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高维拟线性抛物系统能控性的近期进展

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摘要: 本文综述高维拟线性抛物型方程、拟线性复Ginzburg-Landau方程以及只含一个控制变量的高维耦合拟线性抛物型方程组的能控性方面的一些近期的结果. 通过使用不动点技术, 采用主部具有C^1系数的线性抛物型方程或方程组一些新的精细的Carleman估计. 这一方法的要点是在古典解的框架下考虑能控性问题, 并且当给定的数据具有一定的正则性时, 线性抛物型方程或方程组在Hölder空间中来选取控制函数. 利用类似的方法, 还建立了拟线性抛物型方程不灵敏控制的存在性, 其关键是将不灵敏问题转化为由拟线性抛物型方程和线性抛物型方程构成的耦合方程组在单个控制下一个非标准的能控性问题.

关键词: 零能控性; 近似能控性; 拟线性抛物型方程; 复Ginzburg-Landau方程; 不灵敏控制

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Recent progress in controllability of multidimensional quasilinear parabolic systems

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Abstract: We overview some recent controllability results for multidimensional quasilinear parabolic equations, quasilinear complex Ginzburg-Landau equations, and coupled quasilinear parabolic systems with single control variable. When using the fixed point technique, we employ the main tools to investigate some new and delicate Carleman estimates for suitable linear parabolic equations/systems with C^1coefficients in principal parts. The key points of the approach are to formulate the controllability problems in the frame of classical solutions and to seek the control functions in the Hölder spaces for linear parabolic equations/systems with given data having certain regularity. By means of a similar approach, the existence of insensitizing controls for quasilinear parabolic equations is also established. The key point is to transform this insensitizing problem into a nonstandard controllability problem for some nonlinear cascade system governed by a quasilinear parabolic equation and a linear parabolic equation with single control variable.

Key words: null controllability; approximate controllability; quasilinear parabolic equation; complex Ginzburg-Landau equation; insensitizing controls

1 Introduction

Many physical processes can be described by parabolic-type partial differential equations. For example, suppose that there is a body, whose temperature at the location $x \in \mathbb{R}^n$ ($n \in \mathbb{N}$) and at the time $t \in [0, \infty)$ is denoted by $y(x, t)$. By the conservation law, one has $y_t + \mathrm{div} J = 0$, where J denotes the heat flux. Since the temperature propagates along the fast descending direction, a simple model for the diffusion process can be formulated by $J = -a\nabla y$, where a is the diffusion coefficient. If a is a constant, then we obtain the standard heat equation $y_t - a\Delta y = 0$. If, instead, the heat conduction coefficient depends on the distribution in a way such as $a = a(y)$, then the corresponding equation becomes a quasilinear parabolic equation:

$$y_t = \mathrm{div}(a(y)\nabla y). \tag{1}$$

In the last decades, there are many works addressing the controllability problems for linear and semilinear parabolic equations/systems (e.g. [1–7] and the rich references therein). A natural problem is whether the quasilinear parabolic equations like (1) are controllable. This problem was first considered by M. Beceanu[8]. Nevertheless, in [8], the author only obtained the local null controllability of the diffusion equation in one space dimension.

In the literature, there exist many interesting works on the controllability problems for the time-reversible quasi-

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linear systems, say [9–11] and the references therein. However, these works use essentially the time-reversibility of the underlying systems, and therefore, it seems that their approach cannot be applied to time-irreversible systems. For the time-irreversible systems, we refer to [12–13] for the controllability of Newtonian filtration equation, which is a typical quasilinear degenerate parabolic equation ($a(y) = y^m,\ m > 1$ in (1)). In these two papers, the special structure of $a(y)$ plays a crucial role. Hence, the methods in [12–13] cannot be applied to the general quasilinear parabolic equations, either.

It is well known that in general, for a linear parabolic equation, Carleman estimates require that the coefficient of the principal operator belongs to the space $W^{1,1}_\infty$. Therefore, in order to apply the fixed point technique to the quasilinear problems, one has to search for a fixed point in a space at least being contained in the space $W^{1,1}_\infty$. Moreover, this space must have some compactness and can guarantee that the solution of the linearized system has good estimates for the fixed point mapping. Notice that this limitation is not required for semilinear parabolic equations. This is the main difficulty in solving the controllability problems for quasilinear parabolic equations, compared to the semilinear case. In order to establish the desired estimates for the weak solutions of the corresponding linearized equation, [8] essentially makes use of a special Sobolev embedding relation, which is valid only for one space dimension. Therefore, the same argument in [8] does not work in the multidimensional space. The purpose of this paper is to present our recent works on controllability of multidimensional quasilinear parabolic equations/systems and some related control problems.

The rest of this paper is organized as follows. Section 2 is devoted to the controllability of multidimensional quasilinear parabolic equations. In Section 3, the local controllability results for coupled quasilinear parabolic systems by one control are established. Section 4 addresses the controllability of quasilinear complex Ginzburg-Landau equations. Finally, in Section 5, the existence of insensitizing controls for quasilinear parabolic equations is addressed.

2 Local controllability of multidimensional quasilinear parabolic equations

In this section, we review the local controllability results of multidimensional quasilinear parabolic equations. The detailed proofs of these results can be found in [14].

To begin with, we introduce some notations. Let Ω be a nonempty bounded domain of $\mathbb{R}^n$ with C^3 boundary Γ, and ω, $\mathcal{O}$, ω_0 and ω_1 are given nonempty open subsets of Ω such that $\bar\omega_0 \subseteq \omega_1$, $\bar\omega_1 \subseteq \omega$ and $\bar\omega \subseteq \Omega$. For any given $T > 0$, put $Q = \Omega \times (0,T)$ and $\Sigma = \Gamma \times (0,T)$. Denote by χ_ω the characteristic function of ω. For any $k,m \in \mathbb{N}$, $\theta \in (0,1)$ and $p \in [1,\infty]$, we refer to the book [15] for definitions of the spaces $C^{k,m}(\bar Q)$, $C^k(\bar\Omega)$, $C^{k+\theta,m+\frac{\theta}{2}}(\bar Q)$, $C^{2+\theta}(\bar\Omega)$, $L^p(Q)$ and $W^{k,m}_p(Q)$. Denote by spaces $C^{k+\theta,m+\frac{\theta}{2}}(\bar Q;\ \mathbb{C})$ and $C^{k,m}(\bar Q;\ \mathbb{R}^{n\times n})$ the counterparts taking value in $\mathbb{C}$ and $\mathbb{R}^{n\times n}$, respectively. In the sequel, C denotes a generic positive constant which may change from line to line.

Throughout this paper, $f(\cdot) \in C^2(\mathbb{R})$ is a given function with $f(0) = 0$, and $a^{jk}(\cdot) \in C^3(\mathbb{R})$ $(j,k=1,2,\cdots,n)$ are given functions satisfying $a^{jk}(s) = a^{kj}(s)$, $\forall\ s \in \mathbb{R}$, and for some constant $\rho > 0$,

$$\sum_{j,k=1}^n a^{jk}(s)\xi_j\bar\xi_k \geqslant \rho|\xi|^2,\quad \forall\,(s,\xi) \equiv (s,\xi_1,\cdots,\xi_n) \in \mathbb{R}\times\mathbb{C}^n.$$

Let us consider the following quasilinear parabolic equation with an internal controller:

$$\begin{cases} y_t - \sum\limits_{j,k=1}^n (a^{jk}(y)y_{x_j})_{x_k} + f(y) = \chi_\omega u, \text{ in } Q,\\ y = 0, \text{ on } \Sigma,\\ y(0) = y_0, \text{ in } \Omega, \end{cases} \tag{2}$$

where u is the control variable and y is the state variable.

Unlike the case of one space dimension in [8], the controllability of the system (2) is analyzed in the frame of classical solutions. For this purpose, we need to establish a global Carleman estimate for general linear parabolic equations with C^1 coefficients of the principal parts.

2.1 A pointwise estimate and global Carleman estimate for linear parabolic equations

First, we establish a pointwise inequality for second order parabolic operators with symmetric coefficients.

Consider the following linear parabolic equation:

$$\begin{cases} v_t + \mathrm{div}(\mathcal{B}\nabla v) + \mathrm{div}(\mathcal{C}v) + dv = g, \text{ in } Q,\\ v = 0, \text{ on } \Sigma,\\ v(T) = v_T, \text{ in } \Omega, \end{cases} \tag{3}$$

where $\mathcal{C} = (c^j)_{1\leqslant j\leqslant n} \in C^{1,0}(\bar Q;\mathbb{R}^n)$, $d \in L^\infty(Q)$ and $g \in L^2(Q)$. Throughout this paper, $\mathcal{B} = (b^{jk})_{1\leqslant j,k\leqslant n} \in C^1(\bar Q;\mathbb{R}^{n\times n})$ is a given matrix satisfying $b^{jk}(x,t) = b^{kj}(x,t)$ $(j,k = 1,2,\cdots,n)$ and for some constant $\rho > 0$, $\sum\limits_{j,k=1}^n b^{jk}(x,t)\xi_j\bar\xi_k \geqslant \rho|\xi|^2$, $\forall\ (x,t,\xi) \in \bar Q\times\mathbb{C}^n$. Put

$$B = 1 + |\mathcal{B}|^2_{C^1(\bar Q;\mathbb{R}^{n\times n})},\ H = B + |d|^2_{L^\infty(Q)}$$

and

$$D = |\mathcal{C}|^2_{C^{1,0}(\bar Q;\mathbb{R}^n)} + |d|^2_{L^\infty(Q)}.$$

Then one has the following pointwise ineuqality for a general parabolic-like operator.

Lemma 1 Assume that $v,\ell \in C^{2,1}(\bar Q)$, and $\mathcal{D} = (d^{jk})_{1\leqslant j,\ k\leqslant n} \in C^1(\bar Q;\ \mathbb{R}^{n\times n})$ is a symmetric matrix. Set $z = \mathrm{e}^{\ell}v$. Then

$$\begin{aligned}
&\frac{1}{2}\mathrm{e}^{2\ell}|v_t + \mathrm{div}(\mathcal{D}\nabla v)|^2 \geqslant \\
&\sum_{j,k=1}^n [d^{jk}z_{x_j}z_t - \sum_{j',k'=1}^n (2d^{jk}d^{j'k'}\ell_{x_{j'}}z_{x_j}z_{x_{k'}} - \\
&d^{jk}d^{j'k'}\ell_{x_j}z_{x_{j'}}z_{x_{k'}}) - Ed^{jk}\ell_{x_j}z^2]_{x_k} - \\
&\frac{1}{2}(\sum_{j,k=1}^n d^{jk}z_{x_j}z_{x_k} - Ez^2)_t + \sum_{j,k=1}^n c^{jk}z_{x_j}z_{x_k} - \\
&2\sum_{j,k=1}^n\sum_{j',k'=1}^n (d^{jk}z_{x_j})_{x_k}d^{j'k'}\ell_{x_{j'}x_{k'}}z + Fz^2,
\end{aligned}$$

where

$$\Psi = -\sum_{j,k=1}^{n}(d^{jk}_{x_k}\ell_{x_j} + 2d^{jk}\ell_{x_jx_k}),$$

$$E = -\ell_t + \sum_{j,k=1}^{n}(d^{jk}\ell_{x_j}\ell_{x_k} - d^{jk}_{x_k}\ell_{x_j} - d^{jk}\ell_{x_jx_k}) - \Psi,$$

$$F = -\frac{1}{2}E_t + \sum_{j,k=1}^{n}[(Ed^{jk}\ell_{x_j})_{x_k} - \frac{1}{2}(d^{jk}_{x_k}\ell_{x_j})^2 - 2Ed^{jk}\ell_{x_jx_k}],$$

$$c^{jk} = \sum_{j',k'=1}^{n}[2d^{jk'}(d^{j'k}\ell_{x_{j'}})_{x_{k'}} - (d^{jk}d^{j'k'}l_{x_{j'}})_{x_{k'}}] + \frac{1}{2}d^{jk}_t.$$

By [4], one can find a function $\psi \in C^2(\bar{\Omega})$ such that $\psi(x) > 0$ in Ω, $\psi(x) = 0$ on Γ, and $|\nabla\psi(x)| > 0$ in $\bar{\Omega}\setminus\omega_0$. For any given parameter $\mu > 0$, put

$$\varphi(x,t) = \frac{\mathrm{e}^{\mu\psi(x)}}{t(T-t)} \text{ and } \alpha(x,t) = \frac{\mathrm{e}^{\mu\psi(x)} - \mathrm{e}^{2\mu|\psi|_{C(\bar{\Omega})}}}{t(T-t)}.$$

Based on Lemma 1, we have the following global Carleman estimate for the system (3).

Lemma 2 There exists a constant $C > 0$, depending only on ρ, n, ω, Ω and T, such that for any $\mu \geqslant CB$ and $\lambda \geqslant C(D + \mathrm{e}^{2\mu|\psi|_{C(\bar{\Omega})}})$, solutions of Eq.(3) satisfy

$$\int_Q\Big(\lambda\mu^2\mathrm{e}^{2\lambda\alpha}\varphi|\nabla v|^2 + \lambda^3\mu^4\mathrm{e}^{2\lambda\alpha}\varphi^3v^2\Big)\mathrm{d}x\mathrm{d}t \leqslant C\lambda^3\mu^4(B+D)\int_0^{\mathrm{T}}\int_{\omega_0}\mathrm{e}^{2\lambda\alpha}\varphi^3v^2\mathrm{d}x\mathrm{d}t + C\int_Q\mathrm{e}^{2\lambda\alpha}g^2\mathrm{d}x\mathrm{d}t,\ \forall v_T \in L^2(\Omega).$$

Remark 1 Compared with the known results on global Carleman estimates for the linear parabolic equation (e.g. [2]), Lemma 2 provides an explicit estimate on the constant (in the right side of Carleman inequality) with respect to C^1 coefficients on the principal operator. Such kind of weighted inequalities are quite useful in the study of control theory and inverse problems for various kinds of partial differential equations (see [6] and the references therein).

2.2 Null controllability of linear parabolic equations

In order to establish the local controllability of (2) by means of the fixed point technique, one needs to consider the following linearized system of (2):

$$\begin{cases} y_t - \mathrm{div}(\mathcal{B}\nabla y) - dy = \varrho u, \text{ in } Q,\\ y = 0, \text{ on } \Sigma,\\ y(0) = y_0, \text{ in } \Omega, \end{cases} \tag{4}$$

where $\varrho \in C_0^\infty(\omega)$ satisfying $\varrho = 1$ in ω_0 and $y_0 \in L^2(\Omega)$. The key point is to construct a Hölder continuous control function starting from an L^2–control function. This strategy makes it possible to solve the nonlinear controllability problem in the frame of classical solutions.

To this aim, let us consider the following parabolic equation:

$$\begin{cases} p_t + \mathrm{div}(\mathcal{B}\nabla p) + dp = 0, \text{ in } Q,\\ p = 0, \text{ on } \Sigma,\\ p(T) = p_T, \text{ in } \Omega. \end{cases} \tag{5}$$

Using Lemma 2, one can show the following explicit observability estimate for the linear parabolic equation (5).

Proposition 1 Solutions of Eq.(5) satisfy the following inequality

$$\int_\Omega p^2(x,0)\mathrm{d}x \leqslant C\mathrm{e}^{\mathrm{e}^{CH}}\int_0^{\mathrm{T}}\int_\omega \mathrm{e}^{2\lambda\alpha}\varphi^3p^2\mathrm{d}x\mathrm{d}t,$$

for any $p_T \in L^2(\Omega)$, $\mu \geqslant CH$ and $\lambda \geqslant C\mathrm{e}^{CH}$.

Furthermore, by Proposition 1 and an iteration method, we have the following null controllability result for the linear parabolic equation (4). Also, we give an explicit cost estimate for the control function.

Proposition 2 For any $y_0 \in L^2(\Omega)$, there exists a control $u \in C^{\frac{1}{2},\frac{1}{4}}(\bar{Q})$ such that the corresponding solution y of the system (4) satisfies $y(T) = 0$ in Ω. Moreover,

$$|u|_{C^{\frac{1}{2},\frac{1}{4}}(\bar{Q})} \leqslant C\mathrm{e}^{\mathrm{e}^{CH}}|y_0|_{L^2(\Omega)}.$$

2.3 Main results

By Proposition 2 and Kakutani's fixed point theorem, one can have the following local controllability result for Eq.(2).

Theorem 1(Local null controllability) There is a positive constant δ_1, such that for any given initial value $y_0 \in C^{2+\frac{1}{2}}(\bar{\Omega})$ satisfying $|y_0|_{C^{2+\frac{1}{2}}(\bar{\Omega})} \leqslant \delta_1$ and the first order compatibility condition, one can find a control $u \in C^{\frac{1}{2},\frac{1}{4}}(\bar{Q})$ with supp $u \subseteq \omega\times[0,T]$ so that the corresponding solution y of the system (2) satisfies $y(T) = 0$ in Ω. Moreover, $|u|_{C^{\frac{1}{2},\frac{1}{4}}(\bar{Q})} \leqslant C\mathrm{e}^{\mathrm{e}^{CA}}|y_0|_{L^2(\Omega)}$, where

$$A = \sum_{j,k=1}^{n}(1 + \sup_{|s|\leqslant 1}|a^{jk}(s)|^2 + \sup_{|s|\leqslant 1}|(a^{jk})'(s)|^2 + \sup_{|s|\leqslant 1}|f'(s)|^2).$$

As a consequence of Theorem 1, we have the following local approximate controllability result.

Theorem 2(Local approximate controllability) There is a positive constant δ_2, such that for any $\varepsilon > 0$, and any given functions $y_0, y_1 \in C^{2+\frac{1}{2}}(\bar{\Omega})$ satisfying $|y_0|_{C^{2+\frac{1}{2}}(\bar{\Omega})} + |y_1|_{C^{2+\frac{1}{2}}(\bar{\Omega})} \leqslant \delta_2$ and the first order compatibility condition, one can find a control $u \in C(\bar{Q})$ with supp $u \subseteq \omega\times[0,T]$ so that the corresponding solution y of the system (2) satisfies $|y(T) - y_1|_{C^{2+\frac{1}{2}}(\bar{\Omega})} < \varepsilon$.

3 Local controllability of coupled quasilinear parabolic systems by one control

Similarly, one can establish the local controllability for coupled quasilinear parabolic systems by one control. Coupled quasilinear parabolic systems can be used to describe the dynamics of two biological groups.

Consider the following coupled quasilinear parabolic system:

$$\begin{cases} y_{1,t} - \sum_{j,k=1}^{n} (a_1^{jk}(y_1, \cdots, y_m) y_{1,x_j})_{x_k} + \\ f_1(y_1, \cdots, y_m) = \chi_\omega u, \text{ in } Q, \\ y_{2,t} - \sum_{j,k=1}^{n} (a_2^{jk}(y_1, \cdots, y_m) y_{2,x_j})_{x_k} + \\ f_2(y_1, \cdots, y_m) = 0, \text{ in } Q, \\ y_{3,t} - \sum_{j,k=1}^{n} (a_3^{jk}(y_1, \cdots, y_m) y_{3,x_j})_{x_k} + \\ f_3(y_2, \cdots, y_m) = 0, \text{ in } Q, \\ \quad \vdots \\ y_{m,t} - \sum_{j,k=1}^{n} (a_m^{jk}(y_1, \cdots, y_m) y_{m,x_j})_{x_k} + \\ f_m(y_{m-1}, y_m) = 0, \text{ in } Q, \\ y_1 = \cdots = y_m = 0, \text{ on } \Sigma, \\ y_1(0) = y_1^0, \cdots, y_m(0) = y_m^0, \text{ in } \Omega, \end{cases} \tag{6}$$

where u is the control variable and $(y_1, \cdots, y_m)$ is the state variable, $f_\nu(\nu = 1, \cdots, m)$ are given C^2 functions defined on $\mathbb{R}^r$ with $r = \begin{cases} m, & \text{for } \nu = 1, \\ m-\nu+2, & \text{for } \nu = 2, \cdots, m, \end{cases}$ $f_\nu(0, \cdots, 0) = 0$, and $a_\nu^{jk}(\cdot) \in C^3(\mathbb{R}^m)$ are given functions satisfying $a_\nu^{jk}(s) = a_\nu^{kj}(s)$, $\forall\, s \in \mathbb{R}^m$, $\nu = 1, \cdots, m$ and $j, k = 1, \cdots, n$, and for some constant $\rho > 0$,

$$\sum_{j,k=1}^{n} a_\nu^{jk}(s) \xi_j \xi_k \geqslant \rho |\xi|^2,$$

$\forall\, (s, \xi) \equiv (s, \xi_1, \cdots, \xi_n) \in \mathbb{R}^m \times \mathbb{R}^n$ and $1 \leqslant \nu \leqslant m$.

Furthermore, we assume that f_ν $(\nu = 2, \cdots, m)$ satisfy the following condition:

H) $\qquad \dfrac{\partial f_\nu}{\partial y_{\nu-1}}(0, \cdots, 0) \neq 0, \ \nu = 2, \cdots, m.$

First, we establish a global Carleman estimate for coupled linear parabolic systems.

3.1 Global Carleman estimate for coupled linear parabolic systems

Consider the following coupled linear parabolic system:

$$\begin{cases} -p_{1,t} - \sum_{j,k=1}^{n} (b_1^{jk}(x,t) p_{1,x_j})_{x_k} + \\ a_1^1 p_1 + a_1^2 p_2 = 0, \text{ in } Q, \\ -p_{2,t} - \sum_{j,k=1}^{n} (b_2^{jk}(x,t) p_{2,x_j})_{x_k} + \\ a_2^1 p_1 + a_2^2 p_2 + a_2^3 p_3 = 0, \text{ in } Q, \\ \quad \vdots \\ -p_{m,t} - \sum_{j,k=1}^{n} (b_m^{jk}(x,t) p_{m,x_j})_{x_k} + \\ a_m^1 p_1 + a_m^2 p_2 + \cdots + a_m^m p_m = 0, \text{ in } Q, \\ p_1 = \cdots = p_m = 0, \text{ on } \Sigma, \\ p_1(T) = p_1^{\mathrm{T}}, \cdots, p_m(T) = p_m^{\mathrm{T}}, \text{ in } \Omega, \end{cases} \tag{7}$$

where $a_\nu^l \in L^\infty(Q)(l, \nu = 1, \cdots, m)$, and $b_\nu^{jk} \in C^1(\bar{Q})$ satisfying $b_\nu^{jk}(x,t) = b_\nu^{kj}(x,t)$, $\forall\, j, k = 1, 2, \cdots n$, and for some constant $\rho > 0$,

$$\sum_{j,k=1}^{n} b_\nu^{jk}(x,t) \xi_j \xi_k \geqslant \rho |\xi|^2,$$

$\forall (x, t, \xi) \equiv (x, t, \xi_1, \cdots, \xi_n) \in \bar{Q} \times \mathbb{R}^n$ and $1 \leqslant \nu \leqslant m$. Put $a_0^1 = 0$ and

$$B_1 = \sum_{j,k=1}^{n} \sum_{\nu=1}^{m} |b_\nu^{jk}|^2_{C^1(\bar{Q})} + \sum_{l=1}^{m} \sum_{\nu=l-1}^{m} |a_\nu^l|^2_{L^\infty(Q)}.$$

Moreover, we introduce the following condition:

H_1) There exists a nonempty open subset $\omega^* \subseteq \omega$ and a constant $r_* > 0$, such that for $\nu = 1, \cdots, m-1$, $a_\nu^{\nu+1}(x,t) \geqslant r_*$, in $\omega^* \times (0, T)$.

By Lemma 2, we have the following global Carleman estimate for the system (7).

Lemma 3 Suppose that H_1) holds. Then there exist a constant $C > 0$ and an integer $r^* > 0$, depending only on ρ, n, ω, Ω and T, such that for any $\mu \geqslant CB_1$ and $\lambda \geqslant C(B_1 + \mathrm{e}^{2\mu|\psi|_{C(\bar{\Omega})}})$, solutions of the system (7) satisfy

$$\begin{aligned} &\int_Q [\lambda \mu^2 \mathrm{e}^{2\lambda\alpha} \varphi (|\nabla p_1|^2 + \cdots + |\nabla p_m|^2) + \\ &\lambda^3 \mu^4 \mathrm{e}^{2\lambda\alpha} \varphi^3 (p_1^2 + \cdots + p_m^2)] \mathrm{d}x\mathrm{d}t \leqslant \\ &C B_1^{r^*} \lambda^{r^*} \mu^{r^*} \int_0^{\mathrm{T}} \int_{\omega_0} \mathrm{e}^{2\lambda\alpha} \varphi^{r^*} p_1^2 \mathrm{d}x\mathrm{d}t, \\ &\forall\, (p_1^{\mathrm{T}}, \cdots, p_m^{\mathrm{T}}) \in (L^2(\Omega))^m. \end{aligned}$$

3.2 Null controllability of coupled linear parabolic systems

In order to solve the quasilinear problem, one first needs to establish the null controllability of the following coupled linear parabolic system:

$$\begin{cases} y_{1,t} - \sum_{j,k=1}^{n} (b_1^{jk}(x,t) y_{1,x_j})_{x_k} + \\ a_1^1 y_1 + a_2^1 y_2 + \cdots + a_m^1 y_m = \varrho u, \text{ in } Q, \\ y_{2,t} - \sum_{j,k=1}^{n} (b_2^{jk}(x,t) y_{2,x_j})_{x_k} + \\ a_1^2 y_1 + a_2^2 y_2 + \cdots + a_m^2 y_m = 0, \text{ in } Q, \\ y_{3,t} - \sum_{j,k=1}^{n} (b_3^{jk}(x,t) y_{3,x_j})_{x_k} + \\ a_2^3 y_2 + \cdots + a_m^3 y_m = 0, \text{ in } Q, \\ \quad \cdots \\ y_{m,t} - \sum_{j,k=1}^{n} (b_m^{jk}(x,t) y_{m,x_j})_{x_k} + \\ a_{m-1}^m y_{m-1} + a_m^m y_m = 0, \text{ in } Q, \\ y_1 = \cdots = y_m = 0, \text{ on } \Sigma, \\ y_1(0) = y_1^0, \cdots, y_m(0) = y_m^0, \text{ in } \Omega. \end{cases} \tag{8}$$

By Lemma 3, one can show the following observability estimate for the system (7).

Proposition 3 Suppose that H_1) holds. Then solutions of the system (7) satisfy the following inequality

$$\begin{aligned} &\int_\Omega [p_1^2(x,0) + p_2^2(x,0) + \cdots + p_m^2(x,0)] \mathrm{d}x \leqslant \\ &C \mathrm{e}^{\mathrm{e}^{CB_1}} \int_0^{\mathrm{T}} \int_\omega \mathrm{e}^{2\lambda\alpha} \varphi^{r^*} p_1^2(x,t) \mathrm{d}x\mathrm{d}t, \end{aligned}$$

for any $(p_1^{\mathrm{T}}, \cdots, p_m^{\mathrm{T}}) \in (L^2(\Omega))^m$, $\mu \geqslant CB_1$ and $\lambda \geqslant C\mathrm{e}^{CB_1}$.

By Proposition 3, we have the following null controllability result for the system (8).

Proposition 4 Suppose that H_1) holds. Then for any $(y_1^0, \cdots, y_m^0) \in (L^2(\Omega))^m$, there exists a control $u \in C^{\frac{1}{2}, \frac{1}{4}}(\bar{Q})$, such that the corresponding solution $(y_1, \cdots, y_m)$ of the system (8) satisfies $y_1(T) = \cdots = y_m(T) = 0$ in Ω.

3.3 Main results

Based on the null controllability of the coupled system (8), proceeding similar analysis as [14, Theorem 1.1], one has the following local controllability results for the coupled quasilinear parabolic system (6).

Theorem 3(Local null controllability) Suppose that H) holds. Then there is a positive constant δ_3, such that for any given initial value $(y_1^0,\cdots,y_m^0)\in(C^{2+\frac{1}{2}}(\bar{\Omega}))^m$ satisfying $|(y_1^0,\cdots,y_m^0)|_{(C^{2+\frac{1}{2}}(\bar{\Omega}))^m}\leqslant\delta_3$ and the first order compatibility condition, one can find a control $u\in C^{\frac{1}{2},\frac{1}{4}}(\bar{Q})$ with supp $u\subseteq\omega\times[0,T]$ so that the corresponding solution of the system (6) satisfies $y_1(T)=\cdots=y_m(T)=0$ in Ω.

For some special coupled quasilinear parabolic systems, we can get the local approximate controllability result. To this aim, introduce the following condition:

H$_2$) $a_\nu^{jk}\equiv a_\nu^{kj}(y_\nu,y_{\nu+1},\cdots,y_m)$, $\forall\, j,k=1,\cdots,n$; $\nu=1,\cdots,m$. We have the following result.

Theorem 4(Local approximate controllability) Suppose that H) and H$_2$) hold. Then there is a positive constant δ_4, such that for any $\varepsilon>0$, and any given functions $(y_1^0,\cdots,y_m^0)$, $(y_1^{\mathrm{T}},\cdots,y_m^{\mathrm{T}})\in(C^{2+\frac{1}{2}}(\bar{\Omega}))^m$ satisfying $|(y_1^0,\cdots,y_m^0)|_{(C^{2+\frac{1}{2}}(\bar{\Omega}))^m}+|(y_1^{\mathrm{T}},\cdots,y_m^{\mathrm{T}})|_{(C^{2+\frac{1}{2}}(\bar{\Omega}))^m}\leqslant\delta_4$ and the first order compatibility condition, one can find a control $u\in C(\bar{Q})$ with supp $u\subseteq\omega\times[0,T]$ so that the corresponding solution of the system (6) satisfies $|y_k(T)-y_k^{\mathrm{T}}|_{C^{2+\frac{1}{2}}(\bar{\Omega})}<\varepsilon$, for any $k=1,\cdots,m$.

4 Local controllability for quasilinear complex Ginzburg-Landau equations

In this section, we review the local controllability results of quasilinear complex Ginzburg-Landau equations, which arise in superconductivity.

Let us consider the following controlled quasilinear Ginzburg-Landau equation:

$$\begin{cases}(a_1-ia_2)y_t-\sum\limits_{j,k=1}^n(a^{jk}(|y|^2)y_{x_j})_{x_k}+\\ f(|y|^2)y=\chi_\omega u,\ \text{in } Q,\\ y=0,\ \text{on } \Sigma,\\ y(0)=y_0,\ \text{in } \Omega,\end{cases}\tag{9}$$

where $a_1>0$, $a_2\neq0$ and $i=\sqrt{-1}$. In Eq.(9), u is the control variable, y is the state variable and both of them are complex valued. To begin with, as before, we need to establish a global Carleman estimate for the linear Ginzburg-Landau equation.

4.1 A pointwise estimate and global Carleman estimate for linear Ginzburg-Landau equations

First, define a linear operator by

$$\mathcal{P}z=(a+ib)z_t+\sum_{j,k=1}^n(b^{jk}z_{x_j})_{x_k},$$

where $a,b\in C^1(\mathbb{R}^{n+1})$ and $b^{jk}\in C^1(\mathbb{R}^{n+1})$ satisfying $b^{jk}(x,t)=b^{kj}(x,t)$, $\forall\, j,k=1,2,\cdots,n$.

We have the following weighted identity.

Lemma 4 Let $a^1,b^1,\lambda\in\mathbb{R}$ be three parameters. Let $z\in C^2(\mathbb{R}^{n+1};\mathbb{C})$ and $\varsigma\in C^2(\mathbb{R}^{n+1})$. Put $\ell=\lambda\varsigma$ and $v=\theta z=\mathrm{e}^{\ell}z$. Then

$$\begin{aligned}&\theta(\mathcal{P}z\bar{I}_1+\bar{\mathcal{P}}zI_1)+M_t+\sum_{k=1}^n V^k_{x_k}=\\&2|I_1|^2+\sum_{j,k,j',k'=1}^n[2(b^{j'k}\ell_{x_{j'}})_{x_{k'}}b^{jk'}-\\&(b^{jk}b^{j'k'}\ell_{x_{j'}})_{x_{k'}}+\frac{1}{2}(ab^{jk})_t-a^1b^{jk}b^{j'k'}\ell_{x_{j'}x_{k'}}]\cdot\\&(v_{x_k}\bar{v}_{x_j}+\bar{v}_{x_k}v_{x_j})+\\&[-\sum_{j,k=1}^n b^{jk}_{x_k}\ell_{x_j}+b^1\lambda](I_1\bar{v}+\bar{I}_1v)+\\&i\sum_{j,k=1}^n\{[(bb^{jk}\ell_{x_j})_t+b^{jk}(b\ell_t)_{x_j}](\bar{v}_{x_k}v-v_{x_k}\bar{v})+\\&[(bb^{jk}\ell_{x_j})_{x_k}+a^1bb^{jk}\ell_{x_jx_k}](\bar{v}v_t-v\bar{v}_t)\}-\\&\sum_{j,k=1}^n b^{jk}a_{x_k}(v_{x_j}\bar{v}_t+\bar{v}_{x_j}v_t)+B_2|v|^2-\\&a^1\sum_{j,k,j',k'=1}^n b^{jk}(b^{j'k'}\ell_{x_{j'}x_{k'}})_{x_k}(\bar{v}_{x_j}v+v_{x_j}\bar{v}),\end{aligned}$$

where

$$A_1=\sum_{j,k=1}^n b^{jk}\ell_{x_j}\ell_{x_k}-(1+a^1)\sum_{j,k=1}^n b^{jk}\ell_{x_jx_k}-b^1\lambda,$$

$$I_1=ibv_t-a\ell_tv+\sum_{j,k=1}^n(b^{jk}v_{x_j})_{x_k}+A_1v,$$

$$\begin{aligned}B_2=&(a^2\ell_t+b^2\ell_t-aA_1)_t+2\sum_{j,k=1}^n[(b^{jk}\ell_{x_j}A_1)_{x_k}-\\&(ab^{jk}\ell_{x_j}\ell_t)_{x_k}+a^1(A_1-a\ell_t)b^{jk}\ell_{x_jx_k}],\end{aligned}$$

$$\begin{aligned}M=&[(a^2+b^2)\ell_t-aA_1]|v|^2+a\sum_{j,k=1}^n b^{jk}v_{x_j}\bar{v}_{x_k}+\\&ib\sum_{j,k=1}^n b^{jk}\ell_{x_j}(\bar{v}_{x_k}v-v_{x_k}\bar{v}),\end{aligned}$$

and

$$\begin{aligned}&V^k=\\&\sum_{j,j',k'=1}^n\{-ib[b^{jk}\ell_{x_j}(v\bar{v}_t-\bar{v}v_t)+\\&b^{jk}\ell_t(v_{x_j}\bar{v}-\bar{v}_{x_j}v)]-ab^{jk}(v_{x_j}\bar{v}_t+\bar{v}_{x_j}v_t)+\\&(2b^{jk'}b^{j'k}-b^{jk}b^{j'k'})\ell_{x_j}(v_{x_{j'}}\bar{v}_{x_{k'}}+\bar{v}_{x_{j'}}v_{x_{k'}})-\\&a^1b^{j'k'}\ell_{x_{j'}x_{k'}}b^{jk}(v_{x_j}\bar{v}+\\&\bar{v}_{x_j}v)+2b^{jk}(A_1-a\ell_t)\ell_{x_j}|v|^2\}.\end{aligned}$$

Next, we present a global Carleman estimate for the following complex Ginzburg-Landau equation:

$$\begin{cases}(a_1+ia_2)z_t+\sum\limits_{j,k=1}^n(b^{jk}(x,t)z_{x_j})_{x_k}+qz=0,\ \text{in } Q,\\ z=0,\ \text{on } \Sigma,\\ z(T)=z_T,\ \text{in } \Omega,\end{cases}\tag{10}$$

where $q\in C(\bar{Q})$ and $z_T\in L^2(\Omega;\ \mathbb{C})$.

Based on Lemma 4, we have the following global Carleman estimate for Eq.(10).

Lemma 5 There exists a positive constant $\mu_0=\mu_0(C,a_1,a_2,q,b^{jk})$ such that for any $\mu\geqslant\mu_0$, one can find two constants $C_1=C_1(\mu)>0$ and $\lambda_0=\lambda_0(\mu)>0$

so that for any $\lambda \geqslant \lambda_0$, the corresponding solution z of the system (10) satisfies

$$\lambda\mu^2 \int_Q \mathrm{e}^{2\lambda\alpha}\varphi|\nabla z|^2\mathrm{d}x\mathrm{d}t + \lambda^3\mu^4 \int_Q \mathrm{e}^{2\lambda\alpha}\varphi^3|z|^2\mathrm{d}x\mathrm{d}t \leqslant$$

$$C_1\lambda^3\mu^4 \int_0^{\mathrm{T}} \int_{\omega_0} \mathrm{e}^{2\lambda\alpha}\varphi^3|z|^2\mathrm{d}x\mathrm{d}t,\ \forall\, z_T \in L^2(\Omega;\ \mathbb{C}).$$

4.2 Null controllability of linear complex Ginzburg-Landau equations

Combining Lemma 5 and the usual energy estimate, we obtain the following observability estimate for the equation (10).

Proposition 5 There exists a positive constant $\mu_0 = \mu_0(C, a_1, a_2, q, b^{jk})$ such that for any $\mu \geqslant \mu_0$, one can find two constants $C_1 = C_1(\mu) > 0$ and $\lambda_0 = \lambda_0(\mu) > 0$ so that for any $\lambda \geqslant \lambda_0$, the corresponding solution z of the system (10) satisfies

$$\int_\Omega |z(x,0)|^2\mathrm{d}x \leqslant C_1 \int_0^{\mathrm{T}} \int_\omega \mathrm{e}^{2\lambda\alpha}\varphi^3|z|^2\mathrm{d}x\mathrm{d}t,$$

for any $z_T \in L^2(\Omega;\ \mathbb{C})$.

Next, we consider the following linear complex Ginzburg-Landau equation:

$$\begin{cases} (a_1 - ia_2)y_t - \sum\limits_{j,k=1}^{n} (b^{jk}y_{x_j})_{x_k} - qy = \varrho u, \text{ in } Q, \\ y = 0, \text{ on } \Sigma, \\ y(0) = y_0, \text{ in } \Omega. \end{cases} \tag{11}$$

By Proposition 5, we have the following null controllability result for the system (11) with some control function in certain Hölder space.

Proposition 6 For any $y_0 \in L^2(\Omega;\ \mathbb{C})$, there exists a control $u \in C^{\frac{1}{2},\frac{1}{4}}(\bar{Q};\ \mathbb{C})$ such that the corresponding solution of (11) satisfies $y(T) = 0$ in Ω. Moreover, $|u|_{C^{\frac{1}{2},\frac{1}{4}}(\bar{Q};\ \mathbb{C})} \leqslant C^*|y_0|_{L^2(\Omega;\ \mathbb{C})}$, where C^* is a positive constant depending only on $n, T, \omega, \Omega, a_1, a_2,\ |b^{jk}|_{C^1(\bar{Q})}$ and $|q|_{L^\infty(Q)}$.

4.3 Main results

Based on the null controllability of the system (11), by the fixed point technique, we obtain the following local controllability result for the quasilinear complex Ginzburg-Landau system (9).

Theorem 5 There is a positive constant δ_5 such that for any initial value $y_0 \in C^{2+\frac{1}{2}}(\bar{\Omega};\ \mathbb{C})$ satisfying $|y_0|_{C^{2+\frac{1}{2}}(\bar{\Omega};\ \mathbb{C})} \leqslant \delta_5$ and the first order compatibility condition, one can find a control $u \in C^{\frac{1}{2},\frac{1}{4}}(\bar{Q};\ \mathbb{C})$ with supp $u \subseteq \omega \times [0,T]$ such that the corresponding solution y of Eq.(9) satisfies $y(T) = 0$ in Ω.

As a consequence of Theorem 5, we have the following local approximate controllability result.

Theorem 6 There is a positive constant δ_6, such that for any $\varepsilon > 0$, and any given functions $y_0, y_1 \in C^{2+\frac{1}{2}}(\bar{\Omega};\ \mathbb{C})$ satisfying $|y_0|_{C^{2+\frac{1}{2}}(\bar{\Omega};\ \mathbb{C})} + |y_1|_{C^{2+\frac{1}{2}}(\bar{\Omega};\ \mathbb{C})} \leqslant \delta_6$ and the first order compatibility condition, one can find a control $u \in C(\bar{Q};\ \mathbb{C})$ with supp $u \subseteq \omega \times [0,T]$ so that the corresponding solution y of the equation (9) satisfies $|y(T) - y_1|_{C^{2+\frac{1}{2}}(\bar{\Omega};\ \mathbb{C})} < \varepsilon$.

5 The existence of insensitizing controls for quasilinear parabolic equations

In this section, we review the insensitivity results for quasilinear parabolic equations. The detailed proofs of the results in this section can be found in [16].

Consider the following controlled quasilinear parabolic equation:

$$\begin{cases} y_t - \sum\limits_{j,k=1}^{n} (a^{jk}(y)y_{x_j})_{x_k} + f(y) = \eta + \chi_\omega u, \text{ in } Q, \\ y = 0, \text{ on } \Sigma, \\ y(0) = y_0 + \tau\hat{y}_0, \text{ in } \Omega, \end{cases} \tag{12}$$

where η and y_0 are two known functions, τ is an unknown small real number, and $\hat{y}_0$ is an unknown function. In (12), u is the control variable and y is the state variable.

Next, we define the following (partial) energy functional:

$$\Phi(y) = \frac{1}{2} \int_0^{\mathrm{T}}\int_{\mathcal{O}} |y(x,t;\tau,u)|^2\mathrm{d}x\mathrm{d}t, \tag{13}$$

where $y = y(x,t;\tau,u)$ is the corresponding solution of Eq.(12) associated to τ and u. In this section, we are interested in the existence of a control u (depending on η and y_0 but independent of τ and $\hat{y}_0$), which makes the above functional Φ be insensitive with respect to small perturbations on the initial value y_0. A physical interpretation of this problem is as follows: if the state variable y stands for the temperature of a body, then the equation (12) describes the heat conduction of the body, while the diffusion coefficients depend on the temperature in a manner as $a^{jk}(y)$. In Eq.(12), η can be viewed as a given heat source acting on the body, and one can also act on a local domain ω of the body by means of a heat source u. Roughly speaking, the insensitivity problem means that we are expected to find a local heat source u such that the local energy Φ in $\mathcal{O}$ is almost invariant with respect to small perturbations on the initial temperature.

Since we are treating a nonlinear problem, for given functions $\eta \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ and $y_0 \in C^{2+\theta}(\bar{\Omega})$ satisfying suitable conditions (which will be specified later), we require that the desired insensitizing control $u(\in C^{\theta,\frac{\theta}{2}}(\bar{Q}))$, which depends on η and y_0 but is independent of τ and $\hat{y}_0$, satisfies the following condition:

H_3) There exists a $\tau_0 > 0$ such that for any $|\tau| < \tau_0$ and any $\hat{y}_0 \in C_0^\infty(\Omega)$ with $|\hat{y}_0|_{C^{2+\theta}(\bar{\Omega})} = 1$, the equation (12) admits a unique solution $y(\cdot,\cdot;\tau,u) \in C^{2+\theta,1+\frac{\theta}{2}}(\bar{Q})$. Moreover,

$$|y|_{C^{2+\theta,1+\frac{\theta}{2}}(\bar{Q})} \leqslant$$
$$C(n,\Omega,\Gamma,T,a^{jk},f)(|\eta|_{C^{\theta,\frac{\theta}{2}}(\bar{Q})} +$$
$$|u|_{C^{\theta,\frac{\theta}{2}}(\bar{Q})} + |y_0 + \tau\hat{y}_0|_{C^{2+\theta}(\bar{\Omega})}).$$

Now, we introduce the following notion.

Definition 1 For given functions $\eta \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ and $y_0 \in C^{2+\theta}(\bar{\Omega})$, a control $u \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ with supp $u \subseteq$

$\omega \times [0,T]$ is said to insensitize the functional $\varPhi$ if u satisfies condition H$_3$), and

$$\frac{\partial \varPhi(y(\cdot,\cdot;\tau,u))}{\partial \tau}|_{\tau=0}=0,$$

$\forall\, \hat{y}_0 \in C_0^{\infty}(\varOmega)$ with $|\hat{y}_0|_{C^{2+\theta}(\bar{\varOmega})}=1$.

Insensitivity problem was introduced by J.-L. Lions in [17]. In [18], when $\omega \cap \mathcal{O} \neq \emptyset$, for $y_0=0$ and η satisfying suitable assumptions, the existence of insensitizing controls was proved for some semilinear heat equations with globally Lipschitz continuous nonlinearity and Dirichlet boundary conditions. Later, this result was extended to semilinear heat equations with superlinear nonlinearities and other boundary conditions (see [19] and the references therein).

Our main results can be stated as follows.

Theorem 7 Assume that $\omega \cap \mathcal{O} \neq \varnothing$ and $y_0=0$. Then, there exist two positive constants M_1 and δ depending only on $n,\varOmega,\varGamma,T,f(\cdot)$ and $a^{jk}(\cdot)$, such that for any $\eta \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ satisfying

$$\|\eta\|_{C^{\theta,\frac{\theta}{2}}(\bar{Q})}+\|\exp(\frac{M_1}{t(T-t)})\eta\|_{L^2(Q)} \leqslant \delta, \tag{14}$$

one can find a control $u \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$, which insensitizes the functional $\varPhi$ in the sense of Definition 1.

In order to prove the existence of insensitizing controls, as usual, we reduce the problem to a nonstandard null controllability problem of a nonlinear cascade system governed by a quasilinear parabolic equation and a linear parabolic equation, as stated below.

Theorem 8 Assume that $\eta \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ satisfies (14) and $y_0=0$. If a control $u \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ satisfies the condition H$_3$) and the corresponding solution $(w,h) \in (C^{2+\theta,1+\frac{\theta}{2}}(\bar{Q}))^2$ of the following nonlinear cascade system:

$$\begin{cases} w_t-\sum\limits_{j,k=1}^{n}(a^{jk}(w)w_{x_j})_{x_k}+f(w)=\eta+\chi_{\omega}u, \text{ in } Q,\\ w=0, \text{ on } \varSigma,\\ w(0)=0, \text{ in } \varOmega \end{cases} \tag{15}$$

and

$$\begin{cases} -h_t-\sum\limits_{j,k=1}^{n}(a^{jk}(w)h_{x_j})_{x_k}+f'(w)h+\\ \sum\limits_{j,k=1}^{n}(a^{jk})'(w)w_{x_j}h_{x_k}=\chi_{\mathcal{O}}w, \text{ in } Q,\\ h=0, \text{ on } \varSigma,\\ h(T)=0, \text{ in } \varOmega \end{cases} \tag{16}$$

satisfies $h(0)=0$ in $\varOmega$, then u insensitizes the functional $\varPhi$.

In order to prove this controllability result, we need to establish a global Carleman estimate for linear cascade parabolic systems.

5.1 Global Carleman estimate for a linear cascade parabolic system

Consider the following linear cascade parabolic system:

$$\begin{cases} p_t-\sum\limits_{j,k=1}^{n}(b^{jk}p_{x_j})_{x_k}-\\ \quad\sum\limits_{j=1}^{n}(c^jp)_{x_j}+d_1p=0, \text{ in } Q,\\ -q_t-\sum\limits_{j,k=1}^{n}(b^{jk}q_{x_j})_{x_k}+d_2q=\chi_{\mathcal{O}}p, \text{ in } Q,\\ p=q=0, \text{ on } \varSigma,\\ p(0)=p_0,\ q(T)=0, \text{ in } \varOmega, \end{cases} \tag{17}$$

where $d_1,d_2 \in L^{\infty}(Q)$ and $p_0 \in L^2(\varOmega)$.

Put

$$D_1=\sum_{j=1}^{n}|c^j|^2_{C^{1,0}(\bar{Q})}+|d_1|^2_{L^{\infty}(Q)}+|d_2|^2_{L^{\infty}(Q)}.$$

Then by Lemma 2, we have the following global Carleman estimate for the system (17).

Lemma 6 For any $\mu \geqslant CB$ and $\lambda \geqslant C(D_1+\mathrm{e}^{2\mu|\psi|_{C(\bar{\varOmega})}})$, solutions of the equation (17) satisfy the estimate

$$\begin{aligned}&\int_Q \mathrm{e}^{2\lambda\alpha}(\lambda^{-2}\mu^{-2}\varphi|\nabla p|^2+\varphi^3p^2+\\ &\lambda\mu^2\varphi|\nabla q|^2+\lambda^3\mu^4\varphi^3q^2)\mathrm{d}x\mathrm{d}t \leqslant\\ &C[1+(B+D_1)^3]\int_0^{\mathrm{T}}\int_{\omega_1}\mathrm{e}^{2\lambda\alpha}\lambda^4\mu^4\varphi^7q^2\mathrm{d}x\mathrm{d}t,\\ &\forall\, p_0 \in L^2(\varOmega).\end{aligned}$$

5.2 Controllability of a linear cascade parabolic system in Hölder spaces

As a key preliminary to prove Theorem 8, we need to establish the null controllability for the following linear cascade parabolic system, with a control function in certain Hölder space:

$$\begin{cases} w_t-\sum\limits_{j,k=1}^{n}(b^{jk}w_{x_j})_{x_k}+d_2w=\eta+\varrho u, \text{ in } Q,\\ -h_t-\sum\limits_{j,k=1}^{n}(b^{jk}h_{x_j})_{x_k}+\sum\limits_{j=1}^{n}c^jh_{x_j}+\\ d_1h=\chi_{\mathcal{O}}w, \text{ in } Q,\\ w=h=0, \text{ on } \varSigma,\\ w(0)=0,\ h(T)=0, \text{ in } \varOmega, \end{cases} \tag{18}$$

where $\eta \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$.

By Lemma 6, we have the following null controllability result for the system (18).

Proposition 7 For any given function η satisfying $|\mathrm{e}^{\frac{\widetilde{M}}{t(T-t)}}\eta|_{L^2(Q)}<\infty$ with $\widetilde{M}=C(D_1+\mathrm{e}^{CB})^2$, one can find a control $u \in C^{\theta,\frac{\theta}{2}}(\bar{Q})$ with supp $u \subseteq \omega \times [0,T]$, such that the corresponding solution of the system (18) satisfies $h(0)=0$. Moreover,

$$|u|_{C^{\theta,\frac{\theta}{2}}(\bar{Q})} \leqslant C_*(\int_Q \mathrm{e}^{\frac{2\widetilde{M}}{t(T-t)}}\eta^2\mathrm{d}x\mathrm{d}t)^{\frac{1}{2}},$$

where

$$\begin{aligned}C_*=C\exp\{C(1+\sum_{j,k=1}^{n}|b^{jk}|^8_{C^1(\bar{Q})}+\\ \sum_{j=1}^{n}|c^j|^8_{C^{1,0}(\bar{Q})}+|d_1|^4_{L^{\infty}(Q)}+|d_2|^4_{L^{\infty}(Q)})\}.\end{aligned}$$

Finally, based on the null controllability of the system (18), proceeding similar analysis as [14, Theorem 1.1], one can get the existence of insensitizing controls for the quasilinear parabolic system (12) (Theorem 7).

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