文章编号: 1000−8152(2009)02−0197−06

连续分布时滞Cohen-Grossberg神经网络渐近稳定性准则

李 涛[1], 费树岷[1], 王 婷[2], 朱 清[1]

(1. 东南大学 复杂工程系统测量与控制教育部重点实验室, 江苏 南京 210096;

2. 安庆师范学院 数学与计算科学学院, 安徽 安庆 246011)

摘要: 通过选取一个新颖的Lyapunov-Krasovskii泛函和引入所考虑系统的等价描述系统, 讨论了具有时变和连续分布时滞Cohen-Grossberg神经网络的渐近稳定性问题. 在变时滞导函数有上界时, 给出能判定系统是渐近稳定时滞相关的充分性条件. 该条件以线性矩阵不等式形式给出, 易于用MATLAB工具箱LMI进行检验. 最后, 数值例子说明了所得结论的有效性.

关键词: 渐近稳定性; 描述系统; Cohen-Grossberg神经网络; 连续分布时滞; 线性矩阵不等式(LMI)

中图分类号: TP18 **文献标识码**: A

Criteria on asymptotical stability of Cohen-Grossberg neural networks with continuously distributed time-delay

LI Tao[1], FEI Shu-min[1], WANG Ting[2], ZHU Qing[1]

(1. Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, Southeast University, Nanjing Jiangsu 210096, China;

2. School of Mathematics and Computation Science, Anqing Teachers College, Anqing Anhui 246011, China)

Abstract: Choosing a novel Lyapunov-Krasovskii functional and introducing the equivalent descriptor system, we investigate the asymptotical stability for the Cohen-Grossberg neural networks with time-varying and continuously distributed time-delays. When the derivative of the time-varying delay has an upper bound, a delay-dependent sufficient condition is obtained to guarantee the asymptotical stability of the above systems. The condition is presented in terms of LMIs, which can be checked easily by using LMI in MATLAB toolbox. Finally, numerical examples demonstrate the effectiveness of the proposed methods.

Key words: asymptotical stability; descriptor system; Cohen-Grossberg neural networks; continuously distributed delay; linear matrix inequality(LMI)

1 引言(Introduction)

自Cohen和Grossberg在1983年提出Cohen-Grossberg神经网络模型以来[1], 就因为该模型包含几种著名的神经网络模型以及它在许多技术领域的广泛应用而引起了极大的关注. 同时在生物和人工神经网络系统中, 时滞现象存在是不可避免的且时滞是导致网络系统不稳定的关键因素之一. 近年来, 时滞Cohen-Grossberg神经网络稳定性分析已经取得了很多重要的成果[2∼9]. 文献[2,3]讨论了时滞Cohen-Grossberg神经网络的渐近稳定或指数稳定性. 文献[4,5]给出了判定变时滞Cohen-Grossberg神经网络是渐近稳定的充分性条件. 文献[6∼9]探讨了变时滞Cohen-Grossberg神经网络的指数稳定问题.

然而可以注意到, 文献[2,3]所给的结论不适用于时滞是时变的情形. 虽然在文献[5,6]中, 所考虑时滞是时变的, 但是在时滞导函数上界大于1时, 其结论是不成立的. 文献[6∼9]虽然讨论了变时滞Cohen-Grossberg神经网络的指数稳定性, 但是所给出的稳定性准则不是以LMIs形式给出, 这样的结果不容易用现有的先进算法进行检验. 而且就人脑的特性和神经网络本身特性来说, 连续分布时滞更能反映现象的本质. 因此, 讨论连续分布时滞Cohen-Grossberg神经网络的稳定性更具有现实和理论意义[9]. 而在现有文献中, 针对具有时变和连续分布

收稿日期: 2007−04−16; 收修改稿日期: 2008−05 − 15.

基金项目: 国家自然科学基金资助项目(60574006); 高校博士点专项基金资助项目(20070286039).

时滞Cohen-Grossberg神经网络模型, 在变时滞导函数有上界时, 还没有学者以LMI为工具讨论该系统的渐近稳定问题. 本文正是针对已有结论的不足, 研究了具有时变和连续分布时滞Cohen-Grossberg 神经网络的全局渐近稳定性并基于LMI给出时滞相关的充分性条件. 最后的数值例子说明本文结论具有较小的保守性.

2 预备知识(Preliminary knowledge)

考虑下面形式的Cohen-Grossberg神经网络系统:

$$\dot{z}(t)=-a(z(t))[b(z(t))-Ag_1(z(t))-Bg_2(z(t-\tau(t)))-D\int_{-\infty}^{t}K(t-s)g_3(z(s))\mathrm{d}s+L]. \quad (1)$$

这里:

$z(t)=[z_1(t),\cdots,z_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$是状态向量;

$a(z(t))=\mathrm{diag}\{a_1(z_1(t)),\cdots,a_n(z_n(t))\}$表示放大函数;

$b(z(t))=[b_1(z_1(t)),\cdots,b_n(z_n(t))]^{\mathrm{T}}$是行为函数;

$g_i(z(\cdot))=[g_{i1}(z_1(\cdot)),\cdots,g_{in}(z_n(\cdot))]^{\mathrm{T}}\in\mathbb{R}^n(i=1,2,3)$表示激活函数;

$L=[L_1,\cdots,L_n]^{\mathrm{T}}\in\mathbb{R}^n$是常输入向量;

A,B和D是适当维数常数矩阵;

函数$K(t-s)=[k_{ij}(t-s)]_{n\times n}$且$k_{ij}(\cdot)$在区间$[0,\infty)$上非负实连续同时满足$\int_0^{\infty}k_{ij}(s)\mathrm{d}s=1$;

$\tau(t)$是时变函数且满足

$$0\leqslant\tau_0\leqslant\tau(t)\leqslant\tau_m,\ \dot{\tau}(t)\leqslant\mu, \quad (2)$$

这里τ_0,τ_m,μ是常数.

注 1 文献[4,10,11]的稳定性准则是以$\dot{\tau}(t)\leqslant 1$为前提建立的, 显然本文的限制条件比文献[4,10,11]要弱.

假设 1 $a_i(\cdot)$是连续函数且满足$0<\underline{a}_i\leqslant a_i(\cdot)\leqslant\overline{a}_i$. 函数$b_i:\mathbb{R}\to\mathbb{R}$是局部Lipschitz的并存在常数$\gamma_i$使得$\dot{b}_i(\cdot)\geqslant\gamma_i>0$. 记$\Lambda=\mathrm{diag}\{\underline{a}_1,\cdots,\underline{a}_n\}$, $\Psi=\mathrm{diag}\{\overline{a}_1,\cdots,\overline{a}_n\}$和$\Gamma=\mathrm{diag}\{\gamma_1,\cdots,\gamma_n\}$.

假设 2 激活函数$g_{1i}(\cdot),g_{2i}(\cdot),g_{3i}(\cdot)$有界且满足

$$\sigma_i^-\leqslant\frac{g_{1i}(x)-g_{1i}(y)}{x-y}\leqslant\sigma_i^+,\quad \forall\,x,y\in\mathbb{R},\ i=1,\cdots,n,$$
$$\delta_i^-\leqslant\frac{g_{2i}(x)-g_{2i}(y)}{x-y}\leqslant\delta_i^+,$$
$$\rho_i^-\leqslant\frac{g_{3i}(x)-g_{3i}(y)}{x-y}\leqslant\rho_i^+,$$

这里$\sigma_i^+,\sigma_i^-,\delta_i^+,\delta_i^-,\rho_i^+,\rho_i^-$是常数.

在假设2条件下, 系统(1)有一个平衡点记为z^*. 再通过变换$x(\cdot)=z(\cdot)-z^*$, 把系统(1)平衡点z^*移到原点, 则系统(1)可以等价变换成下面的系统

$$\dot{x}(t)=-\alpha(x(t))[\beta(x(t))-Af_1(x(t))-Bf_2(x(t-\tau(t)))-D\int_{-\infty}^{t}K(t-s)f_3(x(s))\mathrm{d}s], \quad (3)$$

这里: $x(\cdot)=[x_1(\cdot),\cdots,x_n(\cdot)]^{\mathrm{T}}$是变换系统(3)的状态向量; $\alpha(x(\cdot))=\mathrm{diag}\{\alpha_1(x_1(\cdot)),\cdots,\alpha_n(x_n(\cdot))\}$, $\beta(x(\cdot))=[\beta_1(x_1(\cdot)),\cdots,\beta_n(x_n(\cdot))]^{\mathrm{T}}$, $\alpha_i(x_i(\cdot))=a_i(x_i(\cdot)+z_i^*),\beta_i(x_i(\cdot))=b(x_i(\cdot)+z_i^*)-b(z_i^*)$; $f_i(x(\cdot))=[f_{i1}(x_1(\cdot)),\cdots,f_{in}(x_n(\cdot))]^{\mathrm{T}}(i=1,2,3)$; $f_{ij}(x_j(\cdot))=g_{ij}(x_j(\cdot)+z_j^*)-g_{ij}(z_j^*)(j=1,\cdots,n)$. 函数$f_{ij}(\cdot)$满足

$$\sigma_j^-\leqslant\frac{f_{1j}(x)}{x}\leqslant\sigma_j^+,\ \delta_j^-\leqslant\frac{f_{2j}(x)}{x}\leqslant\delta_j^+,$$
$$\rho_j^-\leqslant\frac{f_{3j}(x)}{x}\leqslant\rho_j^+,\ \forall\,x\in\mathbb{R},\ j=1,\cdots,n. \quad (4)$$

容易得到$f_{1j}(0)=0,f_{2j}(0)=0$和$f_{3j}(0)=0$.

3 主要结论(Main results)

首先需要引入下面的引理与记号:

引理 1[10] 任意给定适当维数的向量α,β和矩阵N,X,Y,Z, 如果$\begin{bmatrix}X & Y\\ Y^{\mathrm{T}} & Z\end{bmatrix}\geqslant 0$成立, 则有

$$-2\alpha^{\mathrm{T}}N\beta\leqslant\begin{bmatrix}\alpha\\ \beta\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}X & Y-N\\ Y^{\mathrm{T}}-N^{\mathrm{T}} & Z\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}.$$

引理 2[11] 给定常数矩阵X_1,X_2,X_3满足$X_1^{\mathrm{T}}=X_1$和$0<X_2^{\mathrm{T}}=X_2$, 如果$X_1+X_3^{\mathrm{T}}X_2^{-1}X_3<0$成立, 当且仅当$\begin{bmatrix}X_1 & X_3^{\mathrm{T}}\\ * & -X_2\end{bmatrix}<0$或$\begin{bmatrix}-X_2 & X_3\\ * & X_1\end{bmatrix}<0$.

引理 3[12] $x(t)\in\mathbb{R}^n$是具有1阶导数的向量函数, 则对于任意适当维数矩阵$M_1,M_2\in\mathbb{R}^{n\times n}$, $X^{\mathrm{T}}=X>0$和函数$h:=h(t)\geqslant 0$, 下面不等式成立:

$$-\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)X\dot{x}(s)\mathrm{d}s\leqslant\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}M_1^{\mathrm{T}}+M_1 & -M_1^{\mathrm{T}}+M_2\\ * & -M_2^{\mathrm{T}}-M_2\end{bmatrix}\times\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}+h\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}^{\mathrm{T}}\times\begin{bmatrix}M_1^{\mathrm{T}}\\ M_2^{\mathrm{T}}\end{bmatrix}X^{-1}\begin{bmatrix}M_1 & M_2\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-h)\end{bmatrix}. \quad (5)$$

同时, 记

$$\begin{cases}\sigma_i=|\sigma_i^-|,\ \Sigma=\mathrm{diag}\{\sigma_1,\cdots,\sigma_n\},\\ \Sigma_1=\mathrm{diag}\{\sigma_1^+\sigma_1^-,\cdots,\sigma_n^+\sigma_n^-\},\\ \Sigma_2=\mathrm{diag}\{\dfrac{\sigma_1^++\sigma_1^-}{2},\cdots,\dfrac{\sigma_n^++\sigma_n^-}{2}\},\\ \Sigma_3=\mathrm{diag}\{\delta_1^+\delta_1^-,\cdots,\delta_n^+\delta_n^-\},\\ \Sigma_4=\mathrm{diag}\{\dfrac{\delta_1^++\delta_1^-}{2},\cdots,\dfrac{\delta_n^++\delta_n^-}{2}\},\\ \Sigma_5=\mathrm{diag}\{\rho_1^+\rho_1^-,\cdots,\rho_n^+\rho_n^-\},\\ \Sigma_6=\mathrm{diag}\{\dfrac{\rho_1^++\rho_1^-}{2},\cdots,\dfrac{\rho_n^++\rho_n^-}{2}\}.\end{cases} \tag{6}$$

下面给出本文的主要结论: 定理1.

定理 1　任意给定常数$\tau_m\geqslant\tau_0\geqslant0$和$\mu$, 如果存在矩阵$Q_1>0,Q_2>0,R>0,S>0$, 对角矩阵$P>0,Q>0,K>0,U>0,V>0,W>0,H\geqslant0,E\geqslant0,G_1\geqslant0,G_2\geqslant0$, 适当维数的矩阵$M,N,P_i(i=1,2,3),X,Y,Z$和常数$\lambda>0$使得式(7)(8)中LMIs成立:

$$\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}=\begin{bmatrix}X_1 & X_2 & X_4 & Y_1\\ * & X_3 & X_5 & Y_2\\ * & * & X_6 & Y_3\\ * & * & * & Z\end{bmatrix}\geqslant0, \tag{7}$$

$$\begin{bmatrix}\Omega & \bar\tau_m M & \tau_m N\\ * & -\bar\tau_m S & 0\\ * & * & -\tau_m S\end{bmatrix}<0, \tag{8}$$

则系统(3)是渐近稳定的, 其中Ω如下所示且$\bar\tau_m=\tau_m-\tau_0$, n是系统维数, I是单位矩阵,

$$M=[0\ \ 0\ \ 0\ \ M_1\ \ M_2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^{\mathrm T},$$

$$N=[N_1\ \ 0\ \ 0\ \ N_2\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0]^{\mathrm T},$$

$$\Omega_{11}=Q_1+Q_2+\tau_mX_1+N_1+N_1^{\mathrm T}+Y_1+Y_1^{\mathrm T}-U\Sigma_1-V\Sigma_3-W\Sigma_5-2E\Gamma,$$

$$\Omega_{12}=\tau_mX_2+Y_2^{\mathrm T},$$

$$\Omega_{13}=P-P_1^{\mathrm T}+\tau_mX_4+Y_3^{\mathrm T}+K\Sigma-\Gamma Q,$$

$$\Omega_{14}=-Y_1+N_2-N_1^{\mathrm T},$$

$$\Omega_{22}=G_1-G_2+\tau_mX_3+\tau_m(S+Z),$$

$$\Omega_{23}=-P_2^{\mathrm T}+\tau_mX_5,$$

$$\Omega_{33}=\Psi^{\mathrm T}G_2\Psi-\Lambda^{\mathrm T}G_1\Lambda-P_3^{\mathrm T}-P_3+\tau_mX_6,$$

$$\Omega_{44}=(\mu-1)Q_2+M_1+M_1^{\mathrm T}-N_2-N_2^{\mathrm T}-H\Sigma_3.$$

证　首先, 记

$$S(t)=S(x(t),x(t-\tau(t)))=\mathrm{diag}\{s_1(x_1(t),x_1(t-\tau(t))),\cdots,s_n(x_n(t),x_n(t-\tau(t)))\},$$

其中:

$$s_i(\alpha,\beta)=\begin{cases}\dfrac{f_{2i}(\alpha)-f_{2i}(\beta)}{\alpha-\beta}, & \alpha\neq\beta,\\ \delta_i^+\text{或}\delta_i^-, & \alpha=\beta,\end{cases}$$
$$i=1,\cdots,n.$$

根据式(4), 容易看到$f_2(x(t))-f_2(x(t-\tau(t)))=S(t)(x(t)-x(t-\tau(t)))$. 由文献[10], 系统(3)可以等价地写成下面具有连续分布时滞的描述系统:

$$\Omega=\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & 0 & U\Sigma_2+P_1^{\mathrm T}A & V\Sigma_4 & P_1^{\mathrm T}B & W\Sigma_6 & P_1^{\mathrm T}D & -P_1^{\mathrm T}+E\\
* & \Omega_{22} & \Omega_{23} & -Y_2 & 0 & P_2^{\mathrm T}A & 0 & P_2^{\mathrm T}B & 0 & P_2^{\mathrm T}D & -P_2^{\mathrm T}\\
* & * & \Omega_{33} & -Y_3 & 0 & P_3^{\mathrm T}A+K^{\mathrm T} & 0 & P_3^{\mathrm T}B & 0 & P_3^{\mathrm T}D & -P_3^{\mathrm T}\\
* & * & * & \Omega_{44} & M_2-M_1^{\mathrm T} & 0 & 0 & H\Sigma_4 & 0 & 0 & 0\\
* & * & * & * & -Q_1-(M_2^{\mathrm T}+M_2) & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & -U & 0 & 0 & 0 & 0 & A^{\tau}Q^{\tau}\\
* & * & * & * & * & * & -V+R & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & * & -(1-\mu)R-H & 0 & 0 & B^{\mathrm T}Q^{\mathrm T}\\
* & * & * & * & * & * & * & * & -W+n\lambda I & 0 & 0\\
* & * & * & * & * & * & * & * & * & -\dfrac{1}{n}\lambda I & D^{\mathrm T}Q^{\mathrm T}\\
* & * & * & * & * & * & * & * & * & * & -Q-Q^{\mathrm T}
\end{bmatrix},$$

$$\begin{aligned}&\dot x(t)=y(t),y(t)=\alpha(x(t))w(t),\\ &w(t)=-\beta(x(t))+Af_1(x(t))+Bf_2(x(t))-BS(t)\int_{t-\tau(t)}^{t}\dot x(s)\mathrm ds+D\int_{-\infty}^{t}K(t-s)f_3(x(s))\mathrm ds.\end{aligned} \tag{9}$$

根据假设1和式(4), 构造下面形式的Lyapunov-Krasovskii泛函:

$$V(x_t)=V_1(x_t)+V_2(x_t)+V_3(x_t)+V_4(x_t), \tag{10}$$

其中:

$$V_1(x_t)=\int_{t-\tau_m}^{t}x^{\mathrm T}(s)Q_1x(s)\mathrm ds+2\sum_{i=1}^{n}p_i\int_0^{x_i}\frac{s}{\alpha_i(s)}\mathrm ds+$$

$$2\sum_{i=1}^{n}k_i\int_0^{x_i}\frac{f_{1i}(s)+\sigma_i s}{\alpha_i(s)}\mathrm{d}s+$$
$$2\sum_{i=1}^{n}q_i\int_0^{x_i}\frac{\beta_i(s)-\gamma_i s}{\alpha_i(s)}\mathrm{d}s,$$
$$V_2(x_t)=$$
$$\int_{t-\tau(t)}^{t}[x^{\mathrm{T}}(s)Q_2x(s)+f_2^{\mathrm{T}}(x(s))Rf_2(x(s))]\mathrm{d}s,$$
$$V_3(x_t)=\int_{-\tau_m}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)(S+Z)\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$
$$V_4(x_t)=$$
$$\lambda\sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{\infty}k_{ij}(\theta)\int_{t-\theta}^{t}f_{3j}^2(x_j(s))\mathrm{d}s\mathrm{d}\theta,$$

这里: $P=\mathrm{diag}\{p_1,\cdots,p_n\}>0, K=\mathrm{diag}\{k_1,\cdots,k_n\}>0, Q=\mathrm{diag}\{q_1,\cdots,q_n)\}>0$和$Q_1>0, Q_2>0, R>0, S>0, Z\geqslant 0$ 和 $\lambda>0$ 是待定矩阵和常数.

根据假设1和式(9), 对于任意适当维数的对角矩阵$E\geqslant 0, G_1\geqslant 0, G_2\geqslant 0$, 下面不等式成立:

$$\begin{cases}0\leqslant 2[x^{\mathrm{T}}(t))E\beta(x(t))-x^{\mathrm{T}}(t)E\varGamma x(t)],\\ 0\leqslant[y^{\mathrm{T}}(t)G_1y(t)-w^{\mathrm{T}}(t)\varLambda^{\mathrm{T}}G_1\varLambda w(t)],\\ 0\leqslant[w^{\mathrm{T}}(t)\varPsi^{\mathrm{T}}G_2\varPsi w(t)-y^{\mathrm{T}}(t)G_2y(t)].\end{cases}\tag{11}$$

由文献[9], 存在任意适当维数的对角矩阵$U>0, V>0, H\geqslant 0, W>0$, 使得下面式子成立:

$$0\leqslant\left\{\begin{bmatrix}x(t)\\ f_1(x(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-U\varSigma_1 & U\varSigma_2\\ U\varSigma_2 & -U\end{bmatrix}\begin{bmatrix}x(t)\\ f_1(x(t))\end{bmatrix}+\right.$$
$$\begin{bmatrix}x(t)\\ f_2(x(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-V\varSigma_3 & V\varSigma_4\\ V\varSigma_4 & -V\end{bmatrix}\begin{bmatrix}x(t)\\ f_2(x(t))\end{bmatrix}+$$
$$\begin{bmatrix}x(t)\\ f_3(x(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-W\varSigma_5 & W\varSigma_6\\ W\varSigma_6 & -W\end{bmatrix}\begin{bmatrix}x(t)\\ f_3(x(t))\end{bmatrix}+$$
$$\begin{bmatrix}x(t-\tau(t))\\ f_2(x(t-\tau(t)))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-H\varSigma_3 & H\varSigma_4\\ H\varSigma_4 & -H\end{bmatrix}\times$$
$$\left.\begin{bmatrix}x(t-\tau(t))\\ f_2(x(t-\tau(t)))\end{bmatrix}\right\}.\tag{12}$$

下面计算$V_1(x_t)$沿着系统(9)解轨线的导函数

$$\dot{V}_1(x_t)=$$
$$x^{\mathrm{T}}(t)Q_1x(t)-x^{\mathrm{T}}(t-\tau_m)Q_1x(t-\tau_m)+$$
$$2x^{\mathrm{T}}(t)Pw(t)+2f_1^{\mathrm{T}}(x(t))Kw(t)+$$
$$2x^{\mathrm{T}}(t)K\varSigma w(t)+2\beta^{\mathrm{T}}(x(t))Qw(t)-$$
$$2x^{\mathrm{T}}(t)\varGamma^{\mathrm{T}}Qw(t).\tag{13}$$

由式(9), 可以推出

$$2x^{\mathrm{T}}(t)Pw(t)=$$
$$2\eta^{\mathrm{T}}(t)G^{\mathrm{T}}\left\{\begin{bmatrix}w(t)\\ 0\\ -w(t)-\beta(x(t))\end{bmatrix}+\right.$$
$$\begin{bmatrix}0\\ 0\\ A\end{bmatrix}f_1(x(t))+\begin{bmatrix}0\\ 0\\ B\end{bmatrix}f_2(x(t))-$$
$$\begin{bmatrix}0\\ 0\\ B\end{bmatrix}S(t)\int_{t-\tau(t)}^{t}\dot{x}(s)\mathrm{d}s+$$
$$\left.\begin{bmatrix}0\\ 0\\ D\end{bmatrix}\int_{-\infty}^{t}K(t-s)f_3(x(s))\mathrm{d}s\right\},\tag{14}$$

其中:

$$G=\begin{bmatrix}P & 0 & 0\\ 0 & 0 & 0\\ P_1 & P_2 & P_3\end{bmatrix},$$
$$\eta^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t),y^{\mathrm{T}}(t),w^{\mathrm{T}}(t)],$$

$P_i(i=1,2,3)$是适当维数的矩阵. 根据引理1和式(7), 可以得到

$$-2\eta^{\mathrm{T}}(t)G^{\mathrm{T}}[0\ \ 0\ \ B^{\mathrm{T}}]^{\mathrm{T}}S(t)\int_{t-\tau(t)}^{t}\dot{x}(s)\mathrm{d}s\leqslant$$
$$\tau_m\eta^{\mathrm{T}}(t)X\eta(t)+2\eta^{\mathrm{T}}(t)\{Y(x(t)-x(t-\tau(t)))-$$
$$G^{\mathrm{T}}[0\ \ 0\ \ B^{\mathrm{T}}]^{\mathrm{T}}(f_2(x(t))-f_2(x(t-\tau(t)))\}+$$
$$\int_{t-\tau_m}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s.\tag{15}$$

根据式(13)~(15)和引理3, 计算$\dot{V}_i(x_t)(i=2,3,4)$得到

$$\dot{V}(x_t)\leqslant$$
$$2f_1^{\mathrm{T}}(x(t))Kw(t)+2x^{\mathrm{T}}(t)K\varSigma w(t)+$$
$$2\beta^{\mathrm{T}}(x(t))Q[-\beta(x(t))+Af_1(x(t))+$$
$$Bf_2(x(t-\tau(t)))+D\int_{-\infty}^{t}K(t-s)\times$$
$$f_3(x(s))\mathrm{d}s]-2x^{\mathrm{T}}(t)\varGamma^{\mathrm{T}}Qw(t)+$$
$$2x^{\mathrm{T}}(t)Pw(t)+2[x^{\mathrm{T}}(t)P_1^{\mathrm{T}}+y^{\mathrm{T}}(t)P_2^{\mathrm{T}}+$$
$$w^{\mathrm{T}}(t)P_3^{\mathrm{T}}][-w(t)-\beta(x(t))+Af_1(x(t))+$$
$$Bf_2(x(t-\tau(t)))+D\int_{-\infty}^{t}K(t-s)\times$$
$$f_3(x(s))\mathrm{d}s]+\tau_m\eta^{\mathrm{T}}(t)X\eta(t)+2\eta^{\mathrm{T}}(t)Y\times$$
$$(x(t)-x(t-\tau(t)))+x^{\mathrm{T}}(t)Q_1x(t)-$$
$$x^{\mathrm{T}}(t-\tau_m)Q_1x(t-\tau_m)+x^{\mathrm{T}}(t)Q_2x(t)+$$
$$f_2^{\mathrm{T}}(x(t))Rf_2(x(t))-(1-\mu)[x^{\mathrm{T}}(t-\tau(t))\times$$

$$Q_2x(t-\tau(t))+f_2^{\mathrm T}(x(t-\tau(t)))R\times$$
$$f_2(x(t-\tau(t)))]+\tau_m y^{\mathrm T}(t)(S+Z)y(t)+$$
$$\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_m)\end{bmatrix}^{\mathrm T}\begin{bmatrix}M_1^{\mathrm T}+M_1 & -M_1^{\mathrm T}+M_2\\ * & -M_2^{\mathrm T}-M_2\end{bmatrix}\times$$
$$\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_m)\end{bmatrix}+\bar\tau_m\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_m)\end{bmatrix}^{\mathrm T}\times$$
$$\begin{bmatrix}M_1^{\mathrm T}\\M_2^{\mathrm T}\end{bmatrix}S^{-1}\begin{bmatrix}M_1\ M_2\end{bmatrix}\begin{bmatrix}x(t-\tau(t))\\x(t-\tau_m)\end{bmatrix}+$$
$$\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}^{\mathrm T}\begin{bmatrix}N_1^{\mathrm T}+N_1 & -N_1^{\mathrm T}+N_2\\ * & -N_2^{\mathrm T}-N_2\end{bmatrix}\times$$
$$\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}+\tau_m\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}^{\mathrm T}\times$$
$$\begin{bmatrix}N_1^{\mathrm T}\\N_2^{\mathrm T}\end{bmatrix}S^{-1}\begin{bmatrix}N_1\ N_2\end{bmatrix}\begin{bmatrix}x(t)\\x(t-\tau(t))\end{bmatrix}+$$
$$f_3^{\mathrm T}(x(t))(n\lambda I)f_3(x(t))-$$
$$(\int_{-\infty}^{t}K(t-s)f_3(x(s))\mathrm ds)^{\mathrm T}\times$$
$$(\frac{1}{n}\lambda I)(\int_{-\infty}^{t}K(t-s)f_3(x(s))\mathrm ds).\tag{16}$$

再综合式(11)(12)和(16), 进一步可以得到

$$\dot V(x_t)\leqslant$$
$$\zeta^{\mathrm T}(t)\big[\varOmega+\bar\tau_m MS^{-1}M^{\mathrm T}+\tau_m NS^{-1}N^{\mathrm T}\big]\zeta(t),$$

其中

$$\zeta^{\mathrm T}(t)=$$
$$[\eta^{\mathrm T}(t),x^{\mathrm T}(t-\tau(t)),x^{\mathrm T}(t-\tau_m),f_1^{\mathrm T}(x(t)),$$
$$f_2^{\mathrm T}(x(t)),\ f_2^{\mathrm T}(x(t-\tau(t))),\ f_3^{\mathrm T}(x(t)),$$
$$(\int_{-\infty}^{t}K(t-s)f_3(x(s))\mathrm ds)^{\mathrm T},\ \beta^{\mathrm T}(x(t))]$$

和$M,N,\varOmega$如式(8)所示. 根据引理2, 式(8)能保证

$$\varOmega+\bar\tau_m MS^{-1}M^{\mathrm T}+\tau_m NS^{-1}N^{\mathrm T}<0$$

成立. 则对于任意$\zeta(t)\neq 0$, 都有$\dot V(x_t)<0$成立, 则满足式(2)和(4)的系统(3)是渐近稳定的. 证毕.

注 2 在假设1,2的条件下, 定理1给出了保证系统(3)是全局渐近稳定的充分性条件. 稳定性准则式(7)(8)很容易用MATLAB的工具箱LMI进行检验.

4 举例(Numerical examples)

例 1 当$f_i(\cdot)=f(\cdot)$时, 考虑具有下面参数Cohen-Grossberg神经网络:

$$\begin{cases}\alpha(x(t))=\begin{bmatrix}1&0\\0&1\end{bmatrix},\\ \beta(x(t))=\begin{bmatrix}x_1^3(t)+x_1(t)\\0.8x_2(t)\end{bmatrix},\\ A=\begin{bmatrix}1&-1.7\\-1.6&1\end{bmatrix},\ B=\begin{bmatrix}1&0.6\\0.5&0.8\end{bmatrix},\\ D=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ f(x)=\begin{bmatrix}0.2\tanh x_1\\0.2\tanh x_2\end{bmatrix},\\ \tau(t)=1.5\sin^2 t,\end{cases}\tag{17}$$

则有

$$\varPsi=\varLambda=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ \varGamma=\begin{bmatrix}1&0\\0&0.8\end{bmatrix},$$
$$\varSigma=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ \varSigma_1=\varSigma_3=\begin{bmatrix}-0.04&0\\0&-0.04\end{bmatrix},$$
$$\varSigma_2=\varSigma_4=0,\tau_m=\mu=1.5.$$

根据定理1, 容易验证系统(17)是渐近稳定的. 然而定理[1,5]不能证明系统(17)是渐近稳定的.

例 2 考虑如下时滞Cohen-Grossberg神经网络系统:

$$\begin{cases}\alpha(x(t))=\begin{bmatrix}1&0\\0&1\end{bmatrix},\\ \beta(x(t))=\begin{bmatrix}\frac{1}{3}x_1^3(t)+x_1(t)\\x_2(t)\end{bmatrix},\\ A=\begin{bmatrix}1.1&-1.7\\-1.6&1.1\end{bmatrix},\ B=\begin{bmatrix}0&0\\0&0\end{bmatrix},\\ D=\begin{bmatrix}0.4&0.3\\0.1&0.39\end{bmatrix},\ \varGamma=\begin{bmatrix}1&0\\0&1\end{bmatrix},\\ \varSigma=\begin{bmatrix}0.3&0\\0&0.3\end{bmatrix},\ \varSigma_2=\varSigma_6=\begin{bmatrix}0&0\\0&0\end{bmatrix},\\ \varSigma_1=\varSigma_5=\begin{bmatrix}-0.09&0\\0&-0.09\end{bmatrix},\\ K(t-s)=\begin{bmatrix}\mathrm e^{-t}&3\mathrm e^{-3t}\\4\mathrm e^{-4t}&2\mathrm e^{-2t}\end{bmatrix}.\end{cases}\tag{18}$$

根据定理1, 容易验证系统(18)是渐近稳定的. 根据定理[8],

$$\nabla=\varGamma-|A|\varSigma-|D|\varSigma=\begin{bmatrix}0.550&-0.600\\-0.510&0.553\end{bmatrix}.$$

矩阵∇的顺序主子式分别为0.55, −0.0018. 由

文[9]可知, 矩阵∇不是非奇异的M–矩阵. 则定理[1,9]不能证明系统(18)是渐近稳定的.

5 结语(Conclusion)

本文通过选取适当的Lyapunov-Krasovskii泛函, 引入与所考虑系统等价的描述系统, 得出了使具有时变和连续分布时滞Cohen-Grossberg神经网络在平衡点是渐近稳定时滞相关的充分性条件. 数值例子说明了本文结果较以往文献有很大的改进.

参考文献(References):

[1] COHEN M A, GROSSBERG S. Absolute stability of global pattern formation and parallel memory storage by competitive neural networks[J]. *IEEE Transactions on Systems, Man and Cybernetics*, 1983, 13(1): 815 – 826.

[2] JI C, ZHANG H, YI P. Analysis for Cohen-Grossberg neural networks with multiple delays[J]. *Journal of Control Theory and Applications*, 2006, 4(3): 392 – 396.

[3] CHEN Z, ZHAO D, RUAN J. Dynamic analysis of high-order Cohen-Grossberg neural networks with time delay[J]. *Chaos, Solitons and Fractals*, 2007, 32(4): 1538 – 1546.

[4] XIONG W, CUI B. Some criteria for robust stability of Cohen-Grossberg neural networks with delays[J]. *Chaos, Solitons and Fractals*, 2008, 36(5): 1357 – 1365.

[5] WU W, CUI B, LOU X. Some criteria for asymptotic stability of Cohen-Grossberg neural networks with time-varying delays[J]. *Neurocomputing*, 2007, 70(4/6): 1085 – 1088.

[6] JIANG M, SHEN Y, LIAO X. Boundedness and global exponential stability for generalized Cohen-Grossberg neural networks with variable delay[J]. *Applied Mathematics and Computation*, 2006, 172(1): 379 – 393.

[7] CHEN W, ZHENG W. Stability analysis for Cohen-Grossberg neural networks with time-varying delays[J]. *IEEE International Symposium on Circuits and Systems*, 2006, 21(1): 3630 – 3633.

[8] SONG Q, CAO J. Stability analysis of Cohen-Grossberg neural network with both time-varying and continuously distributed delays[J]. *Journal of Computational and Applied Mathematics*, 2006, 197(1):188 – 203.

[9] HUANG T, LI C, CHEN G. Stability of Cohen-Grossberg neural networks with unbounded distributed delays[J]. *Chaos, Solitons and Fractals*, 2007, 34(3): 992 – 996.

[10] CHEN W, LU X. Delay-dependent exponential stability of neural networks with variable delay: an LMI approach[J]. *IEEE Transactions on Circuits and Systems-II*, 2006, 53(3): 837 – 842.

[11] XU S, LAM J. Novel global stability criteria for interval neural networks with multiple time-varying delays[J]. *Physics Letters A*, 2005, 342(4): 322 – 330.

[12] ZHANG X, WU M, SHE J, et al. Delay-dependent stabilization of linear systems with time-varying state and input delays[J]. *Automatica*, 2005, 41(8): 1405 – 1412.

作者简介:

李 涛 (1979—), 男, 博士研究生, 研究方向为神经网络动力学分析与综合, E-mail: fengtailitao@yahoo.com.cn;

费树岷 (1961—), 男, 教授, 博士生导师, 研究方向为非线性系统几何理论、切换系统与时滞系统的稳定性分析, E-mail: smfei@seu.edu.cn;

王 婷 (1980—), 女, 讲师, 研究方向为神经网络计算与优化;

朱 清 (1976—), 男, 博士研究生, 研究方向为自适应控制等.