第 31 卷第 4 期
2014 年 4 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 31 No. 4
Apr. 2014

DOI: 10.7641/CTA.2014.21207

随机Markov跳跃时滞系统的鲁棒H_∞指数滤波器设计

毛卫华[1,2], 邓飞其[2], 万安华[3†]

(1. 华南农业大学 数学系, 广东 广州 510640; 2. 华南理工大学 自动化科学与工程学院, 广东 广州 510640;

3. 中山大学 数学与计算科学学院, 广东 广州 510275)

摘要: 研究了一类不确定随机Markov跳跃时滞系统的鲁棒H_∞指数滤波问题. 应用Lyapunov-Krasovskii稳定性理论和广义Finsler引理, 建立了滤波器存在的时滞相关条件, 避免了使用模型变换方法和自由权矩阵方法所带来的保守性和繁冗的计算, 鲁棒H_∞指数滤波器可通过求解联立线性矩阵不等式设计. 数值例子说明了所采用方法的有效性.

关键词: 随机系统; 时滞; 不确定系统; Markov跳跃; 鲁棒H_∞滤波; 均方指数稳定性; 广义Finsler引理; 线性矩阵不等式

中图分类号: TP273; TP13; TP202　　　**文献标识码**: A

Robust H_∞ exponential filtering for stochastic Markovian jump time-varying delay systems

MAO Wei-hua[1,2], DENG Fei-qi[2], WAN An-hua[3†]

(1. Department of Mathematics, South China Agricultural University, Guangzhou Guangdong 510640, China;
2. College of Automation Science and Technology, South China University of Technology, Guangzhou Guangdong 510640, China;
3. School of Mathematics and Computational Science, Sun Yat-sen University, Guangzhou Guangdong 510275, China)

Abstract: The robust H_∞ exponential filtering problem is investigated for a class of uncertain stochastic systems with Markovian jump parameters and interval mode-dependent time-varying delays. By Lyapunov-Krasovskii theory and generalized Finsler lemma, novel delay-dependent sufficient conditions are obtained to guarantee the existence of desired exponential H_∞ filter, which can be constructed by solving simultaneous linear matrix inequalities. Neither model transformations nor free-weighting matrices are involved; therefore, conservatism and computation burden resulting from them can be avoided. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed method.

Key words: stochastic systems; time delay; uncertain systems; Markovian jumps; robust H_∞ filtering; mean-square exponential stability; generalized Finsler lemma; linear matrix inequalities

1 引言(Introduction)

在控制系统的设计及其信号处理中, H_∞滤波问题具有重要的理论和实际意义, 受到了广泛的关注[1–15]. H_∞滤波通过噪声测量来估计无法获得的状态变量, 使其对应的滤波误差关于参数、外部噪声等不确定性具有一定的鲁棒抗干扰性能. 与Kalman滤波相比, H_∞滤波的优势在于并不需要知道噪声信号的确切统计特性[16].

时滞经常出现在诸如工程系统, 种群生态系统, 经济金融系统等实际系统中, 往往会导致系统性能降低甚至不稳定[17], 根据所获得的充分条件是否依赖时滞信息, 关于H_∞控制与滤波问题的现有成果分为两类: 时滞无关条件[1–2]和时滞相关条件[3–14, 18–24]. 时滞系统通常还会受到随机因素的影响, 使得随机建模在发挥重要作用, 因此, 随机时滞系统的H_∞滤波问题引起广泛关注[6, 8, 11–14, 20, 24]. 由于建模和测量误差, 近似误差等因素影响, 动态系统经常会产生系数不确定性扰动, 同样会导致系统性能恶化, 所以鲁棒H_∞滤波问题的研究也很重要[2–4, 9–14].

指数稳定性可以确保达到可接受的精度(合理的估计误差方差)和实现快速收敛[5–6, 13–14, 25–28]. 模型变换方法[29–30]、松弛变量方法[10, 30]和自由权矩阵方法[31–33]广泛应用于处理时滞系统. 然而, 这些方法可能对降低保守性效果不佳, 所引入的自由变量确实增加了计算复杂程度[34–35]. 为了同时降低保守性和计算成本, 基于Finsler引理, 文献 [36–38] 得到了确定性

收稿日期: 2012−11−30; 录用日期: 2013−12−31.
†通信作者. E-mail: wananhua@mail.sysu.edu.cn.
基金项目: 国家自然科学基金资助项目(61273126, 11201495); 中央高校基本科研业务费专项资金资助项目(13lgpy31); 教育部高校博士点专项资助项目(20130172110027).

时滞系统稳定性的时滞相关条件. 陈云等[26]提出随机时滞系统的广义Finsler引理(generalized Finsler lemma, GFL), 研究了随机常时滞系统的均方指数稳定性问题[26–28].

Markov跳跃系统(Markovian jump systems, MJSs)的状态变量包含两部分: 一个是用于描述模态的连续时间有限状态的马尔科夫过程; 另一个是微分(差分)方程的状态变量. 包括MJSs在内的切换时滞系统适合于结构和系统参数发生突变的动态系统的建模. 因此, 有关切换时滞系统的控制与滤波的研究引起广泛关注[5–6, 9–10, 13–14, 18–23, 38]. 文献[21–22]利用平均驻留时间方法研究了切换区间时变时滞系统的H_∞滤波问题, 时滞类型更为一般, 时滞可以是可导且导数有界或者连续不可导的. 近年来, 非线性随机系统H_∞滤波与控制引起很多学者的兴趣[5, 13–15, 23–24], 文献[23–24]通过求解哈密尔顿–雅克比不等式, 分别研究了一般随机MJSs的状态反馈及输出反馈控制问题, 和一般随机时滞系统的H_∞滤波问题.

目前, 关于具有模态依赖的区间时变时滞的不确定Itô随机Markov 跳跃系统的鲁棒H_∞指数滤波问题的研究成果还很少, 特别是基于GFL对这一问题的研究, 目前尚未见到相关文献讨论. 本文将对这类系统的鲁棒H_∞指数滤波问题开展研究, 应用Lyapunov-Krasovskii泛函方法、GFL、线性矩阵不等式(linear matrices inequalities, LMIs)方法等, 建立这类系统的随机有界实引理, 给出滤波器存在的时滞相关充分条件, 基于有界实引理设计滤波器, 在没有外部干扰情况下, 对给定的H_∞干扰衰减度$\gamma > 0$和所有容许不确定性, 使得滤波误差系统均方指数稳定. 由于采用了引理1的前两个等价条件, 避免了使用模型变换方法, 松弛变量方法和自由权矩阵方法, 有效地降低了保守性和计算复杂程度. 去掉了时滞导数上界小于"1"的保守性条件, 比如$\dot{\tau}(t) \leqslant \mu < 1$[11–12, 36]或者$\dot{\tau}(t) \leqslant \mu_i < 1$[13–14, 20]. 最后, 通过数值例子说明了方法的有效性.

说明: $A > 0$表示对称正定矩阵. "$*$"表示对称矩阵的对称元素. sym(A)表示$A + A^{\mathrm{T}}$. $\lambda_m\{\cdot\}$表示矩阵的最小特征值. $L_2[0,\infty)$是平方可积向量空间.

$$C([-\tau,0];\mathbb{R}^n) = \{\varphi|\varphi:[-\tau,0]\to\mathbb{R}^n\},$$

具有范数$\|\varphi\| = \sup\limits_{-\tau\leqslant s\leqslant 0}|\varphi(s)|$, $|\cdot|$表示欧几里德向量范数, $\|\cdot\|_2$表示$L_2[0,\infty)$范数. 设$(\omega(t), F, \{F\}_{t\geqslant 0}, P)$是具有$\sigma$–代数流$\{F_t\}_{t\geqslant 0}$的完备概率空间, 满足通常条件: 它是右连续的, 并且F_{t_0}包含了所有概率测度为零的集合. $L^p_{F_0}([-\tau,0];\mathbb{R}^n)$表示$F_0$–可测且满足$\mathrm{E}|\xi|^p < \infty$的$C([-\tau,0];\mathbb{R}^n)$–值随机变量$\xi$所构成的空间, $\mathrm{E}\{\cdot\}$是定义于概率空间上的数学期望算子. 除非声明, 矩阵都是适维的.

2 问题描述和预备知识(Problem formulation and preliminaries)

考虑以下具有模态依赖的区间时变时滞不确定Itô随机Markov跳跃时滞系统:

$$\begin{aligned}\mathrm{d}x(t) = {} & [A_0(t,r_t)x(t) + A_1(t,r_t)x(t-\tau_{r_t}(t)) + \\ & A_2(t,r_t)\omega(t)]\mathrm{d}t + \\ & [B_0(t)x(t,r_t) + B_1(t,r_t)x(t-\tau_{r_t}(t)) + \\ & B_2(t,r_t)\omega(t)]\mathrm{d}B(t), \end{aligned} \tag{1}$$

$$\begin{aligned} y(t) = {} & C_0(t,r_t)x(t) + C_1(t,r_t)x(t-\tau_{r_t}(t)) + \\ & C_2(t,r_t)\omega(t), \end{aligned} \tag{2}$$

$$z(t) = L_0(r_t)x(t) + L_1(r_t)x(t-\tau_{r_t}(t)), \tag{3}$$

$$x(t) = \phi(t), r(t) = r_0, \forall t\in[-\tau_2, 0], \tag{4}$$

其中: $x(t)\in\mathbb{R}^n$是系统状态; $y(t)\in\mathbb{R}^l$是测量输出; $z(t)\in\mathbb{R}^q$是待估信号; $\omega(t)\in\mathbb{R}^p$是外部干扰输入, 属于$L_2[0,\infty)$; $B(t)$是定义在完备概率空间$(\omega(t), F, \{F\}_{t\geqslant 0}, P)$上的一维标准维纳过程; 初值$\phi(t)\in L^2_{F_0}([-\tau,0];\mathbb{R}^n)$. $\{r_t, t\geqslant 0\}$表示系统模态, 具有右连续轨线, 是取值于有限集$S=\{1,2,\cdots,N\}$齐次Markov过程, 转移概率为

$$\mathrm{P}\{r_{t+h}=j|r_t=i\} = \begin{cases}\pi_{ij}h+\mathrm{o}(h), & i\neq j,\\ 1+\pi_{ii}h+\mathrm{o}(h), & i=j,\end{cases} \tag{5}$$

其中: $h>0, \lim\limits_{h\to 0}(\mathrm{o}(h)/h)=0, \pi_{ij}\geqslant 0,\ j\neq i$, 表示由$t$时刻模态$i$转移到$t+h$时刻模态$j$的概率, 且

$$\pi_{ii} = -\sum_{j=1,\ j\neq i}^{N}\pi_{ij}. \tag{6}$$

$\forall r_t = i,\ i\in S, \tau_i(t)$是模态依赖的, 满足

$$0\leqslant\tau_{1i}\leqslant\tau_i(t)\leqslant\tau_{2i}<\infty,\ \dot{\tau}_i(t)\leqslant\mu_i<+\infty. \tag{7}$$

$\tau_{1i}, \tau_{2i}, \mu_i$已知, 记$t^i_{\tau_r}=t-\tau_{r_t}(t), r_t=i, i\in S, t_{\tau_k}=t-\tau_k, \tau_k=\min\{\tau_{ki}, i\in S\}, k=1,2, \delta=\tau_2-\tau_1$. $\forall r_t\in S, A_j(t,r_t), B_j(t,r_t), C_j(t,r_t)$为Markov跳跃系数. $A_j(t,r_t)=A_j(r_t)+\Delta A_j(t,r_t),\ B_j(t,r_t)=B_j(r_t)+\Delta B_j(t,r_t), C_j(t,r_t)=C_j(r_t)+\Delta C_j(t,r_t)$, 其中: $j=0,1,2$, $A_j(r_t), B_j(r_t), C_j(r_t)$表示标称系统的已知系数矩阵, $\Delta A_j(t,r_t), \Delta B_j(t,r_t), \Delta C_j(t,r_t)$表示时变系数不确定性未知矩阵, 满足

$$\begin{bmatrix}\Delta A_0(t,r_t) & \Delta A_1(t,r_t) & \Delta A_2(t,r_t)\\ \Delta B_0(t,r_t) & \Delta B_1(t,r_t) & \Delta B_2(t,r_t)\\ \Delta C_0(t,r_t) & \Delta C_1(t,r_t) & \Delta C_2(t,r_t)\end{bmatrix} = \begin{bmatrix}U_1(r_t)\\ U_2(r_t)\\ U_3(r_t)\end{bmatrix} F(t,r_t)[S_0(r_t)\ \ S_1(r_t)\ \ S_2(r_t)], \tag{8}$$

$\forall r_t\in S, U_m(r_t)(m=1,2,3), S_n(r_t)(n=0,1,2)$是已知矩阵, $F(t,r_t)$是不确定性时变矩阵, 并且满足:

$F^{\mathrm T}(t,r_t)F(t,r_t)\leqslant I$.

$\forall r_t=i,\ i\in S$, 矩阵$A_j(t,r_t),B_j(t,r_t),C_j(t,r_t)$等分别以矩阵$A_{ji}(t),B_{ji}(t),C_{ji}(t)$等來简记.

为了方便, 引入下列状态变量和扰动向量:

$$\begin{aligned}&\alpha_{r_t}(t)=A_{0i}(t)x(t)+A_{1i}(t)x(t^i_{\tau_r}),\\&\beta_{r_t}(t)=B_{0i}(t)x(t)+B_{1i}(t)x(t^i_{\tau_r}),\\&\bar\alpha_{r_t}(t)=\alpha_{r_t}(t)+A_{2i}(t)\omega(t),\\&\bar\beta_{r_t}(t)=\beta_{r_t}(t)+B_{2i}(t)\omega(t).\end{aligned}$$

当$\omega(t)=0$时, 记随机Markov跳跃时滞系统(1)和(4)的标称系统为

$$\begin{cases}\mathrm dx(t)=\alpha_{r_t}(t)\mathrm dt+\beta_{r_t}(t)\mathrm dB(t),\\x(t)=\phi(t),\ r(t)=r_0,\ \forall t\in[-\tau_2,0].\end{cases}\tag{9}$$

当$\omega(t)\neq0$时, 随机Markov跳跃时滞系统(1)(3)–(4)重新写为

$$\begin{cases}\mathrm dx(t)=\bar\alpha_{r_t}(t)\mathrm dt+\bar\beta_{r_t}(t)\mathrm dB(t),\\z(t)=L_0(r_t)x(t)+L_1(r_t)x(t^i_{\tau_r}),\\x(t)=\phi(t),\ r(t)=r_0,\ t\in[-\tau_2,0].\end{cases}\tag{10}$$

在本文中, 笔者的目的是通过设计Markov跳跃滤波器, 获得$z(t)$的估计$\tilde z(t)$, 使得对所有的非零$\omega(t)\in L_2[0,\infty)$和容许不确定性, 并且具有较小的估计误差$z(t)-\hat z(t)$. $\forall r_t=i,\ i\in S$, 考虑Markov跳跃滤波器$(\Sigma_{\mathrm f})$如下:

$$\begin{cases}\mathrm d\hat x(t)=A_{\mathrm f}(r_t)\hat x(t)\mathrm dt+B_{\mathrm f}(r_t)y(t)\mathrm dt,\\\hat z(t)=C_{\mathrm f}(r_t)\hat x(t),\end{cases}\tag{11}$$

其中: $\hat x(t)\in\mathbb R^n$是滤波器状态变量, $A_{\mathrm f}(r_t),B_{\mathrm f}(r_t),C_{\mathrm f}(r_t)$是待设计的滤波器增益. $\forall r_t=i,i\in S$, 应用滤波器$(\Sigma_{\mathrm f})$到(1), 得到滤波误差动态系统$(\bar\Sigma_{\mathrm f})$:

$$\begin{cases}\mathrm d\eta(t)=[A_{\mathrm f0i}(t)\eta(t)+A_{\mathrm f1i}(t)E\eta(t^i_{\tau_r})+\\\qquad A_{\mathrm f2i}(t)\omega(t)]\mathrm dt+[B_{\mathrm f0i}\eta(t)+\\\qquad B_{\mathrm f1i}(t)E\eta(t^i_{\tau_r})+B_{\mathrm f2i}(t)\omega(t)]\mathrm dB(t),\\\tilde z(t)=L_{\mathrm f0i}\eta(t)+L_{\mathrm f1i}E\eta(t^i_{\tau_r}),\end{cases}\tag{12}$$

其中$\eta^{\mathrm T}(t)=[x^{\mathrm T}(t)\ \ \hat x^{\mathrm T}(t)]$, 并且

$$A_{\mathrm f0i}=\begin{bmatrix}A_{0i}(t)&0\\B_{fi}C_{0i}(t)&A_{fi}\end{bmatrix},\ A_{\mathrm f1i}=\begin{bmatrix}A_{1i}(t)\\B_{fi}C_{1i}(t)\end{bmatrix},$$

$$A_{\mathrm f2i}=\begin{bmatrix}A_{2i}(t)\\B_{fi}C_{2i}(t)\end{bmatrix},\ B_{\mathrm f0i}=\begin{bmatrix}B_{0i}(t)&0\\0&0\end{bmatrix},$$

$$B_{\mathrm f1i}=\begin{bmatrix}B_{1i}(t)\\0\end{bmatrix},\ B_{\mathrm f2i}=\begin{bmatrix}B_{2i}(t)\\0\end{bmatrix},$$

$$L_{\mathrm f0i}=[L_{0i}\ \ -C_{fi}],\ L_{\mathrm f1i}=L_{1i},\ E=[I\ \ 0].$$

以下给出指数稳定性和鲁棒$\mathrm H_\infty$指数滤波的定义, 以及需要用到的引理.

定义 1[5]　称标称随机Markov跳跃时滞系统(9)均方指数稳定, 如果对所有初值$\phi(\theta)\in L^2_{F_0}([-\tau_2,0];\mathbb R^n)$和初始模态$r_0\in S$, 存在常数$\alpha>0,\beta>0$, 使得

$$\mathrm E\{|x(t,\phi,r_0)|^2\}\leqslant\alpha\mathrm e^{-\beta t}\sup_{-\tau_2\leqslant\theta\leqslant0}\mathrm E\{|\phi(\theta)|^2\},$$

其中$x(t,\phi,r_0)$表示系统(9)在t时刻的解.

定义 2[11]　(鲁棒$\mathrm H_\infty$滤波.) 定义$\mathrm H_\infty$性能指标

$$J_\infty=\mathrm E\{\int_0^\infty[z^{\mathrm T}(t)z(t)-\gamma^2\omega^{\mathrm T}(t)\omega(t)]\mathrm dt\},\tag{13}$$

随机鲁棒$\mathrm H_\infty$滤波描述为: 对不确定随机Markov跳跃时滞系统(1)–(4), 给定干扰衰减度$\gamma>0$, 设计均方指数滤波器(11), 使得对所有的容许不确定性, 满足以下两条: 1) $\omega(t)=0$时, 滤波误差系统(12)均方指数稳定; 2) 零初值条件下, 对任意的非零$\omega(t)\in L_2[0,\infty)$, $J_\infty<0$.

引理 1[26]　(广义Finsler引理.) 对于向量$\theta\in\mathbb R^n$, 对称矩阵$\Theta\in\mathbb R^{n\times n}$, 秩为$r$的矩阵$\mathcal B\in\mathbb R^{m\times n}$. 设$\mathcal B^\perp$为$\mathcal B$右正交补, 即$\mathcal B\mathcal B^\perp=0$, 则以下关系等价:

T_1) $\mathrm E\{\theta^{\mathrm T}\Theta\theta\}<0,\forall\theta\neq0,t>t_0,\int_{t_0}^t\mathrm E\{\mathcal B\theta\}\mathrm ds=0$;

T_2) $\mathcal B^{\perp\mathrm T}\Theta\mathcal B<0$;

T_3) $\exists\epsilon\in\mathbb R:\Theta-\epsilon\mathcal B^{\mathrm T}\mathcal B<0$;

T_4) $\exists\Lambda\in\mathbb R^{n\times m}:\Theta+\Lambda\mathcal B+\mathcal B^{\mathrm T}\Lambda^{\mathrm T}<0$.

引理 2[39]　对任意的向量$x,y\in\mathbb R^n$, 适维矩阵A,D,E,P,F和对称正定矩阵$P>0$, 其中$F^{\mathrm T}F\leqslant I$, 以下不等式成立:

1) $2x^{\mathrm T}DFEy\leqslant\epsilon^{-1}x^{\mathrm T}DD^{\mathrm T}x+\epsilon y^{\mathrm T}EE^{\mathrm T}y$;

2) 对任意的$\epsilon>0$, 如果$P-\epsilon DD^{\mathrm T}>0$, 那么

$$\begin{aligned}&(A+DFE)^{\mathrm T}P^{-1}(A+DFE)\leqslant\\&\epsilon^{-1}E^{\mathrm T}E+A^{\mathrm T}(P-\epsilon DD^{\mathrm T})^{-1}A.\end{aligned}$$

3　稳定性分析和随机有界实引理(Stability analysis and bounded real lemma)

应用Lyapunov-Krasovskii泛函方法和GFL, 给出不确定随机Markov跳跃时滞系统(9)均方指数稳定的时滞相关条件, 建立随机有界实引理.

3.1　稳定性分析(Stability analysis)

定理 1　对于模态依赖的区间时变时滞(7), 和所有的容许不确定性(8), 系统(9)均方指数稳定, 如果存在标量$\epsilon_{ij},\ i\in S,\ j=1,2,3$和对称正定矩阵$P_i>0$ $(i\in S),\ Q>0,\ R>0$使得式(14)成立:

$$\varGamma_i=\begin{pmatrix}\varGamma_{1i}&\varGamma_{2i}\\ *&\varGamma_{3i}\end{pmatrix}<0,\tag{14}$$

其中:

$$\varGamma_{1i}=\varGamma_{0i}+\sum_{k=1}^3\epsilon_{ki}V_{3i}^{\mathrm T}V_{3i},$$

$$\Gamma_{2i}=[\bar{P}_iU_{1i}\ \ \tau_2^2V_{1i}^{\mathrm T}R\ \ 0\ \ V_{2i}^{\mathrm T}P_i\ \ 0],$$

$$\Gamma_{3i}=\mathrm{diag}\{-\epsilon_{1i}I,\begin{pmatrix}-\tau_2^2R & \tau_2^2RU_{1i}\\ * & -\epsilon_{2i}I\end{pmatrix},\begin{pmatrix}-P_i & P_iU_{2i}\\ * & -\epsilon_{3i}I\end{pmatrix}\},$$

$$\Gamma_{0i}=\begin{bmatrix}\Gamma_{11}^i & \Gamma_{12}^i\\ * & \Gamma_{22}^i\end{bmatrix},\ \bar{P}_i^{\mathrm T}=[P_i\ \ 0],\ \rho=\max_{i\in S}\{|\pi_{ii}|\},$$

$$\Gamma_{11}^i=P_iA_{0i}+A_{0i}^{\mathrm T}P_i+\sum_{j=1}^{N}\pi_{ij}P_j+(1+\rho\delta)Q-R,$$

$$\Gamma_{12}^i=PA_{1i}+R,\Gamma_{22}^i=-(1-\mu_i)Q-R,$$

$$V_{1i}=[A_{0i}\ \ A_{1i}],V_{2i}=[B_{0i}\ \ B_{1i}],V_{3i}=[S_{0i}\ \ S_{1i}].$$

证 定义$x_t(s)=x(t+s)$, $t-\tau_{r_t}(t)\leqslant s\leqslant t$, 则$\{(x_t,r_t),t\geqslant 0\}$是新的随机过程, 具有初始状态$(\phi(\cdot),r_0)$.

对$t\geqslant\tau_2$, 选取Lyapunov-Krasovskii泛函

$$\begin{aligned}&V(x_t,r_t,t)=\\&x^{\mathrm T}(t)P(r_t)x(t)+\int_{t-\tau_{r_t}(t)}^{t}x^{\mathrm T}(s)Qx(s)\mathrm ds+\\&\rho\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t}x^{\mathrm T}(s)Qx(s)\mathrm ds\mathrm d\theta+\\&\tau_2\int_{-\tau_2}^{0}\int_{t+\theta}^{t}\alpha_{r_t}^{\mathrm T}(s)R\alpha_{r_t}(s)\mathrm ds\mathrm d\theta.\end{aligned}\tag{15}$$

应用Itô微分公式[40], $\forall r_t=i,i\in S$, 计算Lyapunov-Krasovskii泛函沿着系统(9)任一轨线的随机微分

$$\begin{aligned}&\mathrm dV(x_t,i,t)=\\&\mathcal{L}V(x_t,i,t)\mathrm dt+2x^{\mathrm T}(t)P_i\beta_i(t)\mathrm dB(t),\end{aligned}\tag{16}$$

其中:

$$\begin{aligned}&\mathcal{L}V(x_t,i,t)=\\&x^{\mathrm T}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t)+Tr\{\beta_i^{\mathrm T}(t)P_i\beta_i(t)\}+\\&2x^{\mathrm T}(t)P_i\alpha_i(t)+\sum_{j=1}^{N}\pi_{ij}\int_{t_{\tau_r}^j}^{t}x^{\mathrm T}(s)Qx(s)\mathrm ds+\\&x^{\mathrm T}(t)Qx(t)-(1-\dot\tau_i(t))x^{\mathrm T}(t_{\tau_r}^i)Qx(t_{\tau_r}^i)+\\&x^{\mathrm T}(t)(\rho\delta Q)x(t)-\rho\int_{t_{\tau_2}}^{t_{\tau_1}}x^{\mathrm T}(s)^{\mathrm T}Qx(s)\mathrm ds+\\&\alpha_i^{\mathrm T}(t)(\tau_2^2R)\alpha_i(t)-\tau_2\int_{t_{\tau_2}}^{t}\alpha_i^{\mathrm T}(s)R\alpha_i(s)\mathrm ds.\end{aligned}\tag{17}$$

由$\pi_{ij}\geqslant 0,i\neq j,\pi_{ii}\leqslant 0,\rho=\max_{i\in S}\{|\pi_{ii}|\}$, 得到

$$\begin{aligned}&\sum_{j=1}^{N}\pi_{ij}\int_{t_{\tau_r}^j}^{t}x^{\mathrm T}(s)Rx(s)\mathrm ds\leqslant\\&\sum_{j=1,j\neq i}^{N}\pi_{ij}\int_{t_{\tau_2}}^{t}x^{\mathrm T}(s)Rx(s)\mathrm ds+\\&\pi_{ii}\int_{t_{\tau_1}}^{t}x^{\mathrm T}(s)Rx(s)\mathrm ds\leqslant\\&-\pi_{ii}\int_{t_{\tau_2}}^{t_{\tau_1}}x^{\mathrm T}(s)Rx(s)\mathrm ds\leqslant\\&\rho\int_{t_{\tau_2}}^{t_{\tau_1}}x^{\mathrm T}(s)Rx(s)\mathrm ds.\end{aligned}\tag{18}$$

应用Jensen不等式, 可以得到

$$\begin{aligned}&\int_{t_{\tau_2}}^{t}\alpha^{\mathrm T}(s)(\tau_2R)\alpha(s)\mathrm ds\geqslant\\&\int_{t_{\tau_2}}^{t}\alpha^{\mathrm T}(s)\mathrm ds\,R\int_{t_{\tau_2}}^{t}\alpha(s)\mathrm ds\geqslant\\&\int_{t-\tau_i(t)}^{t}\alpha^{\mathrm T}(s)\mathrm ds\,R\int_{t-\tau_i(t)}^{t}\alpha(s)\mathrm ds.\end{aligned}\tag{19}$$

根据$\dot\tau_i(t)\leqslant\mu_i$和式(17)–(19), 可以得到

$$\mathcal{L}V(x_t,i,t)\mathrm dt\leqslant\xi_i^{\mathrm T}(t)\Theta_i\xi_i(t),\tag{20}$$

其中:

$$\Theta_i=\begin{bmatrix}\tau_2^2R & P_i & 0 & 0\\ * & \Theta_{22}^i & B_{0i}^{\mathrm T}(t)P_iB_{1i}(t) & 0\\ * & * & \Theta_{33}^i & 0\\ * & * & * & -R\end{bmatrix},$$

$$\Theta_{22}^i=\sum_{j=1}^{N}\pi_{ij}P_j+Q+\rho\delta Q+B_{0i}^{\mathrm T}(t)P_iB_{0i}(t),$$

$$\Theta_{33}^i=-(1-\mu_i)Q+B_{1i}^{\mathrm T}(t)P_iB_{1i}(t),$$

$$\xi_i^{\mathrm T}(t)=[\alpha_i^{\mathrm T}(t),x^{\mathrm T}(t),x^{\mathrm T}(t_{\tau_r}^i),\int_{t_{\tau_2}}^{t}\alpha_i^{\mathrm T}(s)\mathrm ds].$$

在式(16)两边取数学期望, 由式(20)得到

$$\begin{aligned}&\mathrm E\{\mathrm dV(x_t,i,t)\}=\mathrm E\{\mathcal{L}V(x_t,i,t)\mathrm dt\}\leqslant\\&\mathrm E\{\xi_i^{\mathrm T}(t)\Theta_i\xi_i(t)\},\end{aligned}\tag{21}$$

显然, 由$\mathrm E\{\xi_i^{\mathrm T}(t)^{\mathrm T}\Theta_i\xi_i(t)\}<0$可得系统(9)的均方稳定性[40]. 证毕.

对式(9)从$t-\tau_i(t)$到t积分, 并取数学期望, 得到

$$\mathrm E\{x(t)-x(t_{\tau_r}^i)-\int_{t_{\tau_r}^i}^{t}\alpha(s)\mathrm ds\}=0.\tag{22}$$

由式(7)和式(22)可以得到

$$\mathrm E\{\mathcal{B}_i\xi_i(t)\}=0,\tag{23}$$

其中

$$\mathcal{B}_i=\begin{bmatrix}-I & A_{0i}(t) & A_{1i}(t) & 0\\ 0 & -I & I & I\end{bmatrix},$$

通过计算, 选取$\mathcal{B}_i$的右正交补如下:

$$\mathcal{B}_i^{\perp}=\begin{bmatrix}A_{0i}^{\mathrm T}(t) & I & 0 & I\\ A_{1i}^{\mathrm T}(t) & 0 & I & -I\end{bmatrix}^{\mathrm T}.$$

由式(23), 根据引理1, 得到$\mathrm E\{\xi_i^{\mathrm T}(t)\Theta_i\xi_i(t)\}<0$成立, 当且仅当以下条件成立:

$$\Lambda_i=\mathcal{B}_i^{\perp\mathrm T}\Theta_i\mathcal{B}_i^{\perp}<0.\tag{24}$$

由式(8)和右正交补$\mathcal{B}_i^{\perp}$表达式, 通过计算, 得到

$$\begin{aligned}\Lambda_i=&\Gamma_{0i}+\mathrm{sym}(\bar{P}_iU_{1i}F_i(t)V_{3i})+\\&(V_{1i}+U_{1i}F_i(t)V_{3i})^{\mathrm T}\tau_2^2R(V_{1i}+U_{1i}F_i(t)V_{3i})+\\&(V_{2i}+U_{2i}F_i(t)V_{3i})^{\mathrm T}P_i(V_{2i}+U_{2i}F_i(t)V_{3i}),\end{aligned}\tag{25}$$

其中$\Gamma_{0i},\bar{P}_i,V_{1i},V_{2i},V_{3i}$定义在定理1中.

根据引理2的1), 对任意的$\epsilon_{1i}>0$, 有

$$\mathrm{sym}(\bar{P}_iU_{1i}F(t)V_{3i})\leqslant \epsilon_{1i}^{-1}\bar{P}_iU_{1i}U_{1i}^{\mathrm{T}}\bar{P}_i^{\mathrm{T}}+\epsilon_{1i}V_{3i}^{\mathrm{T}}V_{3i},\tag{26}$$

根据引理2中的2), 对任意的$\epsilon_{2i},\epsilon_{3i}>0:(\tau_2^2R)^{-1}-\epsilon_{2i}^{-1}U_{1i}U_{1i}^{\mathrm{T}}>0,P_i^{-1}-\epsilon_{3i}^{-1}U_{2i}U_{2i}^{\mathrm{T}}>0$, 得到

$$\begin{cases}(V_{1i}+U_{1i}F_i(t)V_{3i})^{\mathrm{T}}(\tau_2^2R)(V_{1i}+U_{1i}F_i(t)V_{3i})\leqslant\\ V_{1i}^{\mathrm{T}}[(\tau_2^2R)^{-1}-\epsilon_{2i}^{-1}U_{1i}U_{1i}^{\mathrm{T}}]^{-1}V_{1i}+\epsilon_{2i}V_{3i}^{\mathrm{T}}V_{3i},\\ (V_{2i}+U_{2i}F_i(t)V_{3i})^{\mathrm{T}}P_i(V_{2i}+U_{2i}F_i(t)V_{3i})\leqslant\\ V_{2i}^{\mathrm{T}}[P_i^{-1}-\epsilon_{3i}^{-1}U_{2i}U_{2i}^{\mathrm{T}}]^{-1}V_{2i}+\epsilon_{3i}V_{3i}^{\mathrm{T}}V_{3i}.\end{cases}\tag{27}$$

由式(25)–(27), 可以得到

$$\begin{aligned}&\Lambda_i\leqslant\Lambda_{0i}=\\&\Gamma_{0i}+\epsilon_{1i}^{-1}\bar{P}_iU_{1i}U_{1i}^{\mathrm{T}}\bar{P}_i^{\mathrm{T}}+\sum_{k=1}^{3}\epsilon_{ki}V_{3i}^{\mathrm{T}}V_{3i}+\\&V_{1i}^{\mathrm{T}}[(\tau_2^2R)^{-1}-\epsilon_{2i}^{-1}U_{1i}U_{1i}^{\mathrm{T}}]^{-1}V_{1i}+\\&V_{2i}^{\mathrm{T}}[P^{-1}-\epsilon_{3i}^{-1}U_{2i}U_{2i}^{\mathrm{T}}]^{-1}V_{2i}.\end{aligned}\tag{28}$$

对$\Lambda_{0i}<0$应用Schur补引理, 直接得到$\Gamma_i<0$.

以下证明系统(9)均方指数稳定. 由式(21)得到

$$\begin{aligned}&\mathrm{E}\{\mathrm{d}V(x_t,i,t)\}=\mathrm{E}\{\mathcal{L}V(x_t,i,t)\mathrm{d}t\}\leqslant\\&-\lambda_m\mathrm{E}\{|x(t)|^2\},\end{aligned}\tag{29}$$

其中$\lambda_m=\lambda_{\min}\{-\Lambda_i\}>0$. 对式(29)的两边在区间$[0,t]$上使用分部积分法积分, 得到

$$\begin{aligned}&\mathrm{E}\{V(x_t,i,t)\}\leqslant\\&\mathrm{E}\{V(x_0,r_0,0)\}-\lambda_m\int_0^t\mathrm{E}\{|x(s)|^2\}\mathrm{d}s.\end{aligned}\tag{30}$$

另外, 记$\lambda_p=\min\{\lambda_{\min}(P_i)\}>0$, 由式(15)得到

$$\mathrm{E}\{V(x_t,i,t)\}\geqslant\lambda_p\mathrm{E}\{|x(t,\phi,r_0)|^2\}.\tag{31}$$

因此, 根据式(30)–(31), 可以得到

$$\begin{aligned}&\mathrm{E}\{|x(t,\phi,r_0)|^2\}\leqslant\lambda_p^{-1}\mathrm{E}\{V(x_0,r_0,0)\}-\\&\qquad\lambda_p^{-1}\lambda_m\int_0^t\mathrm{E}\{|x(s,\phi,r_0)|^2\}\mathrm{d}s.\end{aligned}\tag{32}$$

由式(32)应用Gronwall-Bellman引理, 得到

$$\mathrm{E}\{|x(t,\phi,r_0)|^2\}\leqslant\lambda_p^{-1}\mathrm{E}\{V(x_0,r_0,0)\}\mathrm{e}^{-\frac{\lambda_m}{\lambda_p}t}.\tag{33}$$

总存在常数$c>0$, 使得

$$\mathrm{E}\{V(x_0,r_0,0)\}\leqslant c\sup_{-\tau_2\leqslant\theta\leqslant0}\mathrm{E}\{\phi(\theta)\}.$$

综上, 得到

$$\mathrm{E}\{|x(t,\phi,r_0)|^2\}\leqslant\alpha\mathrm{e}^{-\beta t}\sup_{-\tau_2\leqslant\theta\leqslant0}\mathrm{E}\{\phi(\theta)\},$$

其中: $\alpha=\lambda_p^{-1}c,\beta=\lambda_p^{-1}\lambda_m$. 由定义1, 系统(9)是均方指数稳定的.

3.2 L_2性能分析(L_2 performance analysis)

接下来, 建立随机Markov跳跃时滞系统(10)的随机有界实引理.

定理 2　给定$\mathcal{L}_2$性能指标水平$\gamma>0$, 对于模态依赖的区间时变时滞(7), 和所有的容许不确定性(8), 系统(10)均方指数稳定, 如果存在标量$\epsilon_{ij}>0,\ i\in S,j=1,2,3$和对称正定矩阵$P_i>0(i\in S),Q>0,R>0$使得式(34)成立:

$$\Phi_i=\begin{bmatrix}\Phi_{1i}&\Phi_{2i}\\ *&\Phi_{3i}\end{bmatrix}<0,\tag{34}$$

其中:

$$\begin{aligned}&\Phi_{1i}=\begin{bmatrix}\Phi_{11}^i&\Phi_{12}^i&P_iA_{2i}\\ *&\Phi_{22}^i&0\\ *&*&-\gamma^2I\end{bmatrix}+\sum_{k=1}^{3}\epsilon_{ki}V_{3i}^{\mathrm{T}}V_{3i},\\&\Phi_{2i}=[\bar{P}_iU_{1i}\ \ \tau_2^2V_{1i}^{\mathrm{T}}R\ \ 0\ \ V_{2i}^{\mathrm{T}}P_i\ \ 0],\\&\Phi_{3i}=\mathrm{diag}\{-\epsilon_{1i}I,\\&\qquad\begin{bmatrix}-\tau_2^2R&\tau_2^2RU_{1i}\\ *&-\epsilon_{2i}I\end{bmatrix},\begin{bmatrix}-P_i&P_iU_{2i}\\ *&-\epsilon_{3i}I\end{bmatrix}\},\\&\Phi_{11}^i=P_iA_{0i}+A_{0i}^{\mathrm{T}}P_i+\sum_{j=1}^{N}\pi_{ij}P_j+\\&\qquad(1+\rho\delta)Q+L_{0i}^{\mathrm{T}}L_{0i}-R,\\&\Phi_{12}^i=PA_{1i}+R,\ \bar{P}_i^{\mathrm{T}}=[P_i\ \ 0\ \ 0],\\&\Phi_{22}^i=L_{1i}^{\mathrm{T}}L_{1i}-(1-\mu_i)Q-R,\\&\rho=\max_{i\in S}\{|\pi_{ii}|\},\ V_{1i}=[A_{0i}\ \ A_{1i}\ \ A_{2i}],\\&V_{2i}=[B_{0i}\ \ B_{1i}\ \ B_{2i}],\ V_{3i}=[S_{0i}\ \ S_{1i}\ \ S_{2i}].\end{aligned}$$

证　实施行(列)初等变换, 对$\Phi_i<0$分别右乘和左乘$K=\prod_{i=3}^{8}C(i,i+1)$和$K^{-1}$, 得到

$$K^{-1}\Phi_iK=\begin{bmatrix}\Gamma_i&\mathcal{A}_i\\ *&-\gamma^2I\end{bmatrix}<0,\tag{35}$$

其中$\mathcal{A}_i^{\mathrm{T}}=[A_{2i}^{\mathrm{T}}P_i\ 0_{n\times7n}]$. 显然, $\Phi_i<0$(即34)蕴含着$\Gamma_i<0$, 从而, 当外部干扰$\omega(t)=0$时, 系统(10)是均方指数稳定的. 类似于式(22), 得到

$$\mathrm{E}\{x(t)-x(t_{\tau_r}^i)-\int_{t_{\tau_r}^i}^{t}\bar{\alpha}_i(s)\mathrm{d}s\}=0.\tag{36}$$

由式(7)和式(36), 得到

$$\mathrm{E}\{\mathcal{D}_i\bar{\xi}_i(t)\}=0,\tag{37}$$

其中: $\bar{\xi}_i^{\mathrm{T}}(t)=[\bar{\alpha}_i^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t)\ \ x^{\mathrm{T}}(t_{\tau_r}^i)\ \ \int_{t_{\tau_2}}^{t}\bar{\alpha}_i^{\mathrm{T}}(s)\mathrm{d}s\ \ \omega^{\mathrm{T}}(t)]$, $\mathcal{D}_i=[\mathcal{B}_i\ \ \mathcal{B}_{1i}]$, $\mathcal{B}_{1i}^{\mathrm{T}}=[A_{2i}^{\mathrm{T}}(t)\ \ 0]$. 计算$\mathcal{D}_i$的右正交补, 记$\mathcal{B}_{2i}^{\mathrm{T}}=[A_{2i}^{\mathrm{T}}(t)\ \ 0_{n\times7n}]$, 得到

$$\mathcal{D}_i^{\perp}=\begin{bmatrix}\mathcal{B}_i^{\perp}&\mathcal{B}_{2i}\\0&I\end{bmatrix},$$

对$t \geqslant \tau_2$, 选取Lyapunov-Krasovskii泛函

$$V(x_t,r_t,t)=\\ x^{\rm T}(t)P(r_t)x(t)+\int_{t^i_{\tau_r}}^t x^{\rm T}(s)Qx(s){\rm d}s+\\ \rho\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^t x^{\rm T}(s)Qx(s){\rm d}s{\rm d}\theta+\\ \tau_2\int_{-\tau_2}^0\int_{t+\theta}^t \bar{\alpha}^{\rm T}_{r_t}(s)R\bar{\alpha}_{r_t}(s){\rm d}s{\rm d}\theta.$$

类似于定理1的证明, 由$\Phi_i<0$推出$J_\infty<0$, 略去. 证毕.

4 鲁棒H_∞指数滤波(Robust H_∞ exponential filtering)

为了方便, 定义如下变量:

$$\tilde{\alpha}_{r_t}(t)=A_{{\rm f}0i}(t)\eta(t)+A_{{\rm f}1i}(t)E\eta(t^i_{\tau_r})+\\ A_{{\rm f}2i}(t)\omega(t),$$

$$\tilde{\beta}_{r_t}(t)=B_{{\rm f}0i}(t)\eta(t)+B_{{\rm f}1i}(t)E\eta(t^i_{\tau_r})+\\ B_{{\rm f}2i}(t)\omega(t).$$

定理 3 给定$\mathcal{L}_2$性能指标水平$\gamma>0$, 对于模态依赖的区间时变时滞(7), 和所有的容许不确定性(8), 系统(12)均方指数稳定, 如果存在标量$\epsilon_{ki}>0, i\in S$, $k=1,2,3$和对称正定矩阵$P_i>0(i\in S)$, $Q>0$, $R>0$使得式(38)成立:

$$\Psi_i=\begin{bmatrix}\Psi_{1i} & \Psi_{2i}\\ * & \Psi_{3i}\end{bmatrix}<0, \tag{38}$$

其中:

$$\Psi_{2i}=[\tilde{P}_{fi}U_{{\rm f}1i}\ \ \tau_2^2\tilde{V}^{\rm T}_{{\rm f}1i}E^{\rm T}R\ \ 0\ \ \tilde{V}^{\rm T}_{{\rm f}2i}P_i\ \ 0],$$

$$\Psi_{3i}={\rm diag}\{-\epsilon_{1i}I,\\ \begin{bmatrix}-\tau_2^2R & \tau_2^2REU_{{\rm f}1i}\\ * & -\epsilon_{2i}I\end{bmatrix},\begin{bmatrix}-P_i & P_iU_{{\rm f}2i}\\ * & -\epsilon_{3i}I\end{bmatrix}\},$$

$$\Psi_{1i}=\begin{bmatrix}\Psi^1_{0i} & \Psi^2_{0i}\\ * & \Psi^3_{0i}\end{bmatrix}+\sum_{k=1}^3\epsilon_{ki}\tilde{V}^{\rm T}_{{\rm f}3i}\tilde{V}_{{\rm f}3i},$$

$$\tilde{V}^{\rm T}_{{\rm f}1i}=[V^{\rm T}_{{\rm f}1i}\ \ 0],\ \tilde{V}^{\rm T}_{{\rm f}2i}=[V^{\rm T}_{{\rm f}2i}\ \ 0],\ \tilde{V}^{\rm T}_{{\rm f}3i}=[V^{\rm T}_{{\rm f}3i}\ \ 0],$$

$$\Psi^1_{0i}=\begin{bmatrix}\Psi^i_{11} & \Psi^i_{12}\\ * & \Psi^i_{22}\end{bmatrix},\ \Psi^2_{0i}=\begin{bmatrix}P_iA_{{\rm f}2i} & L^{\rm T}_{{\rm f}0i}\\ 0 & 0\end{bmatrix},$$

$$\Psi^3_{0i}={\rm diag}\{-\gamma^2I,\ -I\},\ \tilde{P}^{\rm T}_{fi}=[\bar{P}^{\rm T}_{fi}\ \ 0],$$

$$\Psi^i_{11}=P_iA_{{\rm f}0i}+A^{\rm T}_{{\rm f}0i}P_i+E^{\rm T}[(1+\rho\delta)Q-R]E=\\ \sum_{j=1}^N\pi_{ij}P_j,$$

$$\Psi^i_{12}=P_iA_{{\rm f}1i}+E^{\rm T}R,\ \rho=\max_{i\in S}\{|\pi_{ii}|\},$$

$$\Psi^i_{22}=L^{\rm T}_{{\rm f}1i}L_{{\rm f}1i}-(1-\mu_i)Q-R,$$

$$U^{\rm T}_{{\rm f}1i}=[U^{\rm T}_{1i}\ U^{\rm T}_{3i}B^{\rm T}_{fi}],\ U^{\rm T}_{{\rm f}2i}=[U^{\rm T}_{2i}\ \ 0],$$

$$V_{{\rm f}1i}=[A_{{\rm f}0i}\ A_{{\rm f}1i}\ A_{{\rm f}2i}],\ V_{{\rm f}2i}=[B_{{\rm f}0i}\ B_{{\rm f}1i}\ B_{{\rm f}2i}],$$

$$V_{{\rm f}3i}=[S_{{\rm f}0i}\ S_{1i}\ S_{2i}],\ S_{{\rm f}0i}=[S_{0i}\ 0],\ \bar{P}^{\rm T}_{fi}=[P_i\ 0\ 0].$$

证 记$\tilde{\mathcal{A}}^{\rm T}_i=[A^{\rm T}_{{\rm f}2i}P_i\ \ 0_{2n\times 10n}]$. 对$\Psi_i<0$分别右乘和左乘$\tilde{K}=\prod\limits_{i=4}^{11}C(i,i+1)$和$\tilde{K}^{-1}$, 得到

$$\tilde{K}^{-1}\Psi_i\tilde{K}=\begin{bmatrix}\tilde{\Gamma}_i & \tilde{\mathcal{A}}_i\\ * & -\gamma^2I\end{bmatrix}<0. \tag{39}$$

显然, $\Psi_i<0$即式(38)蕴含着$\tilde{\Gamma}_i<0$, 从而, 类似于定理1, 当外部干扰$\omega(t)=0$时, 式(12)均方指数稳定.

对$t\geqslant\tau_2$, 选取Lyapunov-Krasovskii泛函

$$V(\eta_t,r_t,t)=\\ \eta^{\rm T}(t)P(r_t)\eta(t)+\int_{t^i_{\tau_r}}^t\eta^{\rm T}(s)E^{\rm T}QE\eta(s){\rm d}s+\\ \rho\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^t\eta^{\rm T}(s)E^{\rm T}QE\eta(s){\rm d}s{\rm d}\theta+\\ \tau_2\int_{-\tau_2}^0\int_{t+\theta}^t\tilde{\alpha}^{\rm T}_{r_t}(s)E^{\rm T}RE\tilde{\alpha}_{r_t}(s){\rm d}s{\rm d}\theta. \tag{40}$$

应用Itô微分公式[40], 对于每一个$r_t=i,\ i\in S$, $V(\eta_t,r_t,t)$沿着系统(12)任一轨线的随机微分为

$${\rm d}V(\eta_t,i,t)=\\ \mathcal{L}V(\eta_t,i,t){\rm d}t+2\eta^{\rm T}(t)P_i\tilde{\beta}_i(t){\rm d}B(t), \tag{41}$$

其中

$$\mathcal{L}V(\eta_t,i,t)=\\ \eta^{\rm T}(t)\sum_{j=1}^N\pi_{ij}P_j\eta(t)+2\eta^{\rm T}(t)P_i\tilde{\alpha}_i(t)+\\ \tilde{\beta}^{\rm T}_i(t)P_i\tilde{\beta}_i(t)+\sum_{j=1}^N\pi_{ij}\int_{t^j_{\tau_r}}^t\eta^{\rm T}(s)E^{\rm T}QE\eta(s){\rm d}s+\\ \eta^{\rm T}(t)E^{\rm T}QE\eta(t)-(1-\dot{\tau}_i(t))\eta^{\rm T}(t^i_{\tau_r})E^{\rm T}QE\eta(t^i_{\tau_r})+\\ \eta^{\rm T}(t)\rho\delta E^{\rm T}QE\eta(t)-\rho\int_{t_{\tau_2}}^{t_{\tau_1}}\eta^{\rm T}(s)^{\rm T}E^{\rm T}QE\eta(s){\rm d}s+\\ \tilde{\alpha}^{\rm T}_i(t)\tau_2^2E^{\rm T}RE\tilde{\alpha}_i(t)-\int_{t_{\tau_2}}^t\tilde{\alpha}^{\rm T}_i(s)\tau_2E^{\rm T}RE\tilde{\alpha}_i(s){\rm d}s. \tag{42}$$

类似于式(22), 得到

$${\rm E}\{E\eta(t)-E\eta(t^i_{\tau_r})-\int_{t^i_{\tau_r}}^tE\tilde{\alpha}_i(s){\rm d}s\}=0. \tag{43}$$

由式(43), 选取

$$\mathcal{N}_i=\begin{bmatrix}-I & A_{{\rm f}0i}(t) & A_{{\rm f}1i}(t) & 0 & A_{{\rm f}2i}(t)\\ 0 & -E & I & I & 0\end{bmatrix},$$

通过计算, 选取$\mathcal{N}_i$的右正交补如下:

$$\mathcal{N}^\perp_i=\begin{bmatrix}A^{\rm T}_{{\rm f}0i}(t) & I & 0 & E & 0\\ A^{\rm T}_{{\rm f}1i}(t) & 0 & I & -I & 0\\ A^{\rm T}_{{\rm f}2i}(t) & 0 & 0 & 0 & I\end{bmatrix}^{\rm T},$$

由式(7)和式(43)得到以下条件:

$${\rm E}\{\mathcal{N}_i\bar{\eta}_i(t)\}=0, \tag{44}$$

其中:

$$\bar{\eta}_i^{\mathrm{T}}(t) = [\tilde{\alpha}_i^{\mathrm{T}}(t), \eta^{\mathrm{T}}(t), \eta^{\mathrm{T}}(t_{\tau_r}^i)E^{\mathrm{T}}, \int_{t_{\tau_2}}^t \tilde{\alpha}_i^{\mathrm{T}}(s)E^{\mathrm{T}}\mathrm{d}s, \omega^{\mathrm{T}}(t)].$$

考虑到零初始条件和均方指数稳定性, $\forall\omega(t) \neq 0$, 由式(41), 可以得到

$$\mathrm{E}\int_0^\infty \mathcal{L}V(\eta_t, i, t)\mathrm{d}t = \mathrm{E}\int_0^\infty \mathrm{d}V(\eta_t, i, t) = 0.$$

由式(13)和式(42), 得到

$$J_\infty = \mathrm{E}\{\int_0^\infty [\tilde{z}^{\mathrm{T}}(t)\tilde{z}(t) - \gamma^2\omega^{\mathrm{T}}(t)\omega(t) + \mathcal{L}V(\eta_t, i, t)]\mathrm{d}t\} \leqslant \int_0^\infty \mathrm{E}\{\bar{\eta}_i^{\mathrm{T}}(t)\tilde{\Theta}_i\bar{\eta}_i(t)\mathrm{d}t\},$$

其中

$$\tilde{\Theta}_i = \begin{bmatrix} \bar{R} & P_i & 0 & 0 & 0 \\ * & \tilde{\Theta}_{22}^i & \tilde{\Theta}_{23}^i & 0 & B_{\mathrm{f0}i}^{\mathrm{T}}(t)P_iB_{\mathrm{f2}i}(t) \\ * & * & \tilde{\Theta}_{33}^i & 0 & B_{\mathrm{f1}i}^{\mathrm{T}}(t)P_iB_{\mathrm{f2}i}(t) \\ * & * & * & -R & 0 \\ * & * & * & * & B_{\mathrm{f2}i}^{\mathrm{T}}(t)P_iB_{\mathrm{f2}i}(t) - \gamma^2I \end{bmatrix},$$

$$\tilde{\Theta}_{22}^i = \sum_{j=1}^N \pi_{ij}P_j + (1+\rho\delta)E^{\mathrm{T}}QE + L_{\mathrm{f0}i}^{\mathrm{T}}L_{\mathrm{f0}i} + B_{\mathrm{f0}i}^{\mathrm{T}}(t)P_iB_{\mathrm{f0}i}(t),$$

$$\tilde{\Theta}_{23}^i = B_{\mathrm{f0}i}^{\mathrm{T}}(t)P_iB_{\mathrm{f1}i}(t), \bar{R} = \tau_2^2E^{\mathrm{T}}RE,$$

$$\tilde{\Theta}_{33}^i = -(1-\mu_i)Q + B_{\mathrm{f1}i}^{\mathrm{T}}(t)P_iB_{\mathrm{f1}i}(t) + L_{\mathrm{f1}i}^{\mathrm{T}}L_{\mathrm{f1}i}.$$

根据引理1, $\mathrm{E}\{\bar{\eta}_i^{\mathrm{T}}(t)\tilde{\Theta}_i\bar{\eta}_i(t)\} < 0$, 当且仅当

$$\tilde{\Lambda}_i = \mathcal{N}_i^{\perp\mathrm{T}}\tilde{\Theta}_i\mathcal{N}_i^{\perp} < 0. \tag{45}$$

考虑到式(8), 由式(45)计算得

$$\tilde{\Lambda}_i = \bar{\Psi}_{0i} + \mathrm{sym}(\bar{P}_{fi}U_{\mathrm{f1}i}F_i(t)V_{\mathrm{f3}i}) + (V_{\mathrm{f1}i} + U_{\mathrm{f1}i}F_i(t)V_{\mathrm{f3}i})^{\mathrm{T}}\bar{R}(V_{\mathrm{f1}i} + U_{\mathrm{f1}i}F_i(t)V_{\mathrm{f3}i}) + (V_{\mathrm{f2}i} + U_{\mathrm{f2}i}F_i(t)V_{\mathrm{f3}i})^{\mathrm{T}}P_i(V_{\mathrm{f2}i} + U_{\mathrm{f2}i}F_i(t)V_{\mathrm{f3}i}),$$

其中:

$$\bar{\Psi}_{0i} = \begin{bmatrix} \bar{\Psi}_{11}^i & \Psi_{12}^i & P_iA_{\mathrm{f2}i} \\ * & \Psi_{22}^i & 0 \\ * & * & -\gamma^2I \end{bmatrix},$$

$$\bar{\Psi}_{11}^i = \Psi_{11} + L_{\mathrm{f0}i}^{\mathrm{T}}L_{\mathrm{f0}i}.$$

类似于式(28), 得到

$$\tilde{\Lambda}_i \leqslant \tilde{\Lambda}_{0i} = \bar{\Psi}_{0i} + \sum_{k=1}^3 \epsilon_{ki}V_{\mathrm{f3}i}^{\mathrm{T}}V_{\mathrm{f3}i} + \epsilon_{1i}^{-1}\bar{P}_{fi}U_{\mathrm{f1}i}U_{\mathrm{f1}i}^{\mathrm{T}}\bar{P}_{fi}^{\mathrm{T}} + V_{\mathrm{f1}i}^{\mathrm{T}}E^{\mathrm{T}}[(\tau_2^2R)^{-1} - \epsilon_{2i}^{-1}U_{\mathrm{f1}i}U_{\mathrm{f1}i}^{\mathrm{T}}]^{-1}EV_{\mathrm{f1}i} + V_{\mathrm{f2}i}^{\mathrm{T}}[P_{\mathrm{f1}i}^{-1} - \epsilon_{3i}^{-1}EU_{\mathrm{f2}i}U_{\mathrm{f2}i}^{\mathrm{T}}E^{\mathrm{T}}]^{-1}V_{\mathrm{f2}i}.$$

由Schur补引理, $\Psi_i < 0$蕴含$\tilde{\Lambda}_{0i} < 0$, 从而$J_\infty < 0$.

证毕.

注 1　为了方便, 设

$$P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i} & P_{2i} \end{bmatrix} > 0,\ P_{1i} > 0,\ P_{2i} > 0,$$
$$U_i = P_{2i}A_{fi},\ V_i = P_{2i}B_{fi},\ i \in S,$$

将P_i, U_i, V_i代入联立LMIs(38). 根据定理3, 设计Markov跳跃的线性滤波器(11)的增益如下:

$$A_{fi} = P_{2i}^{-1}U_i, B_{fi} = P_{2i}^{-1}V_i, C_{fi}. \tag{46}$$

注 2　LMIs(14)(34)(38)适用于具有模态依赖的区间时变时滞的系统(1), 而且导数上界可以$\mu_i < 1, \mu_i = 1$, 或$\mu_i > 1, i \in S$. 然而$\mu_i < 1$[13–14]和$\mu < 1$[11–12,20,36]是必要条件.

注 3　对于时滞系统的稳定性分析, 模型变换的方法[29–30]会引入保守性, 而松弛变量方法[16]和自由权矩阵方法[31–33]会使得计算变得复杂, 导致很难应用于综合设计中. 定理1, 定理2, 定理3是通过引理1前两个等价条件得到的, 避免了使用模型变换方法和自由权矩阵方法. 因此, 所得结果同时降低了保守性和计算复杂程度. 如果不考虑系统参数不确定性 (8), 则定理1, 定理2, 定理3的推导中就可以避免使用引理2来对交叉项定界. 文献 [10,20,29,31]和本文结果所需决策变量数量列在表1中.

表 1 决策变量数量对比表

Table 1 The numbers of decision variables

稳定性分析	定理1[31] $\frac{17n^2+5n}{2}$	定理1[29] $\frac{37n^2+7n}{2}$	定理1[10] $\frac{6n^2+6n}{2}N$	定理1 $\frac{3n^2+3n+6}{2}N$
$\mathcal{L}_2$性能分析	定理2[10] $\frac{6n^2+6n}{2}N$	定理2[20] $\frac{9n^2+3n}{2}N$	定理2 $\frac{4n^2+4n+6}{2}N$	
滤波器设计	定理4[10] $\frac{9n^2+9n+2}{2}N$	定理3[20] $\frac{21n^2+11n}{2}N$	定理3 $\frac{6n^2+8n+6}{2}N$	

5 例子与仿真(Example and simulation)

例 1　考虑以下具有模态依赖的区间时变时滞, 不确定Itô随机Markov跳跃时滞系统(1), 考虑两个模态.

对于模态1, 系统参数为

$$A_{01} = \begin{bmatrix} -3.0 & 1.0 & 0.0 \\ 0.3 & -4.5 & 1.0 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, A_{21} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1.0 & -0.8 \\ 0.0 & 1.0 & -2.5 \end{bmatrix}, C_{01}^{\mathrm{T}} = \begin{bmatrix} 0.8 \\ 0.3 \\ 0.0 \end{bmatrix},$$

$$B_{01} = B_{11} = \begin{bmatrix} 0.1 & -0.1 & 0.2 \\ 0.3 & 0.3 & -0.4 \\ 0.1 & 0.1 & -0.3 \end{bmatrix}, C_{11}^{\mathrm{T}} = \begin{bmatrix} 0.0 \\ -0.6 \\ 0.2 \end{bmatrix},$$

$$L_{01} = [0.5\ \ -0.1\ \ 1.0],\ C_{21} = 0.2,$$
$$U_{11}^{\mathrm{T}} = U_{21}^{\mathrm{T}} = U_{31}^{\mathrm{T}} = S_{01} = S_{11} = L_{11} = [0\ \ 0\ \ 0],$$
$$S_{21} = 0.$$

对于模态2, 系统参数为

$$A_{02}=\begin{bmatrix}-2.5&0.5&-0.1\\0.1&-3.5&0.3\\-0.1&1.0&-3.2\end{bmatrix},\ A_{22}=\begin{bmatrix}-0.6\\0.5\\0.0\end{bmatrix},$$

$$A_{12}=\begin{bmatrix}0.0&-0.3&0.6\\0.1&0.5&0.0\\-0.6&1.0&-0.8\end{bmatrix},\ C_{02}^{\mathrm T}=\begin{bmatrix}-0.5\\0.2\\0.3\end{bmatrix},$$

$$B_{02}=B_{12}=\begin{bmatrix}0.1&-0.1&0.2\\0.3&0.3&-0.4\\0.1&0.1&-0.3\end{bmatrix},\ C_{12}^{\mathrm T}=\begin{bmatrix}0.0\\-0.6\\0.2\end{bmatrix},$$

$$L_{02}=[0.5\ \ -0.1\ \ 1.0],\ C_{22}=0.5,\ S_{22}=0,$$

$$U_{12}^{\mathrm T}=U_{22}^{\mathrm T}=U_{32}^{\mathrm T}=S_{02}=S_{12}=L_{12}=[0\ \ 0\ \ 0].$$

目的是设计Markov跳跃H$_\infty$指数滤波器(11), 对给定$\mathcal{L}_2$性能指标水平$\gamma=1.2$, 模态依赖的区间时变时滞(7)和所有的容许不确定性(8), 系统(12)均方指数稳定. 设转移概率矩阵为$\begin{bmatrix}-0.5&0.5\\0.3&-0.3\end{bmatrix}$.

当$\tau_{11}=0.4,\tau_{21}=1.0,\mu_1=0.3,\tau_{11}=0.3,\tau_{21}=1.2,\mu_2=1.2$时, 求解联立LMIs(38), 根据定理3, 设计Markov跳跃H$_\infty$指数滤波器增益如下:

$$A_{\rm f1}=\begin{bmatrix}-3.2698&-0.3005&1.7049\\0.0566&-3.6931&0.9087\\-4.8564&-2.9348&-2.6065\end{bmatrix},$$

$$B_{\rm f1}=\begin{bmatrix}-0.5189\\-0.3796\\-6.2932\end{bmatrix},\ C_{\rm f1}^{\mathrm T}=\begin{bmatrix}-0.4764\\-0.0999\\-0.8764\end{bmatrix},$$

$$A_{\rm f2}=\begin{bmatrix}-2.9805&0.2866&0.1095\\0.5719&-3.7256&-0.5702\\-0.3877&0.1085&-4.2676\end{bmatrix},$$

$$B_{\rm f2}=\begin{bmatrix}-0.0401\\-0.1525\\0.4878\end{bmatrix},\ C_{\rm f2}^{\mathrm T}=\begin{bmatrix}-0.3209\\0.0248\\-0.7556\end{bmatrix}.$$

仿真生成Markov链$r(t)$在图1(在MATLAB7.1中, 利用条件循环语句编程, 通过stairs函数画图得到).

图 1 跳变模态

Fig. 1 Jump modes

图2–4给出了状态轨线$x(t)$和估计响应$\hat{x}(t)$, 设初值为

$$x^{\mathrm T}(0)=[-1.8,1.2,1.5],$$
$$\hat{x}^{\mathrm T}(0)=[-1.6,1.5,1.2],$$

外部干扰信号为

$$\omega(t)=\exp(-t)\sin(0.5t).$$

图5是估计误差响应

$$\tilde{z}(t)=z(t)-\hat{z}(t),$$

估计误差快速收敛到"0". 仿真结果表明所设计的Markov跳跃H$_\infty$指数滤波器满足预期的性能指标, 达到了既定目标.

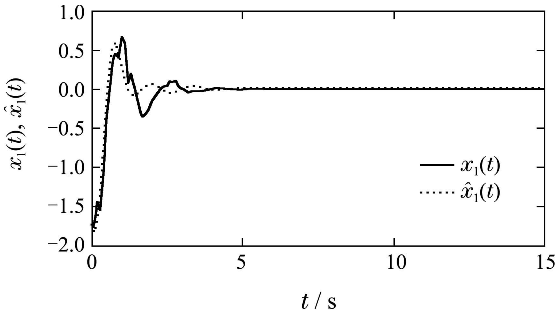

图 2 状态轨线$x_1(t)$和估计响应$\hat{x}_1(t)$

Fig. 2 The state trajectories and estimates response of $x_1(t)$ and $\hat{x}_1(t)$

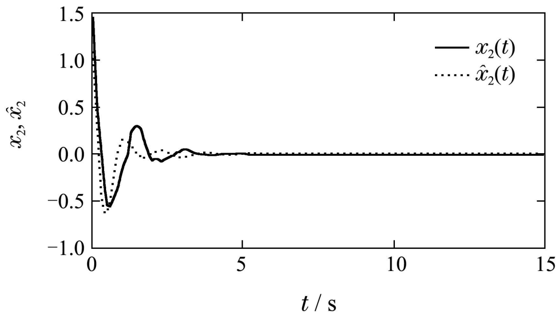

图 3 状态轨线$x_2(t)$和估计响应$\hat{x}_2(t)$

Fig. 3 The state trajectories and estimates response of $x_2(t)$ and $\hat{x}_2(t)$

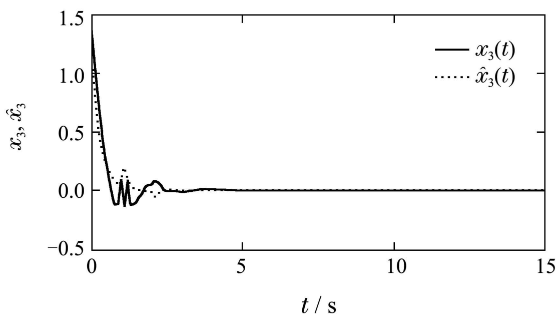

图 4 状态轨线$x_3(t)$和估计响应$\hat{x}_3(t)$

Fig. 4 The state trajectories and estimates response of $x_3(t)$ and $\hat{x}_3(t)$

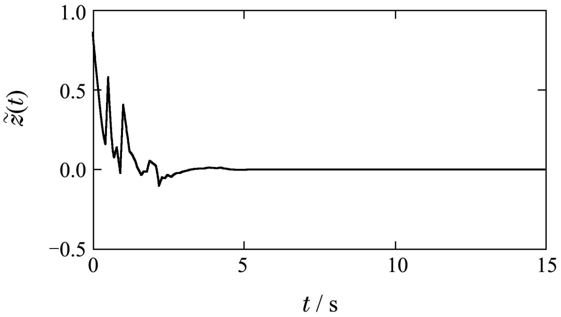

图 5 估计误差响应$\tilde{z}(t)$

Fig. 5 The estimation error response of $\tilde{z}(t)$

当$\tau_{11} = 0.2, \tau_{21} = 0.6, \tau_{12} = 0.3$时, $\tau(t_i), i = 1, 2$的最大允许上界对比值见表2. 在$\tau(t_i), i = 1, 2$的导数上界不同取值情况下, 定理3结果明显优于文献[20]的定理3, 而且后者只适用于慢变时变时滞, 即要求$\dot{\tau}(t_i) \leqslant \mu_i < 1, i = 1, 2$, 本文结果只是要求$\dot{\tau}(t_i) \leqslant \mu_i,\ i = 1, 2$.

表 2 最大允许时滞上界对比表

Table 2 The maximum upper bound of delay

μ_1	$\mu_1 = 0.4$	$\mu_1 = 0.5$	$\mu_1 = 0.6$	$\mu_1 = 0.3$
μ_2	$\mu_2 = 0.7$	$\mu_2 = 0.8$	$\mu_2 = 0.9$	$\mu_2 = 1.2$
定理3[20]	2.4613	1.5386	—	—
定理3	2.5720	1.9751	1.4643	3.2396

6 结论(Conclusions)

应用Lyapunov-Krasovkii泛函和广义Finsler引理, 研究了具有模态依赖的区间时变时滞的不确定随机Markov跳跃时滞系统的鲁棒H_∞滤波问题, 建立了随机有界实引理, 得到了时滞相关条件, 基于有界实引理, 设计了鲁棒H_∞指数滤波器, 在没有外部干扰的情况下, 滤波误差系统是均方指数稳定的. 去掉了时滞导数上界小于1的保守性条件[11–14, 20, 36]. 避免了自由权矩阵和模型变换方法的使用, 所得结果能够同时降低保守性和计算复杂程度. 数值例子说明了方法的有效性.

参考文献(References):

[1] WANG Z, GOODALL D P, BURNHAM K J. On designing observers for time-delay systems with nonlinear disturbances [J]. *Internatinal Journal of Control*, 2002, 75(11): 803 – 811.

[2] YUE D, HAN Q L. Robust H_∞ filter design of uncertain descriptor systems with discrete and distributed delays [J]. *IEEE Transactions on Signal Processing*, 2004, 52(11): 3200 – 3212.

[3] GAO H J, WANG C H. A delay-dependent approach to robust H_∞ filtering for uncertain discrete-time state-delayed systems [J]. *IEEE Transactions on Signal Processing*, 2004, 52(6): 1631 – 1640.

[4] 洪晓芳, 王玉振. 一类变时滞饱和不确定系统鲁棒H_∞滤波器设计 [J]. 控制理论与应用, 2012, 29(10): 1245 – 1252.
(HONG Xiaofang, WANG Yuzhen. Robust H_∞ filter for a class of uncertain systems with time-varying delay and saturation [J]. *Control Theory & Applications*, 2012, 29(10): 1245 – 1252.)

[5] WANG Z, LAM J, LIU X. Exponential filtering for uncertain Markovian jump time−delay systems with nonlinear disturbances [J]. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2004, 51(5): 262 – 268.

[6] WEI G, WANG Z, SHU H, et al. A delay−dependent approach to H_∞ filtering for stochastic delayed jumping systems with sensor nonlinearities [J]. *Internatinal Journal of Control*, 2007, 80(6): 885 – 897.

[7] FRIDMAN E, SHAKED U. An improved delay-dependent H_∞ filtering of linear neutral systems [J]. *IEEE Transactions on Signal Processing*, 2004, 52(3): 668 – 673.

[8] WANG Z D, SHEN B, LIU X H. H_∞ filtering with randomly occurring sensor saturations and missing measurements [J]. *Automatica*, 2012, 48(3): 556 – 562.

[9] XU S, CHEN T, LAM J. Robust H_∞ filtering for uncertain Markovian jump systems with mode-dependent time delays [J]. *IEEE Transactions on Automatic Control*, 2003, 48(5): 900 – 907.

[10] XU S, LAM J, MAO X. Delay-dependent H_∞ control and filtering for uncertain markovian jump systems with time-varying delays [J]. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2007, 54(9): 2070 – 2077.

[11] CHEN G, SHEN Y. Robust H_∞ filter design for neutral stochastic uncertain systems with time-varying delay [J]. *Journal of Mathematical Analysis and Applications*, 2009, 353(1): 196 – 204.

[12] SONG B, XU S, XIA J, et al. Design of robust non-fragile H_∞ filters for uncertain neutral stochastic system with distributed delays [J], *Asian Journal of Control*, 2010, 12(1): 39 – 45.

[13] YAN H C, MENG MAX Q H, ZHANG H, et al. Robust H_∞ exponential filtering for uncertain stochastic time-delay systems with Markovian switching and nonlinearities [J]. *Applied Mathematics and Computation*, 2010, 215(12): 4358 – 4369.

[14] YAN H C, ZHANG H, SHI H B, et al. Robust H_∞ filtering for uncertain nonlinear stochastic systems with mode-dependent time-delays and Markovian jump parameters [J]. *Circuits Systems on Signal Process*, 2011, 30(2): 303 – 321.

[15] ZHANG W H, CHEN B S, TSENG C S. Robust H_∞ filtering for nonlinear stochastic systems [J]. *IEEE Transactions on Signal Processing*, 2005, 53(2): 589 – 598.

[16] 徐伟杰, 李平, 韩波. 扩展Kalman滤波框架下的地平线检测算法 [J]. 控制理论与应用, 2012, 29(2): 225 – 228.
(XU Weijie, LI Ping, HAN Bo. Horizon detection algorithm in extended Kalman filter framework [J]. *Control Theory & Applications*, 2012, 29(2): 225 – 228.)

[17] NICULESCU S. *Delay Effects on Stability: a Robust Control Approach* [M]. New York: Springer, 2001.

[18] 刘健辰, 章兢. 具有时变输入与状态时滞的Markov跳变系统鲁棒H_∞控制 [J]. 控制理论与应用, 2010, 27(6): 809 – 814.
(LIU Jianchen, ZHANG Jing. Robust H_∞ control for Markovian jump systems with time-varying time-delay in input and state [J]. *Control Theory & Applications*, 2010, 27(6): 809 – 814.)

[19] 刘晓华, 吕娜. Markov跳变系统的输出反馈鲁棒预测控制 [J]. 控制理论与应用, 2013, 30(11): 1392 – 1400.
(LIU Xiaohua, LÜ Na. Robust predictive control via output feedback for Markov jump systems [J]. *Control Theory & Applications*, 2013, 30(11): 1392 – 1400.)

[20] WANG G L. Partially mode-dependent design of H_∞ filter for stochastic Markovian jump systems with mode-dependent time delays [J]. *Journal of Mathematical Analysis and Applications*, 2011, 383(2): 573 – 584.

[21] WANG D, WANG W, SHI P. Exponential H_∞ filtering for switched linear systems with interval time-varying delay [J]. *International Journal of Robust and Nonlinear Control*, 2009, 19(5): 532 – 551.

[22] WANG D, SHI P, WANG J L, et al. Delay-dependent exponential H_∞ filtering for discrete-time switched delay systems [J]. *International Journal of Robust and Nonlinear Control*, 2012, 22(13): 1522 – 1536.

[23] LIN Z W, LIN Y, ZHANG W H. A unified design for state and output feedback H_∞ control of nonlinear stochastic Markovian jump systems with state and disturbance-dependent noise [J]. *Automatica*, 2009, 45(12): 2955 – 2962.

[24] ZHANG W, FENG G, LI Q. Robust H_∞ filtering for general nonlinear stochastic state-delayed systems [J]. *Mathematical Problems in Engineering*, 2012, doi: 10.1155/2012/231352.

[25] 华民刚, 邓飞其, 彭云建. 不确定变时滞随机系统的鲁棒均方指数稳定性 [J]. 控制理论与应用, 2009, 26(5): 558 – 561.
(HUA Mingang, DENG Feiqi, PENG Yunjian. Robust mean square exponential stability of uncertain stochastic systems with time-varying delay [J]. *Control Theory & Applications*, 2009, 26(5): 558 – 561.)

[26] CHEN Y, ZHENG W X, XUE A. A new result on stability analysis for stochastic neutral systems [J]. *Automatica*, 2010, 46(12): 2100 – 2104.

[27] CHEN Y, ZHENG W X. Stability and L_2 performance analysis of stochastic delayed neural networks [J]. *IEEE Transactions on Neural Networks*, 2011, 22(10): 1662 – 1668.

[28] CHEN Y, ZHENG W X. On stability of switched time-delay systems subject to nonlinear stochastic perturbations [C] //*Proceedings of the 49th IEEE Conference on Decision and Control*. Atlanta, GA, USA: IEEE, 2010: 2644 – 2649.

[29] CHEN W H, GUAN Z H, LU X. Delay-dependent exponential stability of uncertain stochastic systems with multiple delays: an LMI approach [J]. *Systems & Control Letters*, 2005, 54(6): 547 – 555.

[30] YUE D, HAN Q L, LAM J. Network-based robust H_∞ control of systems with uncertainty [J]. *Automatica*, 2005, 41(6): 999 – 1007.

[31] HUANG L, MAO X. Delay-dependent exponential stability of neutral stochastic delay systems [J]. *IEEE Transactions on Automatic and Control*, 2009, 54(1): 147 – 152.

[32] HE Y, WANG Q, XIE L, et al. Further improvement of free-weighting matrices technique for systems with time varying delay [J]. *IEEE Transactions on Automatic Control*, 2007, 52(2): 293 – 299.

[33] HE Y, WANG Q, LIN C, et al. Delay-range-dependent stability for systems with time-varying delay [J]. *Automatica*, 2007, 43(2): 371 – 376.

[34] GU K, KHARITONOV V L, CHEN J. *Stability of Time-Delay Systems* [M]. Boston: Birkhauser, 2003.

[35] XU S, LAM J. On equivalence and efficiency of certain stability criteria for time-delay systems [J]. *IEEE Transactions on Automatic Control*, 2007, 52(1): 95 – 101.

[36] SUPLIN V, FRIDMAN E, SHAKED U. H_∞ control of linear uncertain time delay systems-a projection approach [J]. *IEEE Transactions on Automatic Control*, 2006, 51(4): 680 – 685.

[37] QIU J, FENG G, YANG J. A new design of delay-dependent robust H_∞ filtering for discrete-time T-S fuzzy systems with time-varying delay [J]. *IEEE Transactions on Fuzzy Systems*, 2009, 17(5): 1044 – 1058.

[38] CHE W, WANG J. Static output feedback H_∞ control for discrete-time Markov jump linear systems [C] //*Proceedings of the 8th IEEE International Conference on Control and Automation*. Piscataway, NJ, USA: IEEE, 2010: 2278 – 2283.

[39] XU S, CHEN T. Robust H_∞ control for uncertain stochastic systems with state delay [J]. *IEEE Transactions on Automatic Control*, 2002, 47(12): 2089 – 2094.

[40] MAO X. *Stochastic Differential Equations with Applications* [M]. 2 Edition. Chichester, UK: Horwood, 2008.

作者简介:

毛卫华 (1977–), 男, 讲师, 研究方向为随机时滞系统的稳定性、鲁棒H_∞控制与滤波, E-mail: weihua.mao@163.com;

邓飞其 (1961–), 男, 教授, 博士生导师, 研究方向为时滞系统、非线性系统和随机系统等复杂系统的稳定性、镇定和鲁棒控制理论, E-mail: aufqdeng@scut.edu.cn;

万安华 (1976–), 女, 副教授, 硕士生导师, 研究方向为非线性控制, E-mail: wananhua@mail.sysu.edu.cn.