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降阶的鲁棒分散自适应反推控制

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摘要: 针对一类含有未建模动态的关联系统, 考虑了降阶的鲁棒分散自适应反推控制问题. 首先通过一系列坐标变换, 将原系统重新参数化, 然后引入降阶观测器, 得到一个误差系统. 基于该系统, 给出了一种降阶自适应反推控制器的设计方案. 证明了自适应控制系统的所有信号全局一致有界, 调节误差渐近收敛到零. 控制器阶次的降低使得本文的设计方案更具应用价值.

关键词: 关联系统; 鲁棒; 降阶; 反推; 自适应控制

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Reduced-order robust decentralized adaptive backstepping control

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Abstract: For a class of interconnected systems with unmodeled dynamics, the problem of reduced-order robust decentralized adaptive backstepping control is considered in this paper. Firstly, by a series of coordinate transformations, the original system is reparameterized. Then, by introducing a reduced-order observer, an error system is obtained. Based on the system, a design scheme of reduced-order adaptive backstepping controller is given. It is also proved that all the signals in the adaptive control system are globally uniformly bounded, and the regulation error converges to zero asymptotically. Due to the order reduction of the controller, the design scheme in this paper has more practical values.

Key words: interconnected systems; robust; reduced-order; backstepping; adaptive control

1 引言(Introduction)

分散自适应控制是关联系统研究中的一种非常重要的控制方案, 近年来倍受人们的关注. 然而, 由于在分析中很难忽略关联项, 因此在这一研究领域仅有少量结果. 文献[1]在这一方向做出了开创性的工作, 设计方案依赖于传统的M-矩阵检测, 但存在3个缺点. 众多文献为减弱这些条件做了大量的工作, 得到了一些很好的结果. 正如文[2]指出的, 根据关联项的形式, 已有方法可分为两类: 一类考虑了范数受多项式函数限制的静态关联项, 另一类则考虑了动态关联项. 因为动态关联项有无限记忆性, 这使得动态关联项不能包含在静态关联项中, 反之亦然. 由于绝大多数文献仅单独考虑了动态或静态关联项, 因此研究同时具有动态和静态关联项和其他未建模动态的关联系统的自适应控制是一项很有意义的工作. 由于这个问题具有一定的难度, 据作者所知, 目前最新的结果属于文献[2,3]. 在文[2]中, 作者利用K-滤波器和反推设计方法, 考虑了这一问题. 文献[3]通过重新定义滤波变换, 降低了控制器的阶次, 证明了除参数估计外所有信号的渐进收敛性.

本文进一步研究了这一问题. 首先通过一系列坐标变换, 将原系统重新参数化, 然后引入降阶观测器, 得到一个误差系统. 基于该系统, 给出了一种降阶自适应反推控制器的设计方案. 证明了自适应控制系统的所有信号全局一致有界, 调节误差渐近收敛到零. 由于本文的观测器是$N(\rho_i-1)$阶的, 自适应律是N阶的, 此控制器的阶次比文献[3]的低$(2n_i+2m_i)N$次, 因此更有应用价值.

2 问题的提出(Problem statement)

考虑一类含有未建模动态的关联系统, 第i个子系统为

$$y_i(t)=\frac{B_i(s)}{A_i(s)}(1+\mu_{ii}\varDelta_{ii}(s))u_i(t)+\frac{D_i(s)}{A_i(s)}(1+\mu_{ii}\varDelta_{ii}(s))\sum_{j=1,j\neq i}^{N}\bar{f}_{ij}(t,y_j)+\sum_{j=1,j\neq i}^{N}\mu_{ij}\varDelta_{ij}(s)y_j, \tag{1}$$

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$i=1,\cdots,N$. u_i, y_i分别是第i个子系统的输入和输出, $A_i(s)=s^{n_i}+a_{i,n_i-1}s^{n_i-1}+\cdots+a_{i0}$, $B_i(s)=b_{i,m_i}s^{m_i}+b_{i,m_i-1}s^{m_i-1}+\cdots+b_{i0}$, a_{ij}和$b_{ik}(j=0,\cdots,n_i-1,\ k=0,\cdots,m_i)$是未知常数. s表示微分算子. $D_i(s)=(s^{n_i-1},\cdots,s,1)$. $\bar f_{ij}(t,y_j)\in\mathbb{R}^{n_i}$和$\Delta_{ij}(s)y_j(i\neq j)$分别表示从第$j$个子系统到第$i$个子系统的静态关联项和动态关联项, $\Delta_{ii}(s)$是第i个子系统的未建模动态. $\mu_{ii}, \mu_{ij}\geqslant 0$分别表示未建模动态和动态关联项的幅值.

控制目标: 对系统设计分散自适应反推控制器, 使得整个系统的所有信号有界, 输出渐进收敛到零.

针对系统(1), 本文需要以下的假设条件:

A1　$B_i(s)$是Hurwitz多项式, 相对阶$\rho_i=n_i-m_i$和b_{i,m_i}的符号已知. 不失一般性, 设$b_{i,m_i}=1$.

A2　存在已知常数$\bar\gamma_{ijk}$和K_{ij}使得$|\bar f_{ij}(t,y_j)|\leqslant\sum\limits_{k=1}^{K_{ij}}\bar\gamma_{ijk}|y_j|^k$.

A3　$\Delta_{ij}(s)$稳定、严格正则且有单位高频增益.

注 1　记$e_1^{\mathrm T}=(1,0,\cdots,0)$, $e_n^{\mathrm T}=(0,\cdots,0,1)$同与之相乘的矩阵有相应的维数. 对任意的向量$x\in\mathbb{R}^n$, 定义$|x|=(\sum\limits_{i=1}^{n}x_i^2)^{\frac12}$. 对任意的矩阵$P$, 定义$\|P\|=[\lambda_{\max}(P^{\mathrm T}P)]^{\frac12}$, $\lambda_{\max}(\cdot)$为矩阵$(\cdot)$的最大特征值.

3　降阶控制器的设计(Design of reduced-order controller)

由附录A知系统(1)有以下的状态空间实现

$$\begin{cases}\dot\chi_i=\bar A_i\chi_i+\bar b_iu_i+f_i,\\ y_i=\chi_{i1}+\aleph_i,\\ \dot\xi_{ii}=A_{\xi ii}\xi_{ii}+b_{\xi ii}\chi_{i1},\\ \dot\xi_{ij}=A_{\xi ij}\xi_{ij}+b_{\xi ij}y_j,\ j=1,\cdots,N,\ j\neq i,\end{cases}\tag{2}$$

其中:

$$\bar A_i=\begin{pmatrix}-a_i & \begin{matrix}I_{n_i-1}\\ 0_{1\times(n_i-1)}\end{matrix}\end{pmatrix},\ \bar b_i=(0,b_i^{\mathrm T})^{\mathrm T},$$
$$c_i^{\mathrm T}=(1,0,\cdots,0),\ a_i=(a_{i,n_i-1},\cdots,a_{i0})^{\mathrm T},$$
$$b_i=(b_{i,m_i},\cdots,b_{i0})^{\mathrm T},$$

满足

$$c_i^{\mathrm T}(sI-\bar A_i)^{-1}\bar b_i=\frac{B_i(s)}{A_i(s)},\ \ c_i^{\mathrm T}(sI-\bar A_i)^{-1}=\frac{D_i(s)}{A_i(s)},$$
$$\chi_{i1}=c_i^{\mathrm T}\chi_i,\ \ \aleph_i=\mu_{ii}\Delta_{ii}(s)\chi_{i1}+\sum_{j=1,j\neq i}^{N}\mu_{ij}\Delta_{ij}(s)y_j,$$
$$f_i=\sum_{j=1,j\neq i}^{N}\bar f_{ij}(t,y_j),$$
$$c_i^{\mathrm T}(sI-A_{\xi ij})^{-1}b_{\xi ij}=\Delta_{ij}(s),\ j=1,\cdots,N.$$

引入相似变换

$$\begin{pmatrix}\bar\chi_{i,\rho_i}\\ \zeta_i\end{pmatrix}=\begin{pmatrix}I_{\rho_i\times\rho_i}\ 0_{\rho_i\times m_i}\\ T_{m_i\times n_i}\end{pmatrix}\chi_i,\tag{3}$$

其中:

$$A_{i1}=\begin{pmatrix}-b_{i,m_i-1}/b_{i,m_i} & \\ \vdots & I_{m_i-1}\\ -b_{i0}/b_{i,m_i} & 0\cdots0\end{pmatrix},$$
$$\bar\chi_{i,\rho_i}=(\chi_{i1},\cdots,\chi_{i,\rho_i})^{\mathrm T},$$
$$A_i=\begin{pmatrix}0_{(n_i-1)\times1} & I_{n_i-1}\\ 0 & 0_{1\times(n_i-1)}\end{pmatrix},$$
$$T_i=(A_{i1}^{\rho_i}e_1,\cdots,A_{i1}e_1,I_{m_i}).$$

容易验证性质:

$$T_i\bar b_i=0,\ T_iA_i=A_{i1}T_i+T_iA_i^{\rho_i}\bar b_ie_1^{\mathrm T}.\tag{4}$$

由式(2)～(4)及$A_i, \bar A_i$的定义得

$$\begin{aligned}\dot\zeta_i=&T_i(\bar A_i\chi_i+\bar b_iu_i+f_i)=\\ &T_iA_i\chi_i-T_ia_i\chi_{i1}+T_if_i=\\ &A_{i1}\zeta_i+B_{i1}\chi_{i1}+T_if_i,\end{aligned}\tag{5}$$

其中$B_{i1}=T_i(A_i^{\rho_i}\bar b_i-a_i)$. 利用式(3)和(5), 重写式(2)为

$$\begin{cases}\dot\chi_{il}=\chi_{i,l+1}-a_{i,n_i-l}\chi_{i1}+f_{il},l=1,\cdots,\rho_i-1,\\ \dot\chi_{i,\rho_i}=b_{i,m_i}u_i-a_{i,m_i}\chi_{i1}+\chi_{i,\rho_i+1}+f_{i,\rho_i},\\ y_i=\chi_{i1}+\aleph_i,\\ \dot\xi_{ii}=A_{\xi ii}\xi_{ii}+b_{\xi ii}\chi_{i1},\\ \dot\xi_{ij}=A_{\xi ij}\xi_{ij}+b_{\xi ij}y_j,j=1,\cdots,N,j\neq i,\\ \dot\zeta_i=A_{i1}\zeta_i+B_{i1}\chi_{i1}+T_if_i.\end{cases}\tag{6}$$

由式(3)和T_i的定义得

$$\zeta_{i1}=e_1^{\mathrm T}A_{i1}^{\rho_i}e_1\chi_{i1}+\cdots+e_1^{\mathrm T}A_{i1}e_1\chi_{i,\rho_i}+\chi_{i,\rho_i+1},\tag{7}$$

即$\chi_{i,\rho_i+1}=c_{i1}\chi_{i1}+\cdots+c_{i,\rho_i}\chi_{i,\rho_i}+\zeta_{i1}$, 其中$c_{i1}=-e_1^{\mathrm T}A_{i1}^{\rho_i}e_1,\cdots,c_{i,\rho_i}=-e_1^{\mathrm T}A_{i1}e_1$. 将其代入式(6)的前两式得

$$\dot{\bar\chi}_{i,\rho_i}=\bar{\bar A}_i\bar\chi_{i,\rho_i}+e_{\rho_i}b_{i,m_i}u_i+e_{\rho_i}\zeta_{i1}+\bar f_{i,\rho_i},\tag{8}$$

其中:

$$\bar{\bar A}_i=\begin{pmatrix}-a_{i,n_i-1} & \\ \vdots & I_{\rho_i-1}\\ -a_{i,m_i+1} & \\ -a_{i,m_i}+c_{i1} & c_{i2}\cdots c_{i,\rho_i}\end{pmatrix},$$
$$\bar f_{i,\rho_i}=(f_{i1},\cdots,f_{i,\rho_i})^{\mathrm T},\ \ e_{\rho_i}=(0,\cdots,0,1)^{\mathrm T}.$$

引入变换

$$x_i=\frac{\Lambda_i}{\|\Lambda_i\|}\bar\chi_{i,\rho_i}\triangleq S_i\bar\chi_{i,\rho_i},\ x_i\in\mathbb{R}^{\rho_i},\tag{9}$$

其中:

$$\Lambda_i=\begin{pmatrix}1 & 0 & \cdots & 0\\ -c_{i,\rho_i} & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & 0\\ -c_{i2} & \cdots & -c_{i,\rho_i} & 1\end{pmatrix},\ S_i=\frac{\Lambda_i}{\|\Lambda_i\|},\ \|S_i\|=1.$$

由附录A得

$$S_i\bar{\bar A}_i=A_{ia}S_i,\ S_ie_{\rho_i}=\frac{1}{\|\Lambda_i\|}e_{\rho_i},\tag{10}$$

其中:

$$A_{ia}=\begin{pmatrix}-\bar{a}_{i1} & \\ \vdots & I_{\rho_i-1}\\ -\bar{a}_{i,\rho_i} & 0\cdots 0\end{pmatrix},\ \bar{a}_{i1}=a_{i,n_i-1}-c_{i,\rho_i},$$

$\bar{a}_{il}=a_{i,n_i-l}-c_{i,\rho_i}a_{i,n_i-l+1}-\cdots-c_{i,\rho_i-l+2}a_{i,n_i-1}-c_{i,\rho_i-l+1},\ 2\leqslant l\leqslant\rho_i.$

由(8)~(10)得

$$\dot{x}_i=S_i(\bar{\bar{A}}_i\bar{\chi}_{i,\rho_i}+e_{\rho_i}u_i+e_{\rho_i}\zeta_{i1}+\bar{f}_{i,\rho_i})=A_{ia}x_i+e_{\rho_i}\frac{b_{i,m_i}}{\|\varLambda_i\|}u_i+e_{\rho_i}\frac{1}{\|\varLambda_i\|}\zeta_{i1}+S_i\bar{f}_{i,\rho_i}.\quad(11)$$

由式(9)知$\chi_{i1}=x_{i1}$. 注意到式(11), 系统(6)等价于

$$\begin{cases}\dot{x}_{il}=x_{i,l+1}-\bar{a}_{il}x_{i1}+g_{il},1\leqslant l\leqslant\rho_i-1,\\ \dot{x}_{i,\rho_i}=\dfrac{b_{i,m_i}}{\|\varLambda_i\|}u_i-\bar{a}_{i,\rho_i}x_{i1}+\dfrac{1}{\|\varLambda_i\|}\zeta_{i1}+g_{i,\rho_i},\\ y_i=x_{i1}+\aleph_i,\\ \dot{\xi}_{ii}=A_{\xi ii}\xi_{ii}+b_{\xi ii}\chi_{i1},\\ \dot{\xi}_{ij}=A_{\xi ij}\xi_{ij}+b_{\xi ij}y_j,j=1,\cdots,N,j\neq i,\\ \dot{\zeta}_i=A_{i1}\zeta_i+B_{i1}\chi_{i1}+T_if_i,\end{cases}\quad(12)$$

其中$\bar{g}_{i,\rho_i}=(g_{i1},\cdots,g_{i,\rho_i})^{\mathrm T}=S_i\bar{f}_{i,\rho_i}$.

当$\dfrac{b_{i,m_i}}{\|\varLambda_i\|}=1$时, 引入降阶观测器

$$\begin{cases}\dot{\hat{x}}_{il}=\hat{x}_{i,l+1}+k_{i,l+1}y_i-k_{il}(\hat{x}_{i1}+k_{i1}y_i),\\ \qquad l=1,\cdots,\rho_i-2,\\ \dot{\hat{x}}_{i,\rho_i-1}=u_i-k_{i,\rho_i-1}(\hat{x}_{i1}+k_{i1}y_i).\end{cases}\quad(13)$$

选取$k_i=(k_{i1},\cdots,k_{i,\rho_i-1})^{\mathrm T}$使得$A_{i0}=\begin{pmatrix}-k_i & I_{\rho_i-2}\\ & 0\cdots 0\end{pmatrix}$稳定. 定义观测误差

$$\varepsilon_{il}=\frac{x_{i,l+1}-\hat{x}_{il}-k_{il}x_{i1}}{p_i},\quad(14)$$

其中:

$$p_i=\max\Big\{1,\sum_{l=1}^{\rho_i-1}|q_{il}|,\sum_{l=1}^{\rho_i-1}|\bar{q}_{il}|,\frac{|P_{i1}B_{i1}|}{\|\varLambda_i\|}\Big\},$$
$$q_{il}=-\bar{a}_{i,l+1}+k_{il}\bar{a}_{i1},$$
$$\bar{q}_{il}=-q_{il}+k_{il}k_{i1}-k_{i,l+1},$$
$$k_{i,\rho_i}=0.$$

由附录A得

$$\dot{\varepsilon}_i=A_{i0}\varepsilon_i+\frac{1}{p_i}(\varDelta_i+e_{\rho_i-1}\frac{1}{\|\varLambda_i\|}\zeta_{i1}+h_i),\quad(15)$$

其中:

$$h_{il}=g_{i,l+1}-k_{il}g_{i1},\varepsilon_i=(\varepsilon_{i1},\cdots,\varepsilon_{i,\rho_i-1})^{\mathrm T},$$
$$\varDelta_i=(\varDelta_{i1},\cdots,\varDelta_{i,\rho_i-1})^{\mathrm T},\varDelta_{il}=q_{il}y_i+\bar{q}_{il}\aleph_i,$$

由式(12)(14)知

$$\dot{y}_i=\dot{x}_{i1}+\dot{\aleph}_i=x_{i2}-\bar{a}_{i1}x_{i1}+g_{i1}+\dot{\aleph}_i=\hat{x}_{i1}+p_i\varepsilon_{i1}+q_ix_{i1}+\dot{\aleph}_i+g_{i1},\quad(16)$$

其中$q_i=k_{i1}-\bar{a}_{i1}$, 得到如下新的参数化模型:

$$\begin{cases}\dot{y}_i=\hat{x}_{i1}+p_i\varepsilon_{i1}+q_ix_{i1}+\dot{\aleph}_i+g_{i1},\\ \dot{\hat{x}}_{il}=\hat{x}_{i,l+1}+k_{i,l+1}y_i-k_{il}(\hat{x}_{i1}+k_{i1}y_i),\\ \qquad l=1,\cdots,\rho_i-2,\\ \dot{\hat{x}}_{i,\rho_i-1}=u_i-k_{i,\rho_i-1}(\hat{x}_{i1}+k_{i1}y_i),\\ \dot{\varepsilon}_i=A_{i0}\varepsilon_i+\dfrac{1}{p_i}(\varDelta_i+e_{\rho_i-1}\dfrac{1}{\|\varLambda_i\|}\zeta_{i1}+h_i),\\ \dot{\xi}_{ii}=A_{\xi ii}\xi_{ii}+b_{\xi ii}\chi_{i1},\\ \dot{\xi}_{ij}=A_{\xi ij}\xi_{ij}+b_{\xi ij}y_j,j=1,\cdots,N,j\neq i,\\ \dot{\zeta}_i=A_{i1}\zeta_i+B_{i1}\chi_{i1}+T_if_i.\end{cases}\quad(17)$$

对式(17), 利用反推技术[4]给出自适应控制器的设计.

第1步 定义坐标变换

$$z_{i1}=y_i,\ z_{i2}=\hat{x}_{i1}-\alpha_{i1},\quad(18)$$

其中α_{i1}是待定的虚拟控制. 由假设A1和A2知A_{i1}, $A_{\xi ii}$和$A_{\xi ij}$稳定. 再由A_{i0}稳定, 则存在$P_{\xi ij}$和$P_{ik}>0$使得$P_{ik}A_{ik}+A_{ik}^{\mathrm T}P_{ik}=-I,k=0,1$, $P_{\xi ij}A_{\xi ij}+A_{\xi ij}^{\mathrm T}P_{\xi ij}=-I,j=1,\cdots,N$. 考虑

$$V_{i1}=\frac{1}{2}y_i^2+\frac{1}{2r_i}\tilde{\theta}_i^2+r_{i0}\varepsilon_i^{\mathrm T}P_{i0}\varepsilon_i+\sum_{j=1}^{N}r_{\xi ij}\xi_{ij}^{\mathrm T}P_{\xi ij}\xi_{ij}+r_{i1}\zeta_i^{\mathrm T}P_{i1}\zeta_i,\quad(19)$$

其中$r_i,r_{i0},r_{\xi ij},r_{i1}>0$为待定参数, $\tilde{\theta}_i=\hat{\theta}_i-\theta_i$, $\hat{\theta}_i$是对$\theta_i=\max\{p_i^2,q_i^2\}$的估计. 利用不等式$ab\leqslant\dfrac{a^2}{d_0}+\dfrac{d_0b^2}{4},\forall a,b\in\mathbb{R},d_{i0}>0$, 由$\theta$和$\tilde{\theta}$的定义得

$$z_{i1}(p_i\varepsilon_{i1}+q_ix_{i1}+\dot{\aleph}_i+g_{i1})\leqslant\bar{\phi}_{i1}\hat{\theta}_iz_{i1}+\phi_{i1}z_{i1}+\frac{r_{i0}}{4\rho_i}|\varepsilon_i|^2+d_{i0}(y_i^2+\aleph_i^2+\dot{\aleph}_i^2+g_{i1}^2)-\bar{\phi}_{i1}\tilde{\theta}_iz_{i1},\quad(20)$$

其中: $\phi_{i1}=\dfrac{1}{2d_{i0}}z_{i1}$, $\bar{\phi}_{i1}=(\dfrac{\rho_i}{r_{i0}}+\dfrac{1}{2d_{i0}})z_{i1}$. 选取$r_{i1}=\dfrac{d_{i1}}{|P_{i1}B_{i1}|^2+|P_{i1}T_i|^2}$, $r_{\xi ij}=\dfrac{d_{i1}}{|P_{\xi ij}b_{\xi ij}|^2}$, $j=1,\cdots,N$. 由$z_{i1}=y_i,y_i=x_{i1}+\aleph_i$得

$$2r_{\xi ii}\xi_{ii}^{\mathrm T}P_{\xi ii}b_{\xi ii}x_{i1}+\sum_{j=1,j\neq i}^{N}2r_{\xi ij}\xi_{ij}^{\mathrm T}P_{\xi ij}b_{\xi ij}y_j+2r_{i1}\zeta_i^{\mathrm T}P_{i1}B_{i1}x_{i1}+2r_{i1}\zeta_i^{\mathrm T}P_{i1}T_if_i\leqslant\frac{1}{2}r_{\xi ii}|\xi_{ii}|^2+\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}|\xi_{ij}|^2+\frac{3}{4}r_{i1}|\zeta_i|^2+8d_{i1}z_{i1}^2+8d_{i1}\aleph_i^2+2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+4d_{i1}f_i^2,\quad(21)$$

其中$d_{i1}>0$为任意的设计参数. 选取$r_{i0}\leqslant\dfrac{d_{i1}d_{i2}}{16\|P_{i0}\|^2}$, 其中$0<d_{i2}<\dfrac{1}{4}$为任意的设计参数.

利用$(a+b+c+d)^2\leqslant 4a^2+4b^2+4c^2+4d^2$, 及$\Delta_i$和$p_i$的定义得

$$\frac{2}{p_i}r_{i0}\varepsilon_i^{\mathrm T}P_{i0}(\Delta_i+e_{\rho_i-1}\frac{1}{\|\Lambda_i\|}\zeta_{i1}+h_i)\leqslant \frac{1}{4}r_{i0}|\varepsilon_i|^2+d_{i1}d_{i2}(z_{i1}^2+\aleph_i^2+h_i^2)+d_{i2}r_{i1}|\zeta_i|^2.\tag{22}$$

将式(17) (18) (20) (21)和(22)代入式(19)的导数得

$$\begin{aligned}\dot V_{i1}\leqslant&-\frac{3}{4}r_{i0}|\varepsilon_i|^2-\frac{1}{2}r_{\xi ii}|\xi_{ii}|^2-\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}|\xi_{ij}|^2-\\&(\frac{1}{4}-d_{i2})r_{i1}|\zeta_i|^2+(\frac{r_{i0}}{4\rho_i}|\varepsilon_i|^2+d_{i0}(y_i^2+\aleph_i^2+\\&\dot\aleph_i^2+g_{i1}^2))+d_{i1}(d_{i2}+8)\aleph_i^2+z_{i1}(z_{i2}+\alpha_{i1}+\\&d_{i1}(d_{i2}+8)z_{i1}+\bar\phi_{i1}(z_{i1})+\phi_{i1}(z_{i1}))-\\&z_{i1}\bar\phi_{i1}(z_{i1})\tilde\theta_i+r_i^{-1}\tilde\theta_i\dot{\hat\theta}_i+2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+\\&4d_{i1}f_i^2+d_{i1}d_{i2}h_i^2.\end{aligned}\tag{23}$$

定义调节函数

$$\begin{cases}\tau_{i1}=r_i\bar\phi_{i1}z_{i1},\\ \alpha_{i1}(y_i,\hat\theta_i)=\\ -c_{i1}z_{i1}-d_{i0}\rho_iz_{i1}-\phi_{i1}(z_{i1})-\bar\phi_{i1}(z_{i1})\hat\theta_i-\\ d_{i1}(d_{i2}+8)z_{i1}-\bar c_{i1}(z_{i1})^{2K-1},\end{cases}\tag{24}$$

其中$K=\max\limits_{1\leqslant i\leqslant N,1\leqslant j\leqslant N}\{k_{ij}\}$. 将式(24)代入式(23)得

$$\begin{aligned}\dot V_{i1}\leqslant& z_{i1}z_{i2}-c_{i1}z_{i1}^2-d_{i0}\rho_iz_{i1}^2-\bar c_{i1}(z_{i1})^{2K}-\\&\frac{3}{4}r_{i0}|\varepsilon_i|^2-\frac{1}{2}r_{\xi ii}|\xi_{ii}|^2-\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}|\xi_{ij}|^2-\\&(\frac{1}{4}-d_{i2})r_{i1}|\zeta_i|^2+\frac{r_{i0}}{4\rho_i}|\varepsilon_i|^2+d_{i0}(y_i^2+\aleph_i^2+\\&\dot\aleph_i^2+g_{i1}^2)+d_{i1}(d_{i2}+8)\aleph_i^2+r_i^{-1}\tilde\theta_i(\dot{\hat\theta}_i-\\&\tau_{i1})+2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+4d_{i1}f_i^2+d_{i1}d_{i2}h_i^2.\end{aligned}\tag{25}$$

第$l(l=2,\cdots,\rho_i)$步 定义坐标变换

$$z_{i,l+1}=\hat x_{il}-\alpha_{il}(y_i,\hat x_{i1},\cdots,\hat x_{i,l-1}),2\leqslant l\leqslant\rho_i,\tag{26}$$

当$l=\rho_i+1$时, $z_{\rho_i+1}=0$, $\hat x_{\rho_i}=u_i$. 选取$V_{i,l-1}=V_{i1}+\sum\limits_{j=1}^{l-1}\frac{1}{2}z_{ij}^2$, 假定已选取$\tau_{i,l-1}$和$\alpha_{i,l-1}$使得

$$\begin{aligned}\dot V_{i,l-1}\leqslant& z_{il}z_{i,l-1}-\sum_{j=1}^{l-1}c_{ij}z_{ij}^2-d_{i0}\rho_iz_{i1}^2-\\&\bar c_{i1}(z_{i1})^{2K}-\frac{3}{4}r_{i0}|\varepsilon_i|^2-\frac{1}{2}r_{\xi ii}|\xi_{ii}|^2-\\&\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}|\xi_{ij}|^2-(\frac{1}{4}-d_{i2})r_{i1}|\zeta_i|^2+\\&(l-1)(\frac{r_{i0}}{4\rho_i}|\varepsilon_i|^2+d_{i0}(y_i^2+\aleph_i^2+\dot\aleph_i^2+\\&g_{i1}^2))+(r_i^{-1}\tilde\theta-\sum_{j=1}^{l-1}z_{ij}\frac{\partial\alpha_{i,j-1}}{\partial\hat\theta})(\dot{\hat\theta}_i-\\&\tau_{i,l-1})+d_{i1}(d_{i2}+8)\aleph_i^2+2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+\\&4d_{i1}f_i^2+d_{i1}d_{i2}h_i^2.\end{aligned}\tag{27}$$

对于类Lyapunov函数

$$V_{il}=V_{i,l-1}+\frac{1}{2}z_{il}^2.\tag{28}$$

类似于式(20)的推导得

$$\begin{aligned}&z_{il}\frac{\partial\alpha_{i,l-1}}{\partial y_i}(p_i\varepsilon_{i1}+q_ix_{i1}+\dot\aleph_i+g_{i1})\leqslant\\&\bar\phi_{il}\hat\theta_iz_{il}+\phi_{il}z_{il}+\frac{r_{i0}}{4\rho_i}|\varepsilon_i|^2+\\&d_{i0}(y_i^2+\aleph_i^2+\dot\aleph_i^2+g_{i1}^2)-\bar\phi_{il}\tilde\theta_iz_{il},\end{aligned}\tag{29}$$

其中:

$$\phi_{il}=\frac{1}{2d_{i0}}z_{il}|\frac{\partial\alpha_{i,l-1}}{\partial y_i}|^2,$$
$$\bar\phi_{il}=(\frac{\rho_i}{r_{i0}}+\frac{1}{2d_{i0}})z_{il}|\frac{\partial\alpha_{i,l-1}}{\partial y_i}|^2.$$

由式(17) (26) (27) (29)得

$$\begin{aligned}\dot V_{il}\leqslant&-\sum_{j=1}^{l-1}c_{ij}z_{ij}^2-d_{i0}\rho_iz_{i1}^2-\bar c_{i1}(z_{i1})^{2K}-\\&\frac{3}{4}r_{i0}|\varepsilon_i|^2-\frac{1}{2}r_{\xi ii}|\xi_{ii}|^2-\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}|\xi_{ij}|^2-\\&(\frac{1}{4}-d_{i2})r_{i1}|\zeta_i|^2+l(\frac{r_{i0}}{4\rho_i}|\xi_i|^2+d_{i0}(y_i^2+\aleph_i^2+\\&\dot\aleph_i^2+g_{i1}^2))+d_{i1}(d_{i2}+8)\aleph_i^2+(r_i^{-1}\tilde\theta_i-\\&\sum_{j=1}^{l-1}z_{ij}\frac{\partial\alpha_{i,j-1}}{\partial\hat\theta_i})(\dot{\hat\theta}-\tau_{i,l-1})-z_{il}\frac{\partial\alpha_{i,l-1}}{\partial\hat\theta_i}\dot{\hat\theta}-\\&\bar\phi_{il}\tilde\theta z_{il}+z_{il}(z_{i,l-1}+z_{i,l+1}+\alpha_{il}+k_{il}y_i-\\&k_{i,l-1}(\hat x_{i1}+k_{i1}y_i)-\frac{\partial\alpha_{i,l-1}}{\partial y_i}\hat x_{i1}-\\&\sum_{j=1}^{l-2}\frac{\partial\alpha_{i,l-1}}{\partial\hat x_{ij}}\dot{\hat x}_{ij}+\phi_{il}+\bar\phi_{il}\hat\theta_i)+\\&2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+4d_{i1}f_i^2+d_{i1}d_{i2}h_i^2.\end{aligned}\tag{30}$$

通过选取如下的调节函数和镇定函数:

$$\begin{cases}\tau_{il}=\tau_{i,l-1}+r_i\bar\phi_{il}z_{il},\\ \alpha_{il}(y_i,\hat x_{i1},\cdots,\hat x_{i,l-1},\hat\theta_i)=\\ -z_{i,l-1}-c_{il}z_{il}-k_{il}z_{i1}+k_{i,l-1}\hat x_{i1}+k_{i,l-1}\\ k_{i1}z_{i1}+\frac{\partial\alpha_{i,l-1}}{\partial y_i}\hat x_{i1}+\sum\limits_{j=1}^{l-2}\frac{\partial\alpha_{i,l-1}}{\partial\hat x_{ij}}\dot{\hat x}_{ij}-\phi_{il}-\\ \bar\phi_{il}\hat\theta_i+\frac{\partial\alpha_{i,j-1}}{\partial\hat\theta_i}\tau_{il}+\sum\limits_{j=1}^{l-2}r_iz_{ij}\frac{\partial\alpha_{j-1}}{\partial\hat\theta_i}\bar\phi_{il}z_{il}.\end{cases}\tag{31}$$

并将其代入式(30)得

$$\begin{aligned}\dot{V}_{il} \leqslant\, & z_{i,l+1}z_{il}-\sum_{j=1}^{l}c_{ij}z_{ij}^2-d_{i0}\rho_i z_{i1}^2-\bar{c}_{i1}(z_{i1})^{2K}-\\ & \frac{3}{4}r_{i0}|\varepsilon_i|^2-\frac{1}{2}r_{\xi ii}|\xi_{ii}|^2-\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}\,|\xi_{ij}|^2-\\ & (\frac{1}{4}-d_{i2})r_{i1}|\zeta_i|^2+l(\frac{r_{i0}}{4\rho_i}|\xi_i|^2+d_{i0}(y_i^2+\aleph_i^2+\\ & \dot{\aleph}_i^2+g_{i1}^2))+d_{i1}(d_{i2}+8)\aleph_i^2+(r_i^{-1}\tilde{\theta}_i-\\ & \sum_{j=1}^{l}z_{ij}\frac{\partial\alpha_{i,j-1}}{\partial\hat{\theta}_i})(\dot{\hat{\theta}}_i-\tau_{il})+2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+\\ & 4d_{i1}f_i^2+d_{i1}d_{i2}h_i^2. \end{aligned}\tag{32}$$

递推地, 当$l=\rho_i$时, 得控制律和自适应律

$$u_i=\hat{x}_{i,\rho_i}=z_{i,\rho_i+1}+\alpha_{i,\rho_i}=\alpha_{i,\rho_i},\ \dot{\hat{\theta}}_i=\tau_{i,\rho_i}.\tag{33}$$

从而

$$\begin{aligned}\dot{V}_{i,\rho_i}\leqslant & -\sum_{j=1}^{\rho_i}c_{ij}z_{ij}^2-\bar{c}_{i1}(z_{i1})^{2K}-\frac{1}{2}r_{i0}|\varepsilon_i|^2-\\ & \frac{1}{2}r_{\xi ii}|\xi_{ii}|^2-\frac{1}{2}\sum_{j=1,j\neq i}^{N}r_{\xi ij}\,|\xi_{ij}|^2-\\ & (\frac{1}{4}-d_{i2})r_{i1}|\zeta_i|^2+\rho_i d_{i0}(\aleph_i^2+\dot{\aleph}_i^2+g_{i1}^2))+\\ & d_{i1}(d_{i2}+8)\aleph_i^2+2\sum_{j=1,j\neq i}^{N}d_{i1}y_j^2+\\ & 4d_{i1}f_i^2+d_{i1}d_{i2}h_i^2.\end{aligned}\tag{34}$$

至此, 由式(34), 利用文[3]的方法可得本文的主要结果.

定理 1 考虑由系统(1), 观测器(13), 控制律和自适应律(33)组成的自适应控制系统. 若假设A1~A3成立, 则一定存在常数$\bar{\mu}>0$, 使得对所有的$\mu_{ij}\in[0,\bar{\mu})(i,j=1,\cdots,N)$和任意的初始条件$x_i(0)$,

1) 闭环系统的所有信号全局一致有界,

2) 除参数估计外的其他信号皆渐进收敛到零.

4 结论性的注(Concluding remarks)

当$b_{i,m_i}/\|\Lambda_i\|=1$时, 本文的观测器是$N(\rho_i-1)$阶的, 自适应律是$N$阶的, 此控制器的动态阶次比文献[3]中基于MT滤波器设计的控制器低$(m_i+3n_i-\rho_i)N=(2n_i+2m_i)N$次, 故本文的控制方案更有应用价值.

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附录A(Appendix A)

式(2)的证明 由$\chi_{i1},\aleph_i,f_i$及$y_i=\chi_{i1}+\aleph_i(t)$得$\chi_{i1}$可由状态空间表示

$$\begin{cases}\dot{\chi}_i=\bar{A}_i\chi_i+\bar{b}_iu_i+f_i,\\ \chi_{i1}=c_i^{\mathrm{T}}\chi_i.\end{cases}$$

考虑$\Delta_{ii}\chi_{i1}$和$\Delta_{ij}y_j$的状态空间表示

$$\begin{cases}\dot{\xi}_{ii}=A_{\xi ii}\xi_{ii}+b_{\xi ii}\chi_{i1},\\ \Delta_{ii}\chi_{i1}=c_i^{\mathrm{T}}\xi_{ii},\end{cases}\quad\begin{cases}\dot{\xi}_{ij}=A_{\xi ij}\xi_{ij}+b_{\xi ij}y_j,\\ \Delta_{ij}y_j=c_i^{\mathrm{T}}\xi_{ij},\end{cases}$$

由假设A3可知$A_{\xi ij}$是稳定的, 所以(2)成立.

式(10)的证明 利用$\bar{a}_{i1}$和$\bar{a}_{il}$的定义得

$$S_i\bar{\bar{A}}_i=\begin{pmatrix}-a_{i,n_i-1} & 1 & & & \\ c_{i,\rho_i}a_{i,n_i-1}-a_{i,n_i-2} & -c_{i,\rho_i} & 1 & & \\ \vdots & \vdots & \ddots & \ddots & \\ c_{i3}a_{i,n_i-1}+\cdots+c_{i,\rho_i}a_{i,m_i+2}-a_{i,m_i+1} & -c_{i3} & \cdots & -c_{i,\rho_i} & 1\\ c_{i2}a_{i,n_i-1}+\cdots+c_{i,\rho_i}a_{i,m_i+1}-a_{i,m_i}+c_{i1} & 0 & \cdots & 0 & 0\end{pmatrix}=A_{ia}S_i.$$

矩阵中未标出的元素为零. $S_ie_{\rho_i}=\dfrac{1}{\|\Lambda_i\|}e_{\rho_i}$易证.

式(15)的证明 由式(12)~(14), q_{il}, $\bar{q}_{il}$和Δ_{il}的定义, 对于$1\leqslant l\leqslant\rho_i-2$,

$$\begin{aligned}&p_i\dot{\varepsilon}_{il}=\dot{x}_{i,l+1}-\dot{\hat{x}}_{il}-k_{il}\dot{x}_{i1}=\\&p_i\varepsilon_{i,l+1}-p_ik_{il}\varepsilon_{i1}+q_{il}y_i+\bar{q}_{il}\aleph_i+g_{i,l+1}-k_{il}g_{i1}=\\&p_i\varepsilon_{i,l+1}-p_ik_{il}\varepsilon_{i1}+\Delta_{il}+g_{i,l+1}-k_{il}g_{i1},\end{aligned}$$

对于$l=\rho_i-1$, 结论类证.

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