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控制方向未知的SISO非仿射系统间接自适应模糊输出反馈控制

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摘要: 针对一类单输入单输出(SISO)非仿射非线性系统控制方向未知时出现的控制器奇异问题, 提出了一种间接自适应模糊控制方案. 利用中值定理将非仿射系统转化为仿射系统, 通过模糊逻辑系统逼近该仿射系统中的未知函数, 并构造模糊控制器, 同时利用Lyapunov稳定性定理设计自适应律, 最终克服了控制器的奇异问题; 在此基础上, 通过构造观测器估计跟踪误差, 设计输出反馈自适应模糊控制器, 解决了状态不可测时系统控制器设计难题, 采用Lyapunov稳定性定理证明控制器能使得跟踪误差收敛同时闭环系统所有信号均有界. 仿真结果验证了所设计控制方案的可行性与有效性.

关键词: 非仿射非线性系统; 控制器奇异; 自适应模糊控制; 输出反馈

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Indirect adaptive fuzzy output-feedback controller for a SISO nonaffine system with unknown control direction

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Abstract: To deal with the singularity problem of the controller for the single-input single-output (SISO) nonaffine nonlinear system when the control direction is unknown, we propose an indirect adaptive fuzzy control scheme. The nonaffine nonlinear system is first transformed into an affine system by using the mean value theory, and then a fuzzy logic system (FLS) is utilized to approximate the unknown functions in the affine system. Using Lyapunov stability theorem, we derive the adaptive laws for FLS parameters. The fuzzy controller which can remove the singularity problem is designed. On the basis of the aforementioned work and by designing an observer to estimate the tracking error, we construct an adaptive fuzzy output-feedback controller. The difficulty in controller design when the system state is unavailable has been overcome. The Lyapunov stability theory is utilized to prove that the tracking error is asymptotic convergent and all signals in closed-loop system are bounded. Finally, the simulation results demonstrate the feasibility and validity of the proposed control schemes.

Key words: nonaffine nonlinear system; controller singularity; adaptive fuzzy control; output-feedback

1 引言(Introduction)

自适应模糊控制是非线性控制领域中一种重要的控制方法, 在应对参数变化、未建模动态以及外部干扰等方面比传统非线性和自适应等控制方法具有更加优越的性能^[1-2], 因此被广泛用于解决复杂非线性系统的控制问题^[3-10]. 目前大部分自适应模糊控制器仅仅适用于仿射非线性系统, 而大多数实际系统如飞控系统、化学反应控制系统等为非仿射非线性系统, 因而非仿射非线性系统控制器的设计成为非线性控制领域重点研究问题.

近年来, 自适应模糊控制在非仿射系统领域取得很多研究成果^[11-15]. Labiod S等首先利用隐函数定理

证明系统理想控制器的存在性, 然后用模糊逻辑系统构造控制器, 同时利用梯度下降算法设计自适应律, 保证由理想控制器与模糊控制器的误差构成的二次代价函数取最小值, 从而使模糊控制器无限逼近理想控制器^[14]. Liu等将自适应模糊控制方法拓展到MIMO非仿射系统^[15]. 以上非仿射系统控制器设计均假设系统控制方向已知, 实际中大部分非线性系统的控制方向是未知的. 控制方向未知时, 间接自适应模糊控制方案通常会出现控制器奇异问题, 而直接自适应模糊控制方案因不能确定控制器参数的更新方向从而无法完成控制器设计. 对此, Labiod S等提出一种间接自适应模糊控制器, 有效克服了控制器奇异问

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题^[16],但是该方法中所引入的函数因为其在零的邻域有两个阶跃函数,这必定会导致控制输入颤振. Liu等利用中值定理将系统转换为仿射形式,设计了一种模糊自适应控制器,利用Nussbaum函数解决了控制方向未知的难题,最终证明跟踪误差收敛到零的某一邻域内以及闭环系统所有信号有界^[17].在文献[14]的基础上, Labiod S等提出一种直接模糊自适应控制方案,用梯度下降算法设计自适应律,同样引入了Nussbaum函数并证明跟踪误差收敛到零的某一邻域内^[18].目前在针对控制方向未知情况下非仿射系统设计控制器时,均假设系统的状态可测,然而在很多情况下系统状态是不可测的.

基于此,本文首先针对系统状态可测情况设计一种间接自适应模糊控制器,克服了控制器奇异问题,相比文献[18],该方法不会引起控制输入颤振.在系统状态不可测的情况下,通过构造观测器来估计跟踪误差,基于估计状态构造了间接自适应模糊输出反馈控制器,最后利用Lyapunov稳定性定理证明了跟踪误差收敛到零的某一邻域同时闭环系统所有信号有界.

2 问题描述(Problem statement)

考虑如下单输入单输出(SISO)非仿射非线性系统:

$$\begin{cases} \dot{x}_i = x_{i+1}, \\ x^{(n)} = F(\mathbf{x}, u), \\ y = x_1, \end{cases} \quad (1)$$

其中: $y \in \mathbb{R}$ 为系统输出量, $u \in \mathbb{R}$ 为系统输入向量, $\mathbf{x} = (x_1, \dots, x_n)^T \in \mathbb{R}^n$ 为系统的状态向量, $F(\mathbf{x}, u)$ 是光滑的未知函数.

控制目的:设计控制器使得系统输出 y 跟踪指定期望轨迹 y_d ,同时闭环系统所有信号均有界.

假设 1 函数 $g(\mathbf{x}) = \frac{\partial F(\mathbf{x}, u)}{\partial u}$ 满足条件 $0 < \left| \frac{\partial F(\mathbf{x}, u)}{\partial u} \right| \leq \bar{g}$, $g(\mathbf{x})$ 为严格正或严格负,但正负未知.

假设 2 $y_d(t)$ 及其对时间导数 $y_d^{(i)}(t)$, $i = 1, 2, \dots, n$ 是有界的光滑函数.

Nussbaum R. D. 提出Nussbaum函数,解决了系统控制方向未知时控制器设计问题^[19].本文中利用Nussbaum函数克服由系统控制方向未知带来的问题.满足如下性质的函数称为Nussbaum函数:

$$\begin{cases} \limsup_{k \rightarrow \infty} \frac{1}{k} \int_0^k N(\varsigma) d\varsigma = +\infty, \\ \liminf_{k \rightarrow \infty} \frac{1}{k} \int_0^k N(\varsigma) d\varsigma = -\infty. \end{cases}$$

本文中采用的Nussbaum函数为 $N(\varsigma) = \varsigma^2 \cos \varsigma$.

引理 1 $V(t), \varsigma(t)$ 均为光滑函数,其中 $V(t) \geq 0, \forall t \in [0, t_f]$. c_0 为常数, c_1 为正数, $g(\mathbf{x}) \in [g_{\min}, g_{\max}]$; 同时 $0 \notin [g_{\min}, g_{\max}]$ ($g(\mathbf{x})$ 为时变函数). 当 $t \in$

$[0, t_f)$ 时, 如果不等式

$$V(t) \leq c_0 + e^{-c_1 t} \int_0^t [g(\mathbf{x}(\tau))N(\varsigma) + 1] \varsigma e^{c_1 \tau} d\tau$$

成立, 则 $V(t), \varsigma(t), \int_0^t g(\mathbf{x}(\tau))N(\varsigma)\varsigma d\tau$ 均有界^[20].

3 自适应模糊控制器设计(Adaptive fuzzy controller design)

首先利用中值定理将非仿射非线性形式转化为仿射非线性形式, 有

$$F(\mathbf{x}, u) = f(\mathbf{x}) + g(\mathbf{x}, u_\lambda)u, \quad (2)$$

其中 $u_\lambda \in [0, u]$.

则系统(1)转换为如下形式:

$$y^{(n)} = f(\mathbf{x}) + g(\mathbf{x}, u_\lambda)u. \quad (3)$$

针对经转化得到的仿射非线性系统, 分系统状态可测与不可测两种情况进行分析并设计控制器.

3.1 系统状态可测(System states available)

因为 $f(\mathbf{x})$ 和 $g(\mathbf{x})$ 为未知光滑函数, 因此首先要对不确定非线性系统进行模糊建模, 即用模糊逻辑系统 $\hat{f}(\mathbf{x}|\hat{\theta}_f) = \hat{\theta}_f^T \xi(\mathbf{x})$ 和 $\hat{g}(\mathbf{x}|\hat{\theta}_g) = \hat{\theta}_g^T \xi(\mathbf{x})$ 来分别逼近 $f(\mathbf{x})$ 和 $g(\mathbf{x})$. 其中: $\hat{\theta}_f = [\hat{\theta}_{f1} \ \hat{\theta}_{f2} \ \dots \ \hat{\theta}_{fM}]^T$, $\hat{\theta}_g = [\hat{\theta}_{g1} \ \hat{\theta}_{g2} \ \dots \ \hat{\theta}_{gM}]^T$ 为自适应参数. $\xi(\mathbf{x}) = [\xi_1(\mathbf{x}) \ \xi_2(\mathbf{x}) \ \dots \ \xi_M(\mathbf{x})]^T$ 为模糊系统的基函数^[3]. 式中:

$$\xi_j(\mathbf{x}) \triangleq \frac{\prod_{i=1}^n \mu_{F_i^j}(x_i)}{\sum_{l=1}^M \left(\prod_{i=1}^n \mu_{F_i^l}(x_i) \right)}, \quad j = 1, 2, \dots, M.$$

根据模糊逻辑系统的万能逼近特性, 存在最优逼近参数 θ_f^* 和 θ_g^* 分别满足

$$\theta_f^* = \arg \min_{\theta_f} (\sup_{\mathbf{x} \in \mathbf{X}} |\hat{f}(\mathbf{x}) - f(\mathbf{x})|),$$

$$\theta_g^* = \arg \min_{\theta_g} (\sup_{\mathbf{x} \in \mathbf{X}} |\hat{g}(\mathbf{x}) - g(\mathbf{x})|).$$

估计误差为

$$\tilde{\theta}_f = \hat{\theta}_f - \theta_f^*, \quad \tilde{\theta}_g = \hat{\theta}_g - \theta_g^*.$$

定义最优逼近误差为

$$\omega = f(\mathbf{x}) - \hat{f}(\mathbf{x}|\theta_f^*) + [g(\mathbf{x}) - \hat{g}(\mathbf{x}|\theta_g^*)]u_c = \varepsilon_f + \varepsilon_g u_c.$$

定义跟踪误差 $e = y_d - y$, 则误差向量 $\mathbf{e} = (e, \dot{e}, \dots, e^{(n-1)})^T$.

选择向量 $\mathbf{K} = (k_n, k_{n-1}, \dots, k_1)^T$ 使得 $s^n + k_1 s^{n-1} + k_2 s^{n-2} + \dots + k_n$ 为Hurwitz多项式. 对式(3)取控制律为 $u = u_c + u_r$. 其中:

$$u_c = \frac{\hat{g}(\mathbf{x})}{\hat{g}^2(\mathbf{x}) + \alpha} (-\hat{f}(\mathbf{x}) + y_r^{(n)} + \mathbf{K}^T \mathbf{e} - \hat{\omega}),$$

式中: α 为给定的一个极小的正常数, $\hat{\omega}$ 用来估计模糊逼近误差.

注1 根据反馈线性化方法可以设计控制律为 $u = \frac{1}{\hat{g}(\mathbf{x}|\hat{\theta}_g)}(-\hat{f}(\mathbf{x}|\hat{\theta}_f) + y_r^{(n)} + \mathbf{K}^T \mathbf{e})$, 但本文中控制方向未知, 即很难确定与 $g(\mathbf{x})$ 正负一致的参数初值, 这样在参数更新过程中容易出现控制器奇异问题, 采用上述控制器可以避免这一问题.

对式(3)进行运算可得如下误差方程:

$$\begin{aligned} \dot{\mathbf{e}}^{(n)} = & -\mathbf{K}^T \mathbf{e} + (\hat{f}(\mathbf{x}) - f(\mathbf{x})) + \\ & (\hat{g}(\mathbf{x}) - g(\mathbf{x}))u_c + u_0 - g(\mathbf{x})u_r = \\ & -\mathbf{K}^T \mathbf{e} + \tilde{\theta}_f^T \xi(\mathbf{x}) + \tilde{\theta}_g^T \xi(\mathbf{x})u_c + \\ & \tilde{\omega} + u_0 - g(\mathbf{x})u_r, \end{aligned} \quad (4)$$

式中 $\tilde{\omega} = \hat{\omega} - \omega$ 为模糊逼近误差的估计误差.

式(4)等价于

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{BK}^T)\mathbf{e} + \mathbf{B}[\tilde{\theta}_f^T \xi(\mathbf{x}) + \tilde{\theta}_g^T \xi(\mathbf{x})u_c + \tilde{\omega} + u_0 - g(\mathbf{x})u_r], \quad (5)$$

式中:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}_{n \times n}, \quad \mathbf{B} = [0 \ 0 \ \cdots \ 0 \ 1]^T_{1 \times n},$$

$$u_0 = \frac{\alpha}{\hat{g}^2(\mathbf{x}) + \alpha}(-\hat{f}(\mathbf{x}) + y_r^{(n)} + \mathbf{K}^T \mathbf{e} - \hat{\omega}).$$

由于 $\mathbf{A} - \mathbf{BK}^T$ 为稳定的矩阵, 因此给定一个正定对称矩阵 \mathbf{P} , 存在正定矩阵 \mathbf{Q} 满足 Lyapunov 方程:

$$(\mathbf{A} - \mathbf{BK}^T)^T \mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{BK}^T) = -\mathbf{Q}. \quad (6)$$

定理1 对于系统(1), 采用如下控制律与自适应律, 可以使得系统输出跟踪给定期望信号, 同时保证闭环系统所有信号均有界.

控制律为

$$u_c = \frac{\hat{g}(\mathbf{x})}{\hat{g}^2(\mathbf{x}) + \alpha}(-\hat{f}(\mathbf{x}) + y_r^{(n)} + \mathbf{K}^T \mathbf{e} - \hat{\omega}), \quad (7)$$

$$u_0 = \frac{\alpha}{\hat{g}^2(\mathbf{x}) + \alpha}(-\hat{f}(\mathbf{x}) + y_r^{(n)} + \mathbf{K}^T \mathbf{e} - \hat{\omega}), \quad (8)$$

$$u_r = \frac{-\alpha N(\varsigma)}{\hat{g}^2(\mathbf{x}) + \alpha}(-\hat{f}(\mathbf{x}) + y_r^{(n)} + \mathbf{K}^T \mathbf{e} - \hat{\omega}). \quad (9)$$

自适应律为

$$\dot{\hat{\theta}}_f = -\gamma_f \hat{\theta}_f - \gamma_f e^T \mathbf{P} \mathbf{B} \xi(\mathbf{x}), \quad (10)$$

$$\dot{\hat{\theta}}_g = -\gamma_g \hat{\theta}_g - \gamma_g e^T \mathbf{P} \mathbf{B} \xi(\mathbf{x}) u_c, \quad (11)$$

$$\dot{\hat{\omega}} = -\gamma_\omega \hat{\omega} - \gamma_\omega e^T \mathbf{P} \mathbf{B}, \quad (12)$$

$$\dot{\varsigma} = \frac{\alpha e^T \mathbf{P} \mathbf{B}}{\hat{g}^2(\mathbf{x}) + \alpha}(-\hat{f}(\mathbf{x}) + y_r^{(n)} + \mathbf{K}^T \mathbf{e} - \hat{\omega}), \quad (13)$$

式中: $\gamma_f, \gamma_g, \gamma_\omega$ 均为正的常数, u_r 为鲁棒控制项.

证 取 Lyapunov 候选函数

$$\mathbf{V} = \frac{1}{2} \mathbf{e}^T \mathbf{P} \mathbf{e} + \frac{1}{2\gamma_f} \tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_g} \tilde{\theta}_g^T \tilde{\theta}_g + \frac{1}{2\gamma_\omega} \tilde{\omega}^2. \quad (14)$$

其对时间的导数为

$$\begin{aligned} \dot{\mathbf{V}} = & \frac{1}{2} \mathbf{e}^T ((\mathbf{A} - \mathbf{BK}^T)^T \mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{BK}^T)) \mathbf{e} + \\ & \mathbf{e}^T \mathbf{P} \dot{\mathbf{e}} + \frac{1}{\gamma_f} \tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \frac{1}{\gamma_g} \tilde{\theta}_g^T \dot{\tilde{\theta}}_g + \frac{1}{\gamma_\omega} \tilde{\omega} \dot{\tilde{\omega}}. \end{aligned}$$

将式(5)–(6)代入上式可得

$$\dot{\mathbf{V}} = -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} + \mathbf{V}_1 + \mathbf{V}_2, \quad (15)$$

其中:

$$\begin{aligned} \mathbf{V}_1 = & \frac{1}{\gamma_f} \tilde{\theta}_f^T (\dot{\tilde{\theta}}_f + \gamma_f e^T \mathbf{P} \mathbf{B} \xi(\mathbf{x})) + \frac{1}{\gamma_\omega} \tilde{\omega} (\dot{\tilde{\omega}} + \\ & \gamma_\omega e^T \mathbf{P} \mathbf{B}) + \frac{1}{\gamma_g} \tilde{\theta}_g^T (\dot{\tilde{\theta}}_g + \gamma_g e^T \mathbf{P} \mathbf{B} \xi(\mathbf{x}) u_c), \end{aligned}$$

$$\mathbf{V}_2 = e^T \mathbf{P} \mathbf{B} (u_0 - g(\mathbf{x})u_r).$$

将自适应律(10)–(12)代入 \mathbf{V}_1 有

$$\mathbf{V}_1 \leq -\frac{1}{2} \tilde{\theta}_f^T \tilde{\theta}_f - \frac{1}{2} \tilde{\theta}_g^T \tilde{\theta}_g - \frac{1}{2} \tilde{\omega}^2 + \beta, \quad (16)$$

式中: $\beta = \frac{1}{2} \|\tilde{\theta}_f^*\|^2 + \frac{1}{2} \|\tilde{\theta}_g^*\|^2 + \frac{1}{2} \omega^2$, 化简过程中用到 $-ab \leq \frac{1}{2} a^2 + \frac{1}{2} b^2$ (此处 a, b 为实数) 性质.

将式(8)–(9)(13)代入 \mathbf{V}_2 有

$$\mathbf{V}_2 = (g(\mathbf{x})N(\varsigma) + 1)\varsigma. \quad (17)$$

将式(16)–(17)代入式(15)有

$$\begin{aligned} \dot{\mathbf{V}} \leq & -\frac{1}{2} \mathbf{e}^T \mathbf{Q} \mathbf{e} - \frac{1}{2} \tilde{\theta}_f^T \tilde{\theta}_f - \frac{1}{2} \tilde{\theta}_g^T \tilde{\theta}_g - \frac{1}{2} \tilde{\omega}^2 + \\ & \beta + (g(\mathbf{x})N(\varsigma) + 1)\varsigma \leq \\ & -\rho \mathbf{V} + \beta + (g(\mathbf{x})N(\varsigma) + 1)\varsigma, \end{aligned} \quad (18)$$

其中 $\rho = \min(\lambda_{\min}(\mathbf{Q}\mathbf{P}^{-1}), \gamma_f, \gamma_g, \gamma_\omega)$.

在不等式(18)左右两端同时乘以 $e^{\rho t}$ (此处 e 为自然对数的底). 然后经过积分运算化简可得

$$\mathbf{V}(t) \leq \delta + e^{-\rho t} \int_0^t (g(\mathbf{x})N(\varsigma) + 1)\varsigma e^{\rho \tau} d\tau,$$

式中 $\delta = \frac{\beta}{\rho} + \mathbf{V}(0)$.

根据引理1有如下结论:

$\mathbf{V}(t), \int_0^t g(\mathbf{x}(\tau))N(\varsigma)\varsigma d\tau, \varsigma(t)$ 均有界. 再由式(7)–(9)(14)可知控制输入有界, 同时跟踪误差 e 有界即系统输出能跟踪给定的期望轨迹 y_r . 证毕.

采用定理1中的控制律以及自适应律可以使得系统输出跟踪给定期望信号, 达到输出跟踪控制目的.

3.2 系统状态不可测(System states unmeasured)

第3.1节分析了在系统状态可测情况下控制器的

设计问题. 但实际工程中很多系统状态是不可测的(系统输出 y 可测). 对于系统(1)设计控制器使得系统输出跟踪给定的期望轨迹 y_r . 此时, 无法得到跟踪误差 e , 可构造如下观测器用 \hat{e} 来估计 e :

$$\begin{cases} \dot{\hat{e}} = \mathbf{A}\hat{e} - \mathbf{BK}_0^T\hat{e} + \mathbf{K}_c(e - \hat{e}), \\ \hat{e} = \mathbf{C}^T\hat{e}, \end{cases} \quad (19)$$

式中: $\mathbf{K}_0 = [k_{0,n} \ k_{0,n-1} \ \dots \ k_{0,1}]^T$, 选取 \mathbf{K}_0 使得 $s^n + k_{0,1}s^{n-1} + k_{0,2}s^{n-2} + \dots + k_{0,n}$ 为Hurwitz多项式. $\mathbf{K}_c^T = [k_{c,1} \ k_{c,2} \ \dots \ k_{c,n}]$ 是使得 $\mathbf{A} - \mathbf{K}_c\mathbf{C}^T$ 为稳定矩阵的观测器增益矩阵. $\hat{e} = [y_d - \hat{x}_1 \ \dot{y}_d - \hat{x}_2 \ \dots \ y_d^{(n-1)} - \hat{x}_n] \in \mathbb{R}^n$, \hat{x}_i 为状态 x_i 的估值($i = 1, 2, \dots, n$), $\mathbf{C} = [1 \ 0 \ \dots \ 0 \ 0]^T_{n \times n}$.

由于系统状态不可测, 则可以用 $\hat{f}(\hat{\mathbf{x}} | \hat{\boldsymbol{\theta}}_f) = \hat{\boldsymbol{\theta}}_f^T \boldsymbol{\xi}(\hat{\mathbf{x}})$ 和 $\hat{g}(\hat{\mathbf{x}} | \hat{\boldsymbol{\theta}}_g) = \hat{\boldsymbol{\theta}}_g^T \boldsymbol{\xi}(\hat{\mathbf{x}})$ 来分别逼近未知函数 $f(\mathbf{x})$ 和 $g(\mathbf{x})$.

同理, 存在最优逼近满足

$$\boldsymbol{\theta}_f^* = \arg \min_{\boldsymbol{\theta}_f} (\sup_{\mathbf{x} \in \mathcal{X}} |\hat{f}(\hat{\mathbf{x}}) - f(\mathbf{x})|),$$

$$\boldsymbol{\theta}_g^* = \arg \min_{\boldsymbol{\theta}_g} (\sup_{\mathbf{x} \in \mathcal{X}} |\hat{g}(\hat{\mathbf{x}}) - g(\mathbf{x})|).$$

参数误差为

$$\tilde{\boldsymbol{\theta}}_f = \hat{\boldsymbol{\theta}}_f - \boldsymbol{\theta}_f^*, \quad \tilde{\boldsymbol{\theta}}_g = \hat{\boldsymbol{\theta}}_g - \boldsymbol{\theta}_g^*.$$

定义 $f(\mathbf{x})$ 和 $g(\mathbf{x})$ 的逼近误差为

$$\begin{aligned} \sigma = & [f(\mathbf{x}) - \hat{f}(\hat{\mathbf{x}} | \boldsymbol{\theta}_f^*)] + [g(\mathbf{x}) - \hat{g}(\hat{\mathbf{x}} | \boldsymbol{\theta}_g^*)] u_c = \\ & \{ [f(\mathbf{x}) - f(\mathbf{x} | \boldsymbol{\theta}_f^*)] + [f(\mathbf{x} | \boldsymbol{\theta}_f^*) - \hat{f}(\hat{\mathbf{x}} | \boldsymbol{\theta}_f^*)] \} + \\ & \{ [g(\mathbf{x}) - g(\mathbf{x} | \boldsymbol{\theta}_g^*)] + [g(\mathbf{x} | \boldsymbol{\theta}_g^*) - \hat{g}(\hat{\mathbf{x}} | \boldsymbol{\theta}_g^*)] \} u_c. \end{aligned}$$

对式(3)取控制律为 $u_1 = u_{c1} + u_{r1}$, 其中

$$u_{c1} = \frac{\hat{g}(\hat{\mathbf{x}})}{\hat{g}^2(\hat{\mathbf{x}}) + \alpha} (-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{K}_0^T \hat{\mathbf{x}} - \hat{\sigma}).$$

将 u_{c1} 代入式(3)并化简可得误差方程

$$\dot{e} = \mathbf{A}e - \mathbf{BK}_0^T\hat{e} + \mathbf{B}[\tilde{\boldsymbol{\theta}}_f^T \boldsymbol{\xi}(\hat{\mathbf{x}}) + \tilde{\boldsymbol{\theta}}_g^T \boldsymbol{\xi}(\hat{\mathbf{x}}) u_{c1} + \tilde{\sigma} + u_{01} - g(\hat{\mathbf{x}}) u_{r1}], \quad (20)$$

式中: $\tilde{\sigma} = \hat{\sigma} - \sigma$ 为逼近误差的估计误差,

$$u_{01} = \frac{\alpha}{\hat{g}^2(\hat{\mathbf{x}}) + \alpha} (-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{K}_0^T \hat{e} - \hat{\sigma}).$$

定义观测误差为 $\tilde{e} = e - \hat{e} = \hat{x}_1 - x_1$, 由式(20)与式(19)可得

$$\begin{cases} \dot{\tilde{e}} = (\mathbf{A} - \mathbf{K}_c\mathbf{C}^T)\tilde{e} + \mathbf{B}[\tilde{\boldsymbol{\theta}}_f^T \boldsymbol{\xi}(\hat{\mathbf{x}}) + \tilde{\boldsymbol{\theta}}_g^T \boldsymbol{\xi}(\hat{\mathbf{x}}) u_{c1} + \tilde{\sigma} + u_{01} - g(\hat{\mathbf{x}}) u_{r1}], \\ \tilde{e} = \mathbf{C}^T \tilde{e}. \end{cases} \quad (21)$$

由于 $\mathbf{A} - \mathbf{BK}_0^T$ 和 $\mathbf{A} - \mathbf{K}_c\mathbf{C}^T$ 均为稳定矩阵, 则给定正定对称矩阵 \mathbf{P}_1 和 \mathbf{P}_2 , 一定存在正定矩阵 \mathbf{Q}_1 和 \mathbf{Q}_2 分别满足

$$(\mathbf{A} - \mathbf{BK}_0^T)^T \mathbf{P}_1 + \mathbf{P}_1 (\mathbf{A} - \mathbf{BK}_0^T) + \mathbf{P}_1 \mathbf{K}_c \mathbf{K}_c^T \mathbf{P}_1 = -\mathbf{Q}_1, \quad (22)$$

$$(\mathbf{A} - \mathbf{K}_c\mathbf{C}^T)^T \mathbf{P}_2 + \mathbf{P}_2 (\mathbf{A} - \mathbf{K}_c\mathbf{C}^T) + \frac{1}{2} \mathbf{C} \mathbf{C}^T = -\mathbf{Q}_2. \quad (23)$$

定理 2 对系统(1), 采用如下控制律与自适应律, 可以使得系统输出跟踪给定期望信号, 同时保证闭环系统所有信号均有界.

控制律为

$$u_{c1} = \frac{\hat{g}(\hat{\mathbf{x}})}{\hat{g}^2(\hat{\mathbf{x}}) + \alpha} (-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{K}_0^T \hat{e} - \hat{\sigma}), \quad (24)$$

$$u_{01} = \frac{\alpha}{\hat{g}^2(\hat{\mathbf{x}}) + \alpha} (-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{K}_0^T \hat{e} - \hat{\sigma}), \quad (25)$$

$$u_{r1} = \frac{-\alpha}{\hat{g}^2(\hat{\mathbf{x}}) + \alpha} (-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{K}_0^T \hat{e} - \hat{\sigma}). \quad (26)$$

自适应律为

$$\dot{\hat{\boldsymbol{\theta}}}_f = -\eta_f \hat{\boldsymbol{\theta}}_f - \eta_f \tilde{e}^T \mathbf{P}_2 \mathbf{B} \boldsymbol{\xi}(\hat{\mathbf{x}}), \quad (27)$$

$$\dot{\hat{\boldsymbol{\theta}}}_g = -\eta_g \hat{\boldsymbol{\theta}}_g - \eta_g \tilde{e}^T \mathbf{P}_2 \mathbf{B} \boldsymbol{\xi}(\hat{\mathbf{x}}) u_{c1}, \quad (28)$$

$$\dot{\hat{\sigma}} = -\eta_\omega \hat{\sigma} - \eta_\omega \tilde{e}^T \mathbf{P}_2 \mathbf{B}, \quad (29)$$

$$\dot{\hat{\sigma}} = \frac{\alpha \tilde{e}^T \mathbf{P}_2 \mathbf{B}}{\hat{g}^2(\hat{\mathbf{x}}) + \alpha} (-\hat{f}(\hat{\mathbf{x}}) + y_r^{(n)} + \mathbf{K}_0^T \hat{e} - \hat{\sigma}), \quad (30)$$

式中: $\eta_f, \eta_g, \eta_\omega$ 均为正的常数, u_{r1} 为鲁棒控制项.

证 选取Lyapunov候选函数

$$\begin{aligned} V = & \frac{1}{2} \tilde{e}^T \mathbf{P}_1 \tilde{e} + \frac{1}{2} \tilde{e}^T \mathbf{P}_2 \tilde{e} + \frac{1}{2\eta_f} \tilde{\boldsymbol{\theta}}_f^T \tilde{\boldsymbol{\theta}}_f + \\ & \frac{1}{2\eta_g} \tilde{\boldsymbol{\theta}}_g^T \tilde{\boldsymbol{\theta}}_g + \frac{1}{2\eta_\sigma} \tilde{\sigma}^2. \end{aligned} \quad (31)$$

其对时间求导并由式(21)-(23)有

$$\dot{V} \leq -\frac{1}{2} \tilde{e}^T \mathbf{Q}_1 \tilde{e} - \frac{1}{2} \tilde{e}^T \mathbf{Q}_2 \tilde{e} + V_3 + V_4, \quad (32)$$

其中:

$$\begin{aligned} V_3 = & \frac{1}{\eta_g} \tilde{\boldsymbol{\theta}}_g^T (\dot{\hat{\boldsymbol{\theta}}}_g + \eta_g \tilde{e}^T \mathbf{P}_2 \mathbf{B} \boldsymbol{\xi}(\hat{\mathbf{x}}) u_{c1}) + \\ & \frac{1}{\eta_f} \tilde{\boldsymbol{\theta}}_f^T (\dot{\hat{\boldsymbol{\theta}}}_f + \eta_f \tilde{e}^T \mathbf{P}_2 \mathbf{B} \boldsymbol{\xi}(\hat{\mathbf{x}})) + \\ & \frac{1}{\eta_\sigma} \tilde{\sigma} (\dot{\hat{\sigma}} + \eta_\sigma \tilde{e}^T \mathbf{P}_2 \mathbf{B}), \end{aligned}$$

$$V_4 = \tilde{e}^T \mathbf{P}_2 \mathbf{B} [u_{01} - g(\hat{\mathbf{x}}) u_{r1}].$$

将式(27)-(29)代入 V_3 , 经计算可得

$$V_3 \leq -\frac{1}{2} \tilde{\boldsymbol{\theta}}_f^T \tilde{\boldsymbol{\theta}}_f - \frac{1}{2} \tilde{\boldsymbol{\theta}}_g^T \tilde{\boldsymbol{\theta}}_g - \frac{1}{2} \tilde{\sigma}^2 + \phi, \quad (33)$$

式中 $\phi = \frac{1}{2} \|\tilde{\boldsymbol{\theta}}_f^*\|^2 + \frac{1}{2} \|\tilde{\boldsymbol{\theta}}_g^*\|^2 + \frac{1}{2} \sigma^2$.

将式(25)-(26)(30)代入 V_4 , 可得

$$V_4 = [g(\mathbf{x}) N(\varsigma) + 1] \dot{\hat{\sigma}}. \quad (34)$$

将式(33)-(34)代入式(32)中有

$$\begin{aligned} \dot{V} \leq & -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2}\tilde{\theta}_f^T \tilde{\theta}_f - \frac{1}{2}\tilde{\theta}_g^T \tilde{\theta}_g - \\ & \frac{1}{2}\tilde{\sigma}^2 + [g(\mathbf{x})N(\varsigma) + 1]\dot{\varsigma} + \phi \leq \\ & -\rho_1 V + [g(\mathbf{x})N(\varsigma) + 1]\dot{\varsigma} + \phi, \end{aligned}$$

其中

$$\rho_1 = \min(\lambda_{\min}(Q_1 P_1^{-1}), \lambda_{\min}(Q_2 P_2^{-1}), \eta_f, \eta_g, \eta_\omega).$$

经过计算有

$$V(t) \leq \delta_1 + e^{-\rho_1 t} \int_0^t (g(\mathbf{x})N(\varsigma) + 1)\dot{\varsigma} e^{\rho_1 \tau} d\tau,$$

式中 $\delta_1 = \frac{\phi}{\rho_1} + V(0)$.

根据引理1有如下结论: $V(t), \varsigma(t), \int_0^t g(\mathbf{x}(\tau)) \cdot N(\varsigma)\dot{\varsigma}d\tau$ 均有界. 再由式(24)(26)(29)-(31)可得控制输入有界, 同时跟踪误差 e 有界, 即系统输出能跟踪给定的期望轨迹 y_r . 证毕.

在系统状态不可测的情况下, 采用定理2中的控制律以及自适应律, 系统输出能跟踪给定期望信号.

4 仿真结果(Simulation results)

本仿真实验分为系统状态可测与系统状态不可测两种情况验证本文所设计控制器的可行性以及有效性. 考虑下面的非线性系统^[21]:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -x_1 + 2x_2 + \frac{u}{\sqrt{|u| + 0.1}} - 2x_1^2 x_2, \\ y = x_1. \end{cases}$$

控制目的是使系统的输出 y 跟踪期望轨迹 $y_r = \sin t$.

当系统状态可测时, 取如下仿真参数:

$$\hat{\theta}_f(0) = [0.1 \ \cdots \ 0.1]_{1 \times 49}^T, \hat{\omega}(0) = 0,$$

$$\varsigma(0) = 0, x_1(0) = 1, x_2(0) = 0,$$

$$P = \begin{bmatrix} 200 & 0.1 \\ 0.1 & 2 \end{bmatrix}, K = \begin{bmatrix} 100 \\ 20 \end{bmatrix}, \gamma_f = 10,$$

$$\gamma_g = 0.001, \gamma_\omega = 0.1, \alpha = 0.001.$$

仿真时间为10s. 仿真结果见图1-4.

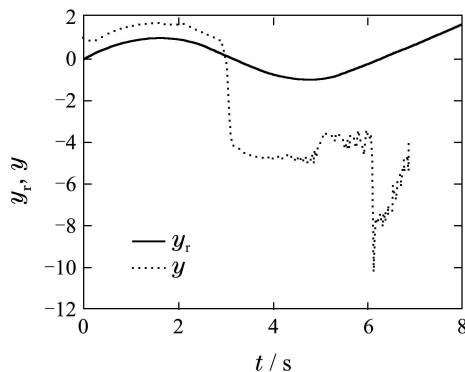


图1 期望输出与实际跟踪曲线

Fig. 1 The reference trajectory and the system output

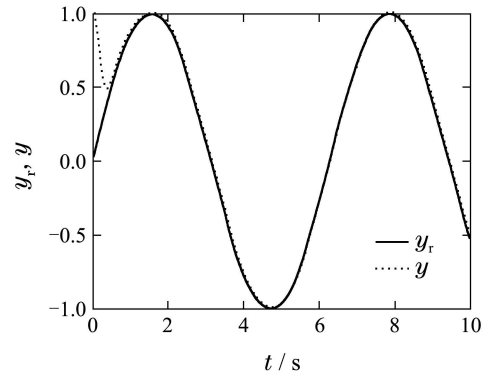


图2 期望输出与实际跟踪曲线

Fig. 2 The reference trajectory and the system output

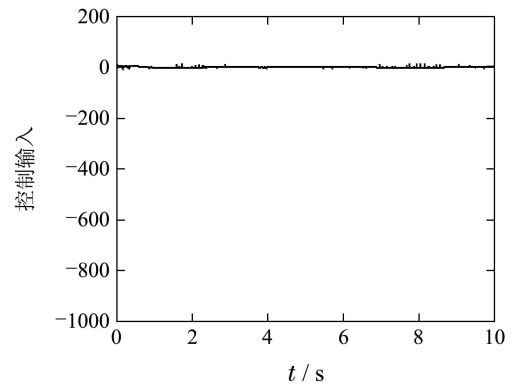


图3 控制输入

Fig. 3 The control input

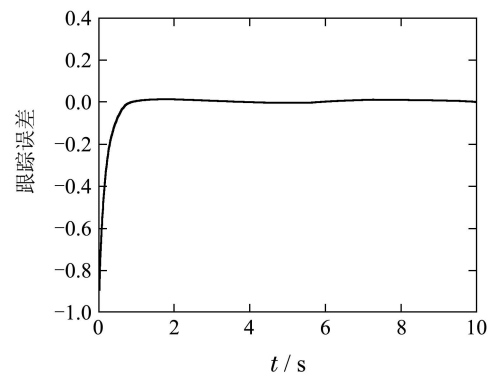


图4 跟踪误差曲线

Fig. 4 The tracking error

当系统状态不可测时, 仿真参数选取如下:

$$\hat{\theta}_g(0) = [0.1 \ \cdots \ 0.1]_{1 \times 49}^T,$$

$$\hat{\sigma}(0) = 0, \varsigma(0) = 0, x_1(0) = 1,$$

$$x_2(0) = 0, \hat{x}_1(0) = 0, \hat{x}_2(0) = 1,$$

$$K_0 = \begin{bmatrix} 400 \\ 40 \end{bmatrix}, K_c = \begin{bmatrix} 40 \\ 400 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 50.1 & -4 \\ -4 & 0.62 \end{bmatrix}, \eta_f = 10,$$

$$\eta_g = 0.001, \eta_\sigma = 0.1, \alpha = 0.001.$$

仿真时间为10s. 仿真结果见图5-9.

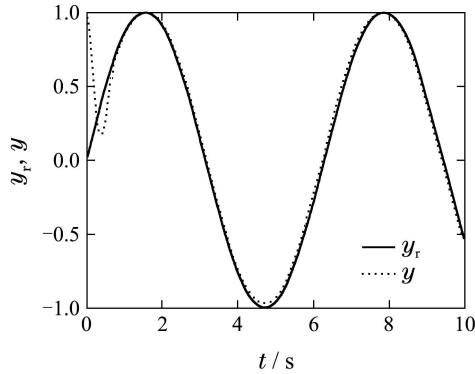


图5 期望输出与实际跟踪曲线

Fig. 5 The reference trajectory and the system output

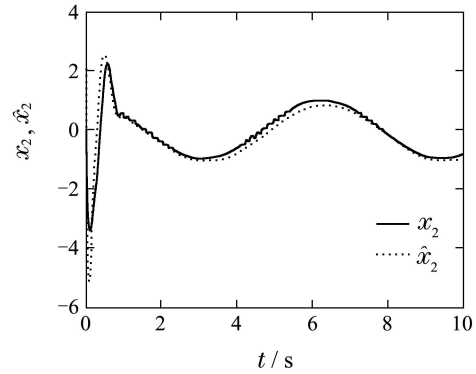


图9 系统状态 x_2 及其估计 \hat{x}_2

Fig. 9 The system state x_2 and its estimate \hat{x}_2

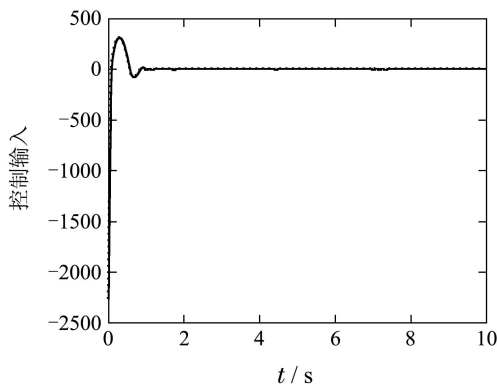


图6 控制输入

Fig. 6 The control input

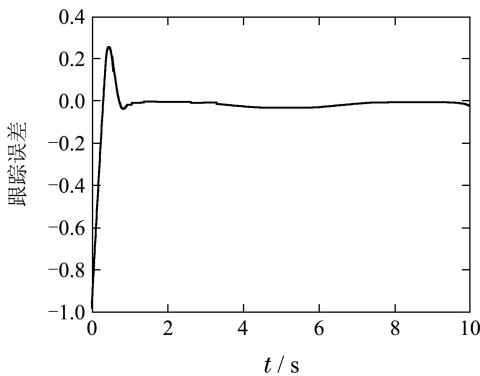


图7 跟踪误差

Fig. 7 The tracking error

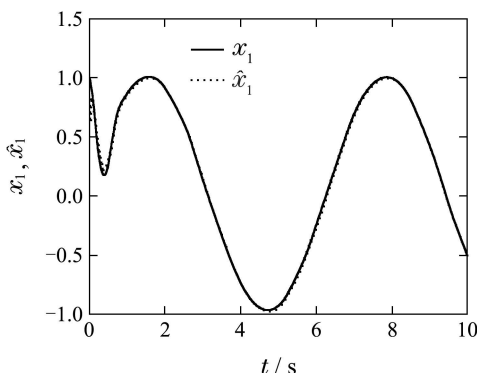


图8 系统状态 x_1 及其估计 \hat{x}_1

Fig. 8 The system state x_1 and its estimate \hat{x}_1

图1为文献[18]中算法仿真结果,在设计控制器时为了克服控制器奇异问题,引入了阶跃项,由此产生控制器输入颤振现象,最终使得系统输出不能跟踪给定的期望输出,跟踪误差发散.

图2与图5表明本文所提出的控制方案可以使系统完成跟踪目标轨迹任务.同时根据图4与图7结果可知,系统跟踪误差收敛于零的某一邻域内.图3和图6表明,两种情况下系统的控制输入均有界.根据图8和图9可知在系统状态不可测时文中所设计的观测器可以有效的估计系统状态.经上述仿真分析可知,本文所设计的控制器能够使得系统输出跟踪给定轨迹,并同时保证了跟踪误差收敛以及控制输入有界.

5 结论(Conclusion)

本文针对一类控制方向未知的SISO非仿射非线性系统,讨论了在状态可测与状态不可测的两种情况下系统控制器的设计问题,所提出的控制方案克服了控制器奇异的问题以及解决了在状态不可测的情况下控制器的设计问题.仿真实验验证了本文控制方案能跟踪给定的期望信号,同时保证闭环系统所有信号均有界.

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