2r}$上进行分析,首先考虑闭环子系统的变换形式,为此引入下述状态变量代换:

$$\begin{cases} \zeta_t(\theta) := \xi_t(0) - \xi_t(\theta), \\ \eta_t(\theta) := \hat{x}_t(0) - \hat{x}_t(\theta), \\ -2r \leqslant \theta \leqslant 0,\ t \geqslant 0. \end{cases} \tag{4}$$

注 2 $\parallel \zeta_t - \zeta_s \parallel = \max\limits_{-2r\leqslant\theta\leqslant0} | \xi_t(0) - \xi_t(\theta) - \xi_s(0) + \xi_s(\theta)\ | \leqslant 2 \parallel \xi_t - \xi_s \parallel$, 即知$\zeta_t, \eta_t, t \geqslant 0$的连续性, 并成立$\zeta_t(0) = 0$, $\eta_t(0) = 0, \forall t \geqslant 0$.

依据式(1)(2)(4)及$\hat{x}(t) = \int_{t-\tau}^t \dot{\hat{x}}(\theta)\mathrm{d}\theta + \hat{x}(t-\tau)$, $t \geqslant 0$得到闭环子系统的变换形式:

$$\begin{cases} \dot{\xi}(t) = \bar{A}_i\xi(t) + \bar{A}_{i\tau}\zeta_t(-\tau) + \bar{A}_{ir}\zeta_t(-r) + \\ \qquad \bar{B}_iw(t) + f_i(t) + g_i(t), \\ z(t) = \bar{E}\xi(t),\ t \geqslant 0,\ i = 1,\cdots,N. \end{cases} \tag{5}$$

其中各项系数矩阵如下:

$$\bar{A}_i := \begin{bmatrix} A_i + A_{i1} + B_i\hat{D}_i & B_i\hat{C}_i \\ \hat{B}_i & \hat{A}_i + \hat{A}_{i1} \end{bmatrix},$$

$$\bar{A}_{i\tau} := \begin{bmatrix} -A_{i1} & A_{i1} \\ 0 & 0 \end{bmatrix},\ \bar{A}_{ir} := \begin{bmatrix} 0 & 0 \\ 0 & -\hat{A}_{i1} \end{bmatrix},$$

$$\bar{B}_i := \begin{bmatrix} B_{iw} \\ 0 \end{bmatrix},\ \bar{E} := \begin{bmatrix} E_i' + \hat{D}_i'F_i' \\ \hat{C}_i'F_i' \end{bmatrix}',$$

$$f_i(t) := \begin{bmatrix} -A_{i1}\int_{t-\tau}^t [(\hat{A}_i + \hat{A}_{i1})\hat{x}(\theta) + \hat{B}_ix(\theta)\mathrm{d}\theta \\ 0 \end{bmatrix},$$
$$t \geqslant 0,$$

$$g_i(t) := \begin{bmatrix} A_{i1}\int_{t-\tau}^t \hat{A}_{i1}\eta(\theta - r)\mathrm{d}\theta \\ 0 \end{bmatrix},\ t \geqslant 0.$$

引理 1 对于一组对称矩阵$\{M_i \in \mathbb{R}^{k\times k} | i = 1,\cdots,l\}$, 如果存在一组非负数$\{\delta_1,\cdots,\delta_l | \sum\limits_{i=1}^l \delta_i = 1\}$, 使得$\sum\limits_{i=1}^l \delta_iM_i$为负定矩阵, 那么对于任意$0 \neq \zeta \in \mathbb{R}^k$, 存在$i(\zeta) \in \{1,\cdots,l\}$, 使得$\zeta'M_i\zeta < 0$.

引理 2 矩阵的正定性是非奇异相似变换下保持不变的性质.

3 主要结果 (Main results)

闭环子系统的变换形式(5)是进一步分析问题, 并最终求解动态控制器(2)与切换控制(3)的出发点.

定理 1 对于闭环子系统(5), 如果存在一组非负数$\{\delta_1,\cdots,\delta_N \mid \sum\limits_{i=1}^N \delta_i = 1\}$及正定对称矩阵$P := \begin{bmatrix} P_{11} & P_{12} \\ P_{12}' & P_{22} \end{bmatrix}$, $M, R \in \mathbb{R}^{2n\times 2n}$; $N, S \in \mathbb{R}^{n\times n}$使得下述Riccati代数方程成立:

$$\sum_{i=1}^N \delta_i\varPi_i < 0, \tag{6}$$

$$\begin{aligned} \varPi_i := {} & \bar{A}_i'P + P\bar{A}_i + rR + rP\bar{A}_{i\tau}M^{-1}\bar{A}_{i\tau}'P + \\ & r\begin{bmatrix} P_{11} \\ P_{12}' \end{bmatrix} A_{i1}\hat{A}_{i1}S^{-1}\hat{A}_{i1}'A_{i1}'[P_{11}\ P_{12}] + \\ & r\begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix} \hat{A}_{i1}N^{-1}\hat{A}_{i1}'[P_{12}'\ P_{22}] + \\ & r\begin{bmatrix} P_{11} \\ P_{12}' \end{bmatrix} A_{i1}[\hat{B}_i\ \hat{A}_i + \hat{A}_{i1}]R^{-1}\begin{bmatrix} \hat{B}_i' \\ \hat{A}_i' + \hat{A}_{i1}' \end{bmatrix} \cdot \\ & A_{i1}'[P_{11}\ P_{12}] + \gamma^{-2}P\bar{B}_i\bar{B}_i'P + \bar{E}_i'\bar{E}_i, \\ & i = 1,\cdots,N. \end{aligned} \tag{7}$$

那么在如下极小化切换序列:

$$s(t) = \arg\min \xi'(t)\varPi_i\xi(t),\ t \geqslant 0 \tag{8}$$

驱动下, 自治系统(5,8)满足H_∞次优控制性能.

证 构造Lyapunov-Krasovskii泛函为

$$V(\xi_t)=V_1(\xi_t)+V_2(\xi_t)+V_3(\xi_t)+V_4(\xi_t)+V_5(\xi_t). \quad (9)$$

其中各项如下:

$$V_1(\xi_t)=\xi'(t)P\xi(t), \quad (10)$$

$$V_2(\xi_t)=r^{-1}\Big[\int_{t-\tau}^{t}\zeta'(\theta)M\zeta(\theta)\mathrm{d}\theta+\int_{t-r}^{t}\eta'(\theta)N\eta(\theta)\mathrm{d}\theta\Big], \quad (11)$$

$$V_3(\xi_t)=\int_{-\tau}^{0}\int_{t+\sigma}^{t}\xi'(\theta)R\xi(\theta)\mathrm{d}\theta\mathrm{d}\sigma, \quad (12)$$

$$V_4(\xi_t)=\int_{-\tau}^{0}\int_{t+\sigma}^{t}\eta'(\theta-r)S\eta(\theta-r)\mathrm{d}\theta\mathrm{d}\sigma, \quad (13)$$

$$V_5(\xi_t)=r\int_{t-r}^{t}\xi'(\theta)\begin{bmatrix}0\\ I_n\end{bmatrix}S[0\ I_n]\xi(\theta)\mathrm{d}\theta= r\int_{t-r}^{t}\eta'(\theta)S\eta(\theta)\mathrm{d}\theta. \quad (14)$$

根据注2, 泛函(9)~(14)中第1项沿系统(5,8)解轨线导数计算如下:

$$D^+V_1(\xi_t)=2\xi'(t)P[\bar{A}_{s(t)}\xi(t)+\bar{A}_{s(t)\tau}\zeta_t(-\tau)+\bar{A}_{s(t)r}\zeta_t(-r)+\bar{B}_{s(t)}w(t)+f_{s(t)}(t)+g_{s(t)}(t)]\leqslant \xi'(t)[\bar{A}_{s(t)}P+P\bar{A}_{s(t)}+rP\bar{A}_{s(t)\tau}M^{-1}\bar{A}'_{s(t)\tau}P+\gamma^{-2}P\bar{B}_{s(t)}\bar{B}'_{s(t)}P]\xi(t)+2\xi'(t)P\bar{A}_{s(t)r}\zeta_t(-r)+2\xi'(t)Pf_{s(t)}(t)+2\xi'(t)Pg_{s(t)}(t)+\gamma^2\mid w(t)\mid^2+r^{-1}\zeta'_t(-\tau)M\zeta_t(-\tau). \quad (15)$$

其中交叉项存在如下估计:

$$2\xi'(t)P\bar{A}_{s(t)r}\zeta_t(-r)=2\xi'(t)P\begin{bmatrix}0\\ -\hat{A}_{s(t)1}\end{bmatrix}\eta_t(-r)\leqslant r\xi'(t)P\begin{bmatrix}0\\ -\hat{A}_{s(t)1}\end{bmatrix}N^{-1}[0\ -\hat{A}'_{s(t)1}]P\xi(t)+r^{-1}\eta'_t(-r)N\eta_t(-t), \quad (16)$$

$$2\xi'(t)Pf_{s(t)}(t)=-\int_{t-\tau}^{t}2\xi'(t)\begin{bmatrix}P_{11}\\ P'_{12}\end{bmatrix}A_{s(t)1}[\hat{B}_{s(t)}\quad \hat{A}_{s(t)}+\hat{A}_{s(t)1}]\cdot\xi(\theta)\mathrm{d}\theta\leqslant\int_{t-\tau}^{t}\{\xi'(t)\begin{bmatrix}P_{11}\\ P'_{12}\end{bmatrix}A_{s(t)1}\cdot[\hat{B}_{s(t)}\quad \hat{A}_{s(t)}+\hat{A}_{s(t)1}]R^{-1}\begin{bmatrix}\hat{B}'_{s(t)}\\ \hat{A}'_{s(t)}+\hat{A}'_{s(t)1}\end{bmatrix}\cdot A'_{s(t)1}[P_{11}\quad P_{12}]\xi(t)+\xi'(\theta)R\xi(\theta)\}\mathrm{d}\theta=\tau\xi'(t)\begin{bmatrix}P_{11}\\ P'_{12}\end{bmatrix}A_{s(t)1}[\hat{B}_{s(t)}\quad \hat{A}_{s(t)}+\hat{A}_{s(t)1}]R^{-1}.\begin{bmatrix}\hat{B}'_{s(t)}\\ \hat{A}'_{s(t)}+\hat{A}'_{s(t)1}\end{bmatrix}A'_{s(t)1}[P_{11}\quad P_{12}]\xi(t)+\int_{t-\tau}^{t}\xi'(\theta)R\xi(\theta)\mathrm{d}\theta, \quad (17)$$

$$2\xi'(t)Pg_{s(t)}(t)\leqslant\tau\xi'(t)\begin{bmatrix}P_{11}\\ P'_{12}\end{bmatrix}A_{s(t)1}\hat{A}_{s(t)1}S^{-1}\hat{A}'_{s(t)1}A'_{s(t)1}\cdot[P_{11}\ P_{12}]\xi(t)+\int_{t-\tau}^{t}\eta'(\theta-r)S\eta(\theta-r)\mathrm{d}\theta. \quad (18)$$

泛函(9)~(14)中其余各项沿系统(5)(8)解轨线导数计算如下:

$$\dot{V}_2(\xi_t)=-r^{-1}[\zeta'_t(-\tau)M\zeta_t(-\tau)+\eta'_t(-r)N\eta_t(-r)], \quad (19)$$

$$\dot{V}_3(\xi_t)=\tau\xi'(t)R\xi(t)-\int_{t-\tau}^{t}\xi'(\theta)R\xi(\theta)\mathrm{d}\theta, \quad (20)$$

$$\dot{V}_4(\xi_t)=\tau\eta'_t(-r)S\eta_t(-r)-\int_{t-\tau}^{t}\eta'(\theta-r)S\eta(\theta-r)\mathrm{d}\theta, \quad (21)$$

$$\dot{V}_5(\xi_t)=-r\eta'_t(-r)S\eta_t(-r). \quad (22)$$

结合(15)~(22)各式, 利用引理1与切换序列(8)的极小化性质推知:

$$|z(t)|^2-\gamma^2|w(t)|^2+D^+V(\xi_t)\leqslant \xi'(t)\varPi_{s(t)}\xi(t)<0,\ t\geqslant 0.$$

依据零初值条件可知自治系统(5)(8)满足给定的L_2-L_2增益性能; 根据Lyapunov-Krasovskii稳定性理论[10], 指数稳定性结论是自明的. 证毕.

为求解动态反馈控制器与泛函参数, 引入增广参数矩阵如下:

$$\begin{cases}\tilde{U}\in\mathbb{R}^{(N\times m)\times 2n},\ \tilde{V}\in\mathbb{R}^{(N\times n)\times 2n},\ \tilde{W}\in\mathbb{R}^{(N\times n)\times n},\\ Q,M,X\in\mathbb{R}^{2n\times 2n},\ Y,Z\in\mathbb{R}^{n\times n}.\end{cases} \quad (23)$$

其中: $Q:=P^{-1}=\begin{bmatrix}Q_{11}\ Q_{12}\\ Q'_{12}\ Q_{22}\end{bmatrix}$, M,X,Y,Z为对称正定矩阵, $\tilde{U},\tilde{V},\tilde{W}$具有如下分块形式:

$$\tilde{U}:=[U'_1\ \cdots\ U'_N]',\ \tilde{V}:=[V'_1\ \cdots\ V'_N]',$$
$$\tilde{W}:=[W'_1\ \cdots\ W'_N]',$$
$$U_i\in\mathbb{R}^{m\times 2n},\ V_i\in\mathbb{R}^{n\times 2n},\ W_i\in\mathbb{R}^{n\times n},$$
$$i=1,\cdots,N.$$

定理 2 记$\varXi:=Q\vec{A}+\vec{B}\tilde{U}+\vec{J}\tilde{V}+rX$, 存在一组非负数$\{\delta_1,\cdots,\delta_N\mid\sum\limits_{i=1}^{N}\delta_i=1\}$与子系统动态反馈控制器(2)使得闭环子系统(5)满足Riccati代数方程(6)(7), 等价于存在参数矩阵(23)使得下列线性矩阵不等式成立:

$$\begin{bmatrix} \Xi'+\Xi & \vec{A}_\tau & \vec{A}_1\tilde{V} & \vec{J}\tilde{W} & \vec{A}_1\tilde{W} & \vec{B}_w & \tilde{U}'\vec{F}'+\vec{Q}'\vec{E}' \\ * & -r^{-1}\vec{M} & 0 & 0 & 0 & 0 & 0 \\ * & * & -r^{-1}\vec{X} & 0 & 0 & 0 & 0 \\ * & * & * & -r^{-1}\vec{Y} & 0 & 0 & 0 \\ * & * & * & * & -r^{-1}\vec{Z} & 0 & 0 \\ * & * & * & * & * & -r^{-1}\vec{I_p} & 0 \\ * & * & * & * & * & * & -r^{-1}\vec{I_q} \end{bmatrix} < 0. \tag{24}$$

“$*$”表示依据对称性得到的矩阵元素, 其中各项如下:

$$\vec{A} := \sum_{i=1}^{N}\delta_i \begin{bmatrix} A_i + A_{i1} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\vec{B} := \left[\delta_1 \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \cdots, \delta_N \begin{bmatrix} B_N \\ 0 \end{bmatrix}\right],$$

$$\vec{J} := \left[\delta_1 \begin{bmatrix} 0 \\ I_n \end{bmatrix}, \cdots, \delta_N \begin{bmatrix} 0 \\ I_n \end{bmatrix}\right],$$

$$\vec{A}_\tau = [\delta_1 \bar{A}_{1\tau}, \cdots, \delta_N \bar{A}_{N\tau}],$$

$$\vec{A}_1 = \mathrm{diag}\left\{\delta_1 \begin{bmatrix} A_{11} \\ 0 \end{bmatrix}, \cdots, \delta_N \begin{bmatrix} A_{N1} \\ 0 \end{bmatrix}\right\},$$

$$\vec{J} := \mathrm{diag}\left\{\delta_1 \begin{bmatrix} 0 \\ I_n \end{bmatrix}, \cdots, \delta_N \begin{bmatrix} 0 \\ I_n \end{bmatrix}\right\},$$

$$\vec{B}_w := [\delta_1 \bar{B}_1, \cdots, \delta_N \bar{B}_N],$$

$$\vec{Q} := [Q, \cdots, Q]_{2n\times(N\times 2n)},$$

$$\vec{F} := \mathrm{diag}\{\delta_1 F_1, \cdots, \delta_N F_N\},$$

$$\vec{E} := \mathrm{diag}\left\{\delta_1 \begin{bmatrix} E_1' \\ 0 \end{bmatrix}', \cdots, \delta_N \begin{bmatrix} E_N' \\ 0 \end{bmatrix}'\right\},$$

$$\vec{\Theta} := \mathrm{diag}\{\delta_1\Theta, \cdots, \delta_N\Theta\},$$

$$\Theta = M, X, Y, Z, I_p, I_q.$$

证　子系统控制器(2)设计为

$$\begin{cases} \lfloor \hat{D}_i\ \hat{C}_i \rfloor = U_i Q^{-1}, \\ \lfloor \hat{B}_i\ \hat{A}_i + \hat{A}_{i1} \rfloor = V_i Q^{-1}, \\ \hat{A}_{i1} = W_i Q_{22}^{-1},\ i = 1, \cdots, N. \end{cases} \tag{25}$$

泛函(9)~(14)中参数取为

$$\begin{cases} N = Q_{22}^{-1} Y Q_{22}^{-1}, \\ R = Q^{-1} X Q^{-1}, \\ S = Q_{22}^{-1} Z Q_{22}^{-1}. \end{cases} \tag{26}$$

依据Schur-complement性质可知, 式(6)(7)等价于

$$\begin{bmatrix} \sum\limits_{i=1}^{N}\delta_i(\bar{A}_i'P+P\bar{A}_i)+rR & P\vec{A}_\tau & \begin{bmatrix} P_{11} \\ P_{12}' \end{bmatrix}\Gamma_1 & \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix}\Gamma_2 & \begin{bmatrix} P_{11} \\ P_{12}' \end{bmatrix}\Gamma_3 & P\bar{B}_w & \Gamma_4 \\ * & -r^{-1}\vec{M} & 0 & 0 & 0 & 0 & 0 \\ * & * & -r^{-1}\vec{R} & 0 & 0 & 0 & 0 \\ * & * & * & -r^{-1}\vec{N} & 0 & 0 & 0 \\ * & * & * & * & -r^{-1}\vec{S} & 0 & 0 \\ * & * & * & * & * & -r^{-1}\vec{I_p} & 0 \\ * & * & * & * & * & * & -r^{-1}\vec{I_q} \end{bmatrix} < 0. \tag{27}$$

其中各项如下:

$$\Gamma_1 := \lfloor \delta_1 A_{11} \lfloor \hat{B}_1\ \hat{A}_1 + \hat{A}_{11} \rfloor, \cdots, \delta_N A_{N1} \lfloor \hat{B}_N\ \hat{A}_N + \hat{A}_{N1} \rfloor \rfloor,$$

$$\Gamma_2 := \lfloor \delta_1 \hat{A}_{11}, \cdots, \delta_N \hat{A}_{N1} \rfloor,$$

$$\Gamma_3 := \lfloor \delta_1 A_{11}\hat{A}_{11}, \cdots, \delta_N A_{N1}\hat{A}_{N1} \rfloor,$$

$$\Gamma_4 := \lfloor \delta_1 \bar{E}_1', \cdots, \delta_N \bar{E}_N' \rfloor,$$

$$\vec{\Theta} := \mathrm{diag}\{\delta_1\Theta, \cdots, \delta_N\Theta\},$$

$$\Theta = N, R, S.$$

选取非奇异相似变换矩阵:

$$T := \mathrm{diag}\{Q, I_{N\times 2n}, \vec{Q}, \vec{Q}_{22}, \vec{Q}_{22}, I_{N\times p}, I_{N\times q}\}. \tag{28}$$

其中$\vec{\Theta} := \mathrm{diag}\{\delta_1\Theta, \cdots, \delta_N\Theta\}$, $\Theta = Q, Q_{22}$, 依据引理2, 式(27)经过式(28)相似变换, 并代入控制器参数(25)及泛函参数(26), 通过矩阵初等运算即知式(24)等价于式(27).　　证毕.

4　示例 (Illustrative example)

给定时滞常数$r = 1.0$ s, 干扰抑止水平$\gamma = 1.0$, 考虑两个二维子系统组成的时滞线性切换系统, 其系数矩阵配置如下:

$$A_1=\begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix},\ A_{11}=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\ B_1=\begin{bmatrix}2\\ 1\end{bmatrix},$$

$$A_2=\begin{bmatrix}-3 & 0\\ -1 & 1\end{bmatrix},\ A_{21}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix},\ B_2=\begin{bmatrix}1\\ -1\end{bmatrix},$$

$$B_{iw}=\begin{bmatrix}1\\ 0\end{bmatrix},\ E_i=\begin{bmatrix}1\\ 1\end{bmatrix}',\ F_i=1,\ i=1,2.$$

选取$\delta_1=0.64$, $\delta_2=0.36$, 解得子系统动态反馈控制器分别为:

$$\dot{\hat{x}}(t)=\begin{bmatrix}-1.56 & 0.01\\ -0.01 & 1.71\end{bmatrix}\hat{x}(t)+\begin{bmatrix}0.21 & 0.31\\ -0.24 & 0.63\end{bmatrix}x(t)+\begin{bmatrix}0.6 & -0.05\\ -0.05 & 0.58\end{bmatrix}\hat{x}(t-1),$$

$$u(t)=\begin{bmatrix}0.11 & -0.12\end{bmatrix}\hat{x}(t)+\begin{bmatrix}-1.31 & -0.8\end{bmatrix}x(t),\ t\geqslant 0,$$

$$\dot{\hat{x}}(t)=\begin{bmatrix}-1.63 & 0.07\\ -0.09 & -1.5\end{bmatrix}\hat{x}(t)+\begin{bmatrix}0.3 & 0.34\\ -0.11 & 0.68\end{bmatrix}x(t)+\begin{bmatrix}0.7 & -0.05\\ -0.06 & 0.65\end{bmatrix}\hat{x}(t-1),$$

$$u(t)=[0.3\ \ 0.2]\hat{x}(t)+[-0.83\ \ -0.6]x(t),\ t\geqslant 0.$$

在干扰输入恒为零的条件下, 自治系统(5,8)状态轨迹如图1所示.

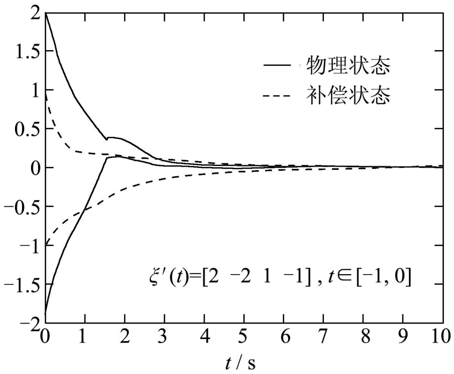

图 1 切换控制驱动下闭环系统状态轨迹

Fig. 1 Forced trajectory of the state of closed system under switching control

5 结束语 (Conclusion)

本文针对切换系统的非连续性, 通过设计子系统动态反馈控制器, 将闭环子系统的形式变换与Lyapunov-Krasovskii泛函的构造有机地相结合, 从而得到与时滞相关的子系统动态反馈控制与H_∞性能优化切换策略的存在性判据, 进而通过参数代换与矩阵相似变换将其转化为LMI, 最终解得泛函与控制器参数. 文末的仿真算例图示了本文方法的有效性.

谨以此文追念亡友, 缅怀他的事迹与品格, 愿他的精神与我们同在.

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