

一类切换系统基于观测器的滑模降阶控制

何召兰^{1,2}, 王 茂¹, 崔 阳²

(1. 哈尔滨工业大学 空间控制与惯性技术研究中心, 黑龙江 哈尔滨 150001;

2. 哈尔滨理工大学 自动化学院, 黑龙江 哈尔滨 150080)

摘要: 针对一类状态不可测的切换系统, 研究了其基于观测器的滑模控制问题. 设计了一类降阶观测器, 并用观测到的状态设计了滑模面函数以及滑模控制器, 使得闭环系统的状态能够到达滑模面上, 产生滑动模态. 并应用Lyapunov函数的方法给出了切换系统的滑动模态可达条件以及确保闭环切换系统渐近稳定的离散切换律. 最后, 数值仿真验证了本文所提方法的有效性.

关键词: 切换系统; Lyapunov函数; 滑模控制; 降阶观测器

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Sliding mode control for a class of switched systems based on reduced-order observer

HE Zhao-lan^{1,2}, WANG Mao¹, CUI Yang²

(1. Space Control and Inertial Technology Research Center, Harbin Institute of Technology, Harbin Heilongjiang 150001, China;

2. School of Automation, Harbin University of Science and Technology, Harbin Heilongjiang 150080, China)

Abstract: The sliding mode control for switched systems with unmeasured states is addressed. Firstly, a reduced-order observer is designed to estimate the unmeasured states, and then a sliding surface function is chosen and a sliding mode control law is synthesized based on the estimated states. The sliding mode control law ensures the reachability of the pre-defined sliding surface, and thus the sliding mode dynamics is resulted. A sufficient condition is also derived to guarantee the existence of the sliding mode dynamics, and a set of switching laws are then devised to insure asymptotic stability of the overall switched closed-loop system. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed approaches.

Key words: switched systems; Lyapunov function; sliding mode control; reduced-order observer

1 引言(Introduction)

切换系统作为一类重要的混杂动态系统, 其稳定性分析和控制器的设计得到了广泛的关注. 文献[1~3]对简单切换系统各种稳定的充分条件作了详细的综述, 总结了切换系统早期的控制方法. Hetel等讨论了不确定时滞时变切换系统的鲁棒控制器综合问题, 通过切换信号的设计确保系统对由反馈延迟引入的不确定性因素具有鲁棒性^[4]. Blanchini等讨论了当凸Lyapunov函数不存在时, 切换线性系统的镇定问题^[5]. 文献[6]基于多Lyapunov函数方法, 应用状态反馈实现了不确定切换系统的鲁棒镇定.

滑模控制具有对参数摄动不敏感、抗干扰能力强和响应速度快等优点^[7], 已被成功用于大量实际

系统, 如机器人操纵装置和电力系统等. 近年来出现了有关切换系统滑模控制的研究^[8~11], Richard等讨论了一类布尔输入切换系统的滑模控制问题, 建立了切换系统滑动模态可达条件的一般表达式, 基于Lyapunov函数得到了几种不同的滑模控制策略^[8]. Wu和Lam研究了时变延迟切换系统的滑模控制问题^[11].

实际切换系统的物理状态往往不易直接测量, 或不可能实际上获得系统全部状态变量, El-Farra用多Lyapunov函数法, 设计了输入受限非线性切换系统的输出反馈控制器, 并基于状态观测值设计了切换策略^[12]. 本文将针对一类非线性扰动切换系统, 引入滑模控制技术, 提出了一种基于观测器的滑模降阶控制方法, 根据滑动模态可达条件, 应用观测状

态设计了系统的滑模控制器和切换律,最后用实例仿真验证了它的有效性.

2 问题描述(Problem formulation)

考虑如下非线性扰动切换系统:

$$\begin{aligned} \dot{x}(t) &= A(\gamma_t)x(t) + B(\gamma_t)u(\gamma_t) + f(x, t, \gamma_t), \quad (1) \\ y(t) &= C(\gamma_t)x(t). \quad (2) \end{aligned}$$

其中: $x(t) \in \mathbb{R}^n$ 是系统的状态向量, $u(\gamma_t) \in \mathbb{R}^m$ 是控制输入向量, $y(t) \in \mathbb{R}^r$ 是输出向量; $A(\gamma_t), B(\gamma_t), C(\gamma_t)$ 是具有适当维数的常值矩阵; $f(x, t, \gamma_t)$ 是未知非线性函数; $\gamma_t: [0, +\infty) \rightarrow \varphi = \{1, \dots, N\}$ 为切换信号, 是一个分段右连续的时间函数, 即 $\forall k, \gamma_{t_k} = \lim_{t \rightarrow t_k^+} \gamma_t$. 系统(1)可看作由 N 个连续时间子系统构成的混杂系统, 各子系统之间在一些规则作用下进行切换, 这些规则定义了用以描述离散状态时间演化的切换序列. 如果用 t_{i_k} 和 $t_{i'_k}$ 表示第 i 个子系统的第 k 次切入和切出时间, 即对所有的 $k \in \mathbb{Z}_+$, $\gamma(t_{i_k}^+) = \gamma(t_{i'_k}^-) = i$, 当 $t_{i_k}^+ \leq t \leq t_{i'_k}^-$ 时第 i 个子系统处于被激活状态.

对任意 $\gamma_t = i \in \varphi$, 设

$$\begin{aligned} A(\gamma_t) &= A_i, \quad B(\gamma_t) = B_i, \quad C(\gamma_t) = C_i, \\ f(x, t, \gamma_t) &= f_i(x, t), \quad u(\gamma_t) = u_i. \end{aligned}$$

假设 1 矩阵对 (A_i, B_i) 和 (A_i, C_i) 分别是可控、可观测的.

由 (A_i, C_i) 的可观测性知, 存在矩阵 L_i 使得 $(A_i - L_i C_i)$ 是稳定的, 因此对于任意的正定矩阵 Q_i , 以下李雅普诺夫方程有唯一正定对称解 P_i :

$$(A_i - L_i C_i)^T P_i + P_i (A_i - L_i C_i) = -Q_i. \quad (3)$$

本文假设输出矩阵满足 $C_i = [I_{ir} \quad 0]$, 因此系统(1)(2)可表示为如下形式($i \in \varphi$):

$$\begin{aligned} \begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \end{bmatrix} &= \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} B_{i1} \\ B_{i2} \end{bmatrix} u + f_i(x), \quad (4) \\ y(t) &= \bar{x}_1, \quad (5) \end{aligned}$$

其中: $\bar{x}_1 \in \mathbb{R}^r, A_{i11} \in \mathbb{R}^{r \times r}, B_{i1} \in \mathbb{R}^{r \times m}$.

假设 2 存在连续函数 $\Gamma_i(\cdot), \beta_i(\cdot)$ 和 $\eta_i(\cdot)$, 使得

$$\begin{aligned} f_i(x, t) &< B_i \Gamma_i(x, t), \\ \|\Gamma_i(x, t)\| &\leq \beta_i(y, t) \eta_i(x, t), \end{aligned}$$

并且对于 (y, \bar{x}_2) 和 (y, \hat{x}_2) 以及 $t \in \mathbb{R}^+$:

$$\|\eta_i(\bar{x}_2, t) - \eta_i(\hat{x}_2, t)\| \leq \mu_i(y, t) \|\bar{x}_2 - \hat{x}_2\|.$$

这里: $\mu_i(\cdot)$ 连续, $\|\cdot\|$ 表示欧几里得范数.

3 主要结果(Main results)

对于系统(4)(5), 存在 K_i 使得 $(A_{i22} + K_i A_{i12})$ 是稳定的, 由式(3)和 $C_i = [I_{ir} \quad 0]$ 知 $K_i = P_{i22}^{-1} P_{i21}$, 其中:

$$P_i = \begin{bmatrix} P_{i11} & P_{i12} \\ P_{i21} & P_{i22} \end{bmatrix}, \quad Q_i = \begin{bmatrix} Q_{i11} & Q_{i12} \\ Q_{i21} & Q_{i22} \end{bmatrix}, \quad (6)$$

并且对于所有的 $i \in \varphi$, 如下李雅普诺夫方程成立:

$$\begin{aligned} (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})^T P_{i22} + P_{i22} (A_{i22} + \\ P_{i22}^{-1} P_{i21} A_{i12}) = -Q_{i22}. \quad (7) \end{aligned}$$

构造状态观测器

$$\begin{aligned} \dot{\hat{z}}_2 &= \\ (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) \hat{z}_2 &+ [A_{i21} + P_{i22}^{-1} P_{i21} \times \\ A_{i11} - (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) P_{i22}^{-1} P_{i21}] y &+ \\ (B_{i2} + P_{i22}^{-1} P_{i21} B_{i1}) u_i, \quad (8) \end{aligned}$$

$$\hat{\bar{x}}_2 = \hat{z}_2 - P_{i22}^{-1} P_{i21} y. \quad (9)$$

定理 1 状态观测器(8)(9)对于任意初始状态 $x(0), \hat{z}(0)$, 存在常数 $\varepsilon > 0$, 使得 $\|\bar{x}_2(t) - \hat{\bar{x}}_2(t)\| \leq \varepsilon$, 其中, $\hat{\bar{x}}_2(t)$ 是 $\bar{x}_2(t)$ 的观测状态.

证 引入线性变换 $z = T_i x$, 这里 T_i 定义为^[13]

$$T_i = \begin{bmatrix} I_r & 0 \\ P_{i22}^{-1} P_{i21} & I_{n-r} \end{bmatrix}, \quad (10)$$

由式(4)(5)和(10)可得以下动态系统:

$$\begin{aligned} \dot{z}_2 &= \\ (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) z_2 &+ [A_{i21} + P_{i22}^{-1} P_{i21} \times \\ A_{i11} - (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) P_{i22}^{-1} P_{i21}] z_1 &+ \\ (B_{i2} + P_{i22}^{-1} P_{i21} B_{i1}) u_i, \quad (11) \end{aligned}$$

$$y = z_1. \quad (12)$$

由 $z = T_i x$ 和式(9), $z_2 = \bar{x}_2 + P_{i22}^{-1} P_{i21} y$. 根据定义 $\hat{z}_2 = \hat{\bar{x}}_2 + P_{i22}^{-1} P_{i21} y$, 可得 $\bar{x}_2 - \hat{\bar{x}}_2 = z_2 - \hat{z}_2$. 设观测误差 $e = \bar{x}_2 - \hat{\bar{x}}_2$, 由式(8)和式(11),

$$\dot{e} = (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12}) e, \quad i \in \varphi, \quad (13)$$

对于误差系统(13), 考虑如下Lyapunov函数:

$$V_e = \sum_{i=1}^N e^T P_{i22} e. \quad (14)$$

对 V_e 求时间的导数, 可得

$$\begin{aligned} \dot{V}_e &= \sum_{i=1}^N e^T [(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})^T P_{i22} + \\ P_{i22} (A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})] e &= \\ - \sum_{i=1}^N e^T Q_{i22} e &< 0. \quad (15) \end{aligned}$$

由Lyapunov稳定性理论知, 误差系统(13)是渐近稳定的, 因此存在正的常数 ε , 使得对于任意初始状态出发的运动满足 $\|\bar{x}_2(t) - \hat{x}_2(t)\| \leq \varepsilon$.

证毕.

现在, 将设计滑模控制以确保系统状态到达滑动模态区并做滑模运动. 系统(1)(2)的滑动模态到达条件为^[8]

$$\dot{V} = \sum_{i=1}^N s_i^T W_i \dot{s}_i < 0, \quad (16)$$

其中: W_i 为正定对称矩阵, s_i 是各子系统的滑模函数, 定义如下:

$$s_i = F_{i1}y + F_{i2}\hat{x}_2, \quad i \in \varphi. \quad (17)$$

在新坐标 (x, e) 下, 式(17)可表示为

$$s_i = F_i x - F_{i2}e, \quad i \in \varphi. \quad (18)$$

这里: $F_i = [F_{i1} \quad F_{i2}]$ 是待设计的参数, $F_{i1} \in \mathbb{R}^{m \times r}$, F_i 的选择应满足 $A_i - B_i(F_i B_i)^{-1} F_i A_i$ 有 $n - m$ 个实部为负的特征值, 并且 $F_i B_i$ 是非奇异的.

由式(13)和式(18)得滑模函数的导数如下:

$$\dot{s}_i = F_i(A_i x + B_i u_i + f_i(x)) - F_{i2}(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})e, \quad i \in \varphi. \quad (19)$$

系统进入滑动模态区时 $\dot{s}_i = 0$, 可得到等效控制

$$u_{ieq}(t) = -(F_i B_i)^{-1} [F_i A_i x + F_i f_i(x) - F_{i2}(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})e], \quad i \in \varphi. \quad (20)$$

将 \dot{V} 表示为如下点乘的形式:

$$\dot{V} = \sum_{i=1}^N (W_i s_i)^T \hat{u}_i, \quad (21)$$

显然, \dot{V} 为负定的一个充分条件是伪控制 \hat{u}_i 取为 $\hat{u}_i = -\rho_i s_i$, 这里 $\rho_i > 0$. 对任意 $i \in \varphi$, 定义如下伪控制 \hat{u}_i :

$$\hat{u}_i = \begin{cases} -\rho_i \frac{W_i s_i}{\|W_i s_i\|}, & s_i \neq 0, \\ 0, & s_i = 0. \end{cases} \quad (22)$$

由于等效控制 u_{ieq} 满足 $\dot{s}_i = 0$, 因此有以下等式:

$$F_i(A_i x + B_i u_{ieq} + f_i(x)) - F_{i2}(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})e = 0, \quad i \in \varphi. \quad (23)$$

用式(19)减式(23), 得到 \dot{s}_i 的另一种表示形式:

$$\dot{s}_i = F_i B_i (u_i - u_{ieq}), \quad i \in \varphi. \quad (24)$$

由式(21)和 \hat{u}_i 的定义, 将式(20)代入式(24)可得实际

控制为

$$u_i = -(F_i B_i)^{-1} [F_i A_i x + F_i f_i(x) - F_{i2}(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})e - \hat{u}_i], \quad i \in \varphi. \quad (25)$$

由于式(25)含有不可测状态向量 \bar{x}_2 , 因此需要对其进行改进, 定理2给出了基于观测器的滑模控制.

定理2 考虑切换系统(1)(2), 对任意 $i \in \varphi$, 定义控制器

$$u_i = -(F_i B_i)^{-1} [(F_{i1} A_{i11} + F_{i2} A_{i21})y + (F_{i1} A_{i12} + F_{i2} A_{i22})\hat{x}_2 + K_i(y, t) + \|F_i B_i\| \beta_i(y, t) \eta_i(y, \hat{x}_2, t) - \hat{u}_i]. \quad (26)$$

其中 \hat{u}_i 如式(22)所定义.

$$K_i(y, t) > \varepsilon (\|F_{i1} A_{i12} - F_{i2} P_{i22}^{-1} P_{i21} A_{i12}\| + \beta_i(y, t) \mu_i(y, t) \|F_i B_i\|). \quad (27)$$

定义区间

$$\Theta_i^* = \{x \in \mathbb{R}^n: V_i(\hat{x}) = 0.5 s_i^T W_i s_i < \zeta_{\hat{x}, i}\}, \\ \zeta_{\hat{x}, i} > 0,$$

如果在任意给定 T 时刻 $x(T) \in \Theta_j^*$, $j \in \varphi$, $j \neq i$, 同时满足 $V_j(\hat{x}(T)) < V_j(\hat{x}(t_{j^*}))$. 这里 $t_{j^*} < T$ 是第 j 个子系统的切入时间, 即 $\gamma(t_{j^*}^-) \neq \gamma(t_{j^*}^+) = j$, 则当 $t \geq T^+$ 时, 设 $\gamma(t) = j$, 使得系统(1)(2)在控制(26)的作用下进入滑动模态区并做滑模运动.

证 当 $s_i(t) = 0$ 时系统已进入滑动模态区做滑模运动. 对于 $s_i(t) \neq 0$ 的情况, 将式(26)代入式(19), 由式(16)可得

$$\dot{V} = \sum_{i=1}^N s_i^T W_i [F_i A_i x - (F_{i1} A_{i11} + F_{i2} A_{i21})y - (F_{i1} A_{i12} + F_{i2} A_{i22})\hat{x}_2 + \hat{u}_i - \|F_i B_i\| \times \beta_i(y, t) \eta_i(y, \hat{x}_2, t) - K_i(y, t) + F_i f_i(x) - F_{i2}(A_{i22} + P_{i22}^{-1} P_{i21} A_{i12})e], \quad (28)$$

其中

$$F_i A_i x - (F_{i1} A_{i11} + F_{i2} A_{i21})y - (F_{i1} A_{i12} + F_{i2} A_{i22})\hat{x}_2 = (F_{i1} A_{i12} + F_{i2} A_{i22})e. \quad (29)$$

由假设2可知

$$F_i f_i(x) - \|F_i B_i\| \beta_i(y, t) \eta_i(y, \hat{x}_2, t) = F_i B_i \Gamma_i(x) - \|F_i B_i\| \beta_i(y, t) \eta_i(y, \hat{x}_2, t) \leq \|F_i B_i\| \|\Gamma_i(x)\| - \|F_i B_i\| \beta_i(y, t) \eta_i(y, \hat{x}_2, t) \leq \beta_i(y, t) \mu_i(y, t) \|F_i B_i\| \|e\|. \quad (30)$$

将式(29)(30)代入式(28), 则

$$\begin{aligned} \dot{V} = & \sum_{i=1}^N s_i^T W_i [\hat{u}_i + (F_{i1} A_{i12} - F_{i2} P_{i22}^{-1} P_{i21} A_{i12}) e + \\ & \beta_i(y, t) \mu_i(y, t) \|F_i B_i\| \|e\| - K_i(y, t)] \leq \\ & \sum_{i=1}^N s_i^T W_i [\hat{u}_i + \varepsilon (\|F_{i1} A_{i12} - F_{i2} P_{i22}^{-1} P_{i21} A_{i12}\| + \\ & \beta_i(y, t) \mu_i(y, t) \|F_i B_i\|) - K_i(y, t)] < \\ & \sum_{i=1}^N s_i^T W_i \hat{u}_i = - \sum_{i=1}^N \rho_i (W_i s_i)^T \frac{W_i s_i}{\|W_i s_i\|} < 0, \quad (31) \end{aligned}$$

因此, 滑动模态到达条件满足, 在控制(26)和给定的切换律作用下, 系统(1)(2)能够进入滑动模态区并做滑模运动. 证毕.

4 仿真实例(Numerical example)

考虑切换系统(4)(5), 其中 $i = 1, 2$,

$$A_1 = \begin{bmatrix} 4 & 1 & 2 \\ 3 & -6 & 2 \\ 1 & 0 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 7 & 3 & 1 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix}.$$

$$B_1 = [0.124 \quad -0.131 \quad -0.892]^T,$$

$$B_2 = [1.013 \quad -0.012 \quad -0.921]^T,$$

$$C_1 = C_2 = [1 \quad 0 \quad 0],$$

$$f_1(x) = B_1 \Gamma_1(y) = B_1(y + \sin t) \sin x_2,$$

$$f_2(x) = B_2 \Gamma_2(y) = B_2(y + \sin t) \sin x_3,$$

扰动满足

$$\|\Gamma_1\| \leq (|y| + 1)|x_2|, \|\Gamma_2\| \leq (|y| + 1)|x_3|,$$

则有

$$\beta_1(y, t) = \beta_2(y, t) = |y| + 1,$$

$$\eta_1(x, t) = |x_2|, \eta_2(x, t) = |x_3|.$$

设

$$L_1 = [12 \quad 4 \quad 0]^T,$$

$$L_2 = [14 \quad 2 \quad -2]^T, Q_1 = Q_2 = 6I_2,$$

求解式(3)得正定对称矩阵 P_i :

$$P_1 = \begin{bmatrix} 0.4476 & 0.0127 & 0.5932 \\ 0.0127 & 0.5021 & 0.2318 \\ 0.5932 & 0.2318 & 4.6500 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.4529 & 0.0190 & 0.2081 \\ 0.0190 & 0.6144 & 0.0919 \\ 0.2081 & 0.0919 & 1.6041 \end{bmatrix}.$$

根据对 F_i 的要求得相应的滑模函数系数矩阵为 $F_1 = [5 \quad 2 \quad 1], F_2 = [6 \quad -1 \quad 1]$. 取初始状态

为 $x(0) = [1 \quad 1 \quad 1]^T$, 图1为切换信号, 值为1表示切换到第1个子系统, 值为2表示切换到第2个子系统; 图2为闭环切换系统状态响应曲线; 图3为控制律随时间的变化曲线. 仿真结果表明, 基于观测器所设计的滑模控制器和切换律使得系统是渐近稳定的.

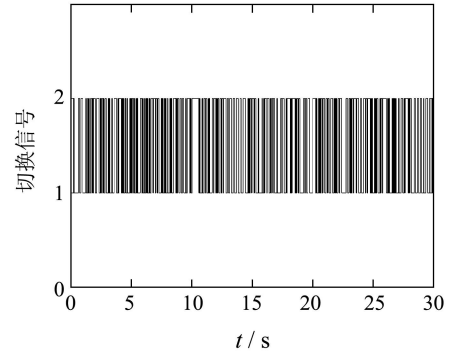


图 1 切换信号

Fig. 1 Switching signal

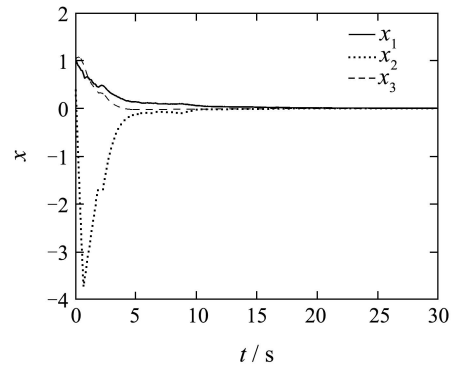


图 2 闭环系统的状态响应曲线

Fig. 2 States of the closed-loop system

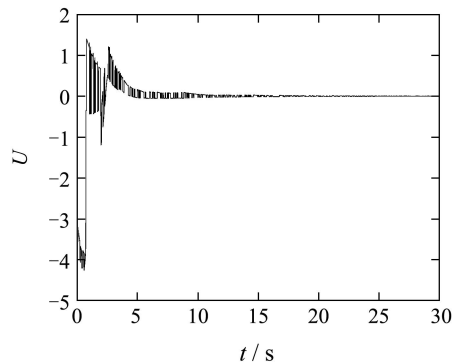


图 3 控制输入曲线

Fig. 3 Control input

5 结论(Conclusion)

对于实际物理状态难于全部测量的切换系统, 本文设计了降阶状态观测器. 结合Lyapunov函数法, 将滑模控制策略应用到切换系统的控制器设计, 给出了切换系统滑动模态可达条件, 同时设计了基于观

测状态的滑模控制器和离散切换策略. 通过仿真验证了本文设计方法使闭环切换系统对于未知非线性扰动具有鲁棒性.

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作者简介:

何召兰 (1969—), 女, 副教授, 博士研究生, 主要研究方向为切换系统、滑模控制以及惯性技术等, E-mail: hezhaolan@sina.com;

王茂 (1965—), 男, 教授, 博士生导师, 主要研究方向为非线性系统控制、惯性技术等, E-mail: wangmao0451@sina.com;

崔阳 (1979—), 女, 讲师, 主要研究方向为理论电工、电机理论与控制等, E-mail: cuiyang97@126.com.