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# 转移概率一般不确知时滞Markov跳变神经网络的同步

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**摘要**: 研究了一类具有一般不确知转移概率的时滞Markov跳变神经网络的渐近同步问题. 此类系统跳变过程的转移概率完全未知或者仅知道其估计值, 因而更具有一般性. 通过选择适当的Lyapunov-Krasovskii函数, 利用线性矩阵不等式(LMIs)方法, 得到了系统均方渐近同步的充分条件. 最后, 数值例子说明了所给结果的有效性.

**关键词**: 同步; 一般不确知转移概率; Markov跳变神经网络; Kronecker积

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# Synchronization for Markov jump neural networks with generally uncertain transition rates

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**Abstract:** This paper investigates the asymptotical synchronization problem for a class of Markov jump neural networks (MJNNs) with generally uncertain transition rates (GUTRs). In the GUTR neural network model, each transition rate may be completely unknown or only its estimate value is known. This new uncertain model could be applied in many practical cases. Based on the Lyapunov-Krasovskii function method, a sufficient condition for the asymptotical synchronization in mean square is derived in terms of linear matrix inequalities (LMIs). Finally, a numerical example is presented to illustrate the effectiveness of the developed method.

**Key words:** synchronization; generally uncertain transition rates; Markov jump neural networks; Kronecker product

## 1　引言(Introduction)

Markov跳变神经网络系统是描述系统在运行过程中受到随机故障, 环境的突变等干扰导致系统中的一些参数产生突变, 使得系统按照一定的切换规则在多个模态之间进行切换的混杂系统. 因为Markov跳变神经网络具有广泛的应用背景, 国内外学者对其进行了大量的研究, 并取得了丰硕的成果[1–8]. 遗憾的是, 在已有的文献中, 大部分成果都假设转移概率完全已知[3–4, 7–8]. 然而, 由于系统在运行过程中往往存在不确定因素以及测量条件的制约, 导致准确获取转移概率全部信息十分困难. 近年来, 有学者也讨论了转移概率不完全确知的情况[9–13], 包括有界不确知和部分不确知两种情形. 其中有界不确知模型中要精确知道未知部分的上下界; 部分不确知模型中仅考虑了转移概率部分完全已知, 部分完全未知的情形, 这使得模型在应用中有一定的局限性. 针对以上问题, 文献[14]将转移概率推广到更一般的情形, 研究了带有一般不确知转移概率Markov跳变系统的稳定性问题. 这种模型中的转移概率或者完全未知, 或者仅知其估计值, 而且这种一般的不确知模型包含了有界不确知和部分不确知两种情况[14], 因而具有更广泛的应用. 另一方面, 由于混沌同步在通讯、物理、信息科学、生物工程等领域有广阔的应用前景, 引起了人们的广泛关注, 已经成为非线性科学领域的一个热门研究课题.

本文主要讨论一类转移概率是一般不确知的时滞Markov跳变神经网络系统的同步问题. 通过选择适当的 Lyapunov-Krasovskii 函数, 利用 Kronecker 积和一些不等式技巧, 得到了系统均方渐近同步的充分条件, 该条件可以利用Matlab的LMI工具箱非常方便的求解. 最后的仿真结果说明文中所给方法的有效性.

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## 2　问题的描述(Discription of the problem)

考虑由$m$个不同节点组成的神经网络系统. 每个节点都可用下面参数依赖于Markov跳的$n$维时滞动力系统来描述:

$$\dot{x}_k(t)=A(r(t))f(x_k(t))+B(r(t))f(x_k(t-\tau(t)))+\sum_{l=1}^{m}g_{kl}(r(t))\Gamma_1 x_l(t)+\sum_{l=1}^{m}h_{kl}(r(t))\Gamma_2\cdot x_l(t-\tau(t))+C(r(t))\int_{t-\sigma(t)}^{t}f(x_k(s))\mathrm{d}s, \tag{1}$$

其中: $x_k(t)=(x_{k1},x_{k2},\cdots,x_{kn})^{\mathrm{T}}\in\mathbb{R}^n$表示第$k$个节点的状态向量; $\Gamma_1,\Gamma_2$表示子系统间内部耦合矩阵; $A_i=A(r(t))\in\mathbb{R}^{n\times n}, B_i=B(r(t))\in\mathbb{R}^{n\times n}$表示第$i$个模态的系数矩阵; $G_i=(g_{kl}(r(t)))\in\mathbb{R}^{m\times m}$, $H_i=(h_{kl}(r(t)))\in\mathbb{R}^{m\times m}$表示第$i$个模态下的线性耦合结构矩阵, 并且满足:

$$g_{kl}=g_{lk}\geqslant 0, l\neq k, g_{kk}(r(t))=-\sum_{l=1,l\neq k}^{m}g_{kl}(r(t)),$$

$$h_{kl}=h_{lk}\geqslant 0, l\neq k, h_{kk}(r(t))=-\sum_{l=1,l\neq k}^{m}h_{kl}(r(t)).$$

时滞满足$0<\tau(t)\leqslant\tau, \dot{\tau}(t)\leqslant\mu, 0<\sigma(t)\leqslant\sigma$, 记$\eta=\max(\tau,\sigma)$. 设$x(t)=[x_1^{\mathrm{T}}(t),x_2^{\mathrm{T}}(t),\cdots,x_m^{\mathrm{T}}(t)]^{\mathrm{T}}$, $F(x(t))=[f^{\mathrm{T}}(x_1(t)),f^{\mathrm{T}}(x_2(t)),\cdots,f^{\mathrm{T}}(x_m(t))]^{\mathrm{T}}$. 通过利用Kronecker积, 系统(1)可写成下面的形式

$$\dot{x}(t)=(I_m\otimes A_i)F(x(t))+(I_m\otimes B_i)F(x(t-\tau(t)))+(G_i\otimes\Gamma_1)x(t)+(H_i\otimes\Gamma_2)x(t-\tau(t))+(I_m\otimes C_i)\int_{t-\sigma(t)}^{t}F(x(s))\mathrm{d}s, \tag{2}$$

设$\{r(t),t\geqslant 0\}$是在有限集合$S=\{1,2,\cdots,s\}$中取值的Markov跳变过程, 其跳变转移概率矩阵是$\Pi=(\pi_{ij})(i,j\in S)$, 满足

$$\Pr\{r(t+\Delta t)=j|r(t)=i\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j,\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j.\end{cases}$$

其中$\Delta t>0$, $\lim\limits_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0, \pi_{ij}\geqslant 0$是从$t$时刻的模态$i$跳变到$t+\Delta t$时刻的模态$j$的转移概率, 当$i\neq j$时, 有$\pi_{ii}=-\sum\limits_{j=1,j\neq i}^{s}\pi_{ij}$. 本文假设Markov跳变过程的转移概率是一般不确知的, 如某个包含$s$个模式的系统具有如下转移概率矩阵:

$$\begin{pmatrix}\hat{\pi}_{11}+\Delta_{11} & ? & ?\cdots & ?\\ ? & ? & ?\cdots & \hat{\pi}_{2s}+\Delta_{2s}\\ \vdots & \vdots & \vdots & \vdots\\ ? & \hat{\pi}_{s2}+\Delta_{s2} & ?\ldots & \hat{\pi}_{ss}+\Delta_{ss}\end{pmatrix}, \tag{3}$$

其中$\hat{\pi}_{ij}$和$\Delta_{ij}\in[-\delta_{ij},\delta_{ij}](\delta_{ij}\geqslant 0)$分别表示未知转移概率$\pi_{ij}$的估计值和误差, 并且$\hat{\pi}_{ij}$和$\delta_{ij}$已知. “?”表示完全未知的转移概率. 为方便起见, $\forall i\in S$, 定义$S^i=S_k^i\bigcup S_{uk}^i$, 并且

$$S_k^i=\{j\in S:\pi_{ij}\text{的估计值已知}\},$$

$$S_{uk}^i=\{j\in S:\pi_{ij}\text{的估计值未知}\}.$$

如果$S_k^i\neq\varnothing$, 进一步设

$$S_k^i=\{k_1^i,k_2^i,\cdots,k_m^i\},$$

其中$k_m^i\in\mathbb{N}^+$表示转移概率矩阵$\Pi$的第$i$行中第$m$个已知的元素.

本文给出以下假设:

**假设 1**　如果$i\in S_k^i, S_{uk}^i\neq\varnothing$, 则$\forall j\in S_k^i, j\neq i$, $\hat{\pi}_{ij}-\delta_{ij}\geqslant 0; \hat{\pi}_{ii}+\delta_{ii}\leqslant 0$, 且$\sum\limits_{j\in S_k^i}\hat{\pi}_{ij}\leqslant 0$.

**假设 2**　如果$i\notin S_k^i, S_{uk}^i\neq S$, 则$\forall j\in S_k^i, \hat{\pi}_{ij}-\delta_{ij}\geqslant 0$.

**假设 3**　如果$S_k^i=S, S_{uk}^i=\varnothing$, 则$\forall j\in S, j\neq i$, $\hat{\pi}_{ij}-\delta_{ij}\geqslant 0; \hat{\pi}_{ii}=-\sum\limits_{j=1,j\neq i}^{s}\hat{\pi}_{ij}\leqslant 0$, 并且

$$\delta_{ii}=\sum_{j=1,j\neq i}^{s}\delta_{ij}>0.$$

**假设 4**[3]　对$\forall x,y\in\mathbb{R}^n$, $f(\cdot)$满足

$$(f(x_k(t))-f(x_l(t))-D_1(x_k(t)-x_l(t)))^{\mathrm{T}}\cdot(f(x_k(t))-f(x_l(t))-D_2(x_k(t)-x_l(t)))\leqslant 0; \tag{4}$$

$$(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t)))-Z_1(x_k(t-\tau(t))-x_l(t-\tau(t))))^{\mathrm{T}}\cdot(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t)))-Z_2(x_k(t-\tau(t))-x_l(t-\tau(t))))\leqslant 0, \tag{5}$$

其中$D_1,D_2,Z_1,Z_2$是已知的常数矩阵.

**定义 1**　对系统(2)任意的解$x(t)$, 如果满足$\lim\limits_{t\to\infty}\mathrm{E}\{|x_k(t)-x_l(t)|^2\}=0, k,l\in\{1,2,\cdots,m\}$, 则称系统(2)为均方渐近同步的.

**引理 1**[15]　Kronecker积满足如下性质:

1) $(\alpha A)\otimes B=A\otimes(\alpha B)$;

2) $(A+B)\otimes C=A\otimes C+B\otimes C$;

3) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$.

**引理 2**[16]　设$U=(u_{kl})_{m\times m}, P\in\mathbb{R}^{n\times n}, x^{\mathrm{T}}=[x_1^{\mathrm{T}},x_2^{\mathrm{T}},\cdots,x_n^{\mathrm{T}}]$和$y^{\mathrm{T}}=[y_1^{\mathrm{T}},y_2^{\mathrm{T}},\cdots,y_n^{\mathrm{T}}]$, 若$U=U^{\mathrm{T}}$且$U$每行的和为零, 则

$$x^{\mathrm{T}}(U\otimes P)y=-\sum_{1\leqslant k<l\leqslant m}u_{kl}(x_k-x_l)^{\mathrm{T}}P(y_k-y_l).$$

**引理 3** (Jensen's inequality)　对任意常数矩阵$G>0$, 标量$a$和$b$且$a<b$, 向量值函数$g(\cdot):[a,b]\to$

$\mathbb{R}^n$使得下列积分是存在的, 则下列不等式成立:

$$[\int_a^b g(s)\mathrm{d}s]^{\mathrm{T}}G[\int_a^b g(s)\mathrm{d}s] \leqslant (b-a)[\int_a^b g^{\mathrm{T}}(s)Gg(s)\mathrm{d}s].$$

**引理 4**[17] 给定任意实数$\varepsilon$和矩阵$R$, 对任意正定矩阵$M$有, $\varepsilon(R+R^{\mathrm{T}}) \leqslant \varepsilon^2 M + RM^{-1}R^{\mathrm{T}}$成立.

**引理 5** (Schur补引理) 设对称矩阵$F=F^{\mathrm{T}}$的分块表示为$F=\begin{bmatrix} A & B^{\mathrm{T}} \\ B & C \end{bmatrix}$. 这里, $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{m\times n}$, $C\in\mathbb{R}^{m\times m}$, 则下面的条件是等价的:

1) $F<0$,

2) $C<0, A-B^{\mathrm{T}}C^{-1}B<0$.

## 3 主要结果(Main results)

**定理 1** 在假设1–4下, 系统(2)具有一般不确知转移概率(3), 则系统是均方渐近同步的, 如果存在正定矩阵$Q_1,Q_2,Q_3$, 一组正定矩阵$P_i$和标量$\beta_1>0$, $\beta_2>0$, 使得下面的线性矩阵不等式成立:

1) 若$i\in S_k^i$, 即$\pi_{ii}$的估计值和估计误差界已知, 且$S_{uk}^i\neq\varnothing$. 设$S_k^i=\{k_1^i,k_2^i,\cdots,k_s^i\}$, 则存在正定矩阵$W_{ij\gamma}\in\mathbb{R}^{n\times n}(i,j\in S_k^i,\gamma\in S_{uk}^i)$, 使得

$$\varPi_1=\begin{bmatrix}
\varPi_{11} & mh_{kl}(i)P_i\varGamma_2 & P_iA_i+\beta_1\bar{D}_2 & P_iB_i & P_iC_i & P_{k_1^i}-P_\gamma & \cdots & P_{k_s^i}-P_\gamma \\
* & (\mu-1)Q_1-\beta_2\bar{Z}_1 & 0 & \beta_2\bar{Z}_2 & 0 & 0 & \cdots & 0 \\
* & * & Q_2+\eta^2Q_3-\beta_1I & 0 & 0 & 0 & \cdots & 0 \\
* & * & * & (\mu-1)Q_2-\beta_2I & 0 & 0 & \cdots & 0 \\
* & * & * & * & -Q_3 & 0 & \cdots & 0 \\
* & * & * & * & * & -W_{ik_1^i\gamma} & \cdots & 0 \\
* & * & * & * & * & * & \ddots & 0 \\
* & * & * & * & * & * & \cdots & -W_{ik_s^i\gamma}
\end{bmatrix}<0. \quad (6)$$

2) 若$i\notin S_k^i$, 即$\pi_{ii}$的估计值和估计误差界未知, 且$S_{uk}^i\neq S$. 设$S_k^i=\{k_1^i,k_2^i,\cdots,k_m^i\}$, 则存在正定矩阵$T_{ij}\in\mathbb{R}^{n\times n}(i\notin S_k^i,j\in S_k^i)$, 使得

$$\varOmega_1=\begin{bmatrix}
\varOmega_{11} & mh_{kl}(i)P_i\varGamma_2 & P_iA_i+\beta_1\bar{D}_2 & P_iB_i & P_iC_i & P_{k_1^i}-P_i & \cdots & P_{k_m^i}-P_i \\
* & (\mu-1)Q_1-\beta_2\bar{Z}_1 & 0 & \beta_2\bar{Z}_2 & 0 & 0 & \cdots & 0 \\
* & * & Q_2+\eta^2Q_3-\beta_1I & 0 & 0 & 0 & \cdots & 0 \\
* & * & * & (\mu-1)Q_2-\beta_2I & 0 & 0 & \cdots & 0 \\
* & * & * & * & -Q_3 & 0 & \cdots & 0 \\
* & * & * & * & * & -T_{ik_1^i} & \cdots & 0 \\
* & * & * & * & * & * & \ddots & 0 \\
* & * & * & * & * & * & \cdots & -T_{ik_m^i}
\end{bmatrix}<0. \quad (7)$$

3) 若$S_k^i=S$, 即$\pi_{ii}$的估计值和估计误差界已知, 且$S_{uk}^i=\varnothing$, 则存在正定矩阵$R_{ij}\in\mathbb{R}^{n\times n}(i,j\in S_k^i)$, 使得

$$\varPsi_1=\begin{bmatrix}
\varPsi_{11} & mh_{kl}(i)P_i\varGamma_2 & P_iA_i+\beta_1\bar{D}_2 & P_iB_i & P_iC_i & P_1-P_i & \cdots & P_{i-1}-P_i & P_{i+1}-P_i & \cdots & P_s-P_i \\
* & (\mu-1)Q_1-\beta_2\bar{Z}_1 & 0 & \beta_2\bar{Z}_2 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & Q_2+\eta^2Q_3-\beta_1I & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & (\mu-1)Q_2-\beta_2I & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & * & -Q_3 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & * & * & -R_{i1} & \cdots & 0 & 0 & \cdots & 0 \\
* & * & * & * & * & * & \ddots & 0 & 0 & \cdots & 0 \\
* & * & * & * & * & * & \cdots & R_{i(i-1)} & 0 & \cdots & 0 \\
* & * & * & * & * & * & \cdots & * & R_{i(i+1)} & \cdots & 0 \\
* & * & * & * & * & * & \cdots & * & * & \ddots & 0 \\
* & * & * & * & * & * & \cdots & * & * & \cdots & -R_{is}
\end{bmatrix}<0. \quad (8)$$

其中:

$$\varPi_{11}=\varXi+\sum_{j\in S_k^i}(\hat{\pi}_{ij}(P_j-P_\gamma)+\frac{\delta_{ij}^2W_{ij\gamma}}{4}),\ \ \varOmega_{11}=\varXi+\sum_{j\in S_k^i}(\hat{\pi}_{ij}(P_j-P_i)+\frac{\delta_{ij}^2T_{ij}}{4}),$$

$$\Psi_{11}=\Xi+\sum_{j=1,j\neq i}^{s}(\hat{\pi}_{ij}(P_j-P_i)+\frac{\delta_{ij}^2R_{ij}}{4}),$$

$$\Xi=Q_1+mg_{kl}(i)P_i\varGamma_1+mg_{kl}(i)\varGamma_1^{\mathrm{T}}P_i^{\mathrm{T}}-\beta_1\bar{D}_1,$$

$$\bar{D}_1=\frac{D_1^{\mathrm{T}}D_2+D_2^{\mathrm{T}}D_1}{4},\bar{D}_2=\frac{D_1^{\mathrm{T}}+D_2^{\mathrm{T}}}{2},$$

$$\bar{Z}_1=\frac{Z_1^{\mathrm{T}}Z_2+Z_2^{\mathrm{T}}Z_1}{4},\bar{Z}_2=\frac{Z_1^{\mathrm{T}}+Z_2^{\mathrm{T}}}{2}.$$

**证**　选择如下的Lyapunov-Krasovskii函数:

$$\begin{aligned}&V(x(t),t,i)=\\&x^{\mathrm{T}}(t)(U\otimes P_i)x(t)+\\&\int_{t-\tau(t)}^{t}x^{\mathrm{T}}(s)(U\otimes Q_1)x(s)\mathrm{d}s+\\&\int_{t-\tau(t)}^{t}F^{\mathrm{T}}(x(s))(U\otimes Q_2)F(x(s))\mathrm{d}s+\\&\eta\int_{0}^{\eta}\int_{t-s}^{t}F^{\mathrm{T}}(x(\theta))(U\otimes Q_3)F(x(\theta))\mathrm{d}\theta\mathrm{d}s,\end{aligned}\tag{9}$$

其中

$$U=\begin{bmatrix}m-1&-1&\cdots&-1\\-1&m-1&\cdots&-1\\\vdots&\vdots&&\vdots\\-1&-1&\cdots&m-1\end{bmatrix}_{m\times m}.$$

设$L$是弱无穷小算子, 则

$$\begin{aligned}&LV(x(t),t,i)=\\&\dot{x}^{\mathrm{T}}(t)\frac{\partial V}{\partial x}+\frac{\partial V}{\partial t}+\sum_{j=1}^{s}\pi_{ij}V(x(t),t,j)=\\&x^{\mathrm{T}}(t)(U\otimes Q_1)x(t)-\\&(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))(U\otimes Q_1)x(t-\tau(t))+\\&F^{\mathrm{T}}(x(t))(U\otimes Q_2)F(x(t))-(1-\dot{\tau}(t))\cdot\\&F^{\mathrm{T}}(x(t-\tau(t)))(U\otimes Q_2)F(x(t-\tau(t)))+\\&\eta^2F^{\mathrm{T}}(x(t))(U\otimes Q_3)F(x(t))-\\&\eta\int_{t-\eta}^{t}F^{\mathrm{T}}(x(s))(U\otimes Q_3)F(x(s))\mathrm{d}s+\\&\dot{x}^{\mathrm{T}}(t)(U\otimes P_i)x(t)+x^{\mathrm{T}}(t)(U\otimes P_i)\dot{x}(t)+\\&x^{\mathrm{T}}(t)\sum_{j=1}^{s}\pi_{ij}(U\otimes P_j)x(t)\leqslant\\&x^{\mathrm{T}}(t)((U\otimes Q_1)+\sum_{j=1}^{s}\pi_{ij}(U\otimes P_j))x(t)-\\&(1-\mu)x^{\mathrm{T}}(t-\tau(t))(U\otimes Q_1)x(t-\tau(t))+\\&F^{\mathrm{T}}(x(t))(U\otimes Q_2+\eta^2U\otimes Q_3)F(x(t))-\\&(1-\mu)F^{\mathrm{T}}(x(t-\tau(t)))(U\otimes Q_2)F(x(t-\tau(t)))-\\&\eta\int_{t-\eta}^{t}F^{\mathrm{T}}(x(s))(U\otimes Q_3)F(x(s))\mathrm{d}s+\\&2x^{\mathrm{T}}(t)[(U\otimes P_iA_i)F(x(t))+\\&(U\otimes P_iB_i)F(x(t-\tau(t)))+(mG_i\otimes P_i\varGamma_1)x(t)+\\&(mH_i\otimes P_i\varGamma_2)x(t-\tau(t))+\\&(U\otimes P_iC_i)\int_{t-\sigma(t)}^{t}F(x(s))\mathrm{d}s],\end{aligned}$$

由引理1-3, 得

$$\begin{aligned}&LV(x(t),t,i)\leqslant\\&\sum_{1\leqslant k<l\leqslant m}^{s}\{(x_k(t)-x_l(t))^{\mathrm{T}}(Q_1+mg_{kl}(i)P_i\varGamma_1+\\&mg_{kl}(i)\varGamma_1^{\mathrm{T}}P_i^{\mathrm{T}}+\sum_{j=1}^{s}\pi_{ij}P_j)(x_k(t)-x_l(t))-\\&(1-\mu)(x_k(t-\tau(t))-x_l(t-\tau(t)))^{\mathrm{T}}\cdot\\&Q_1(x_k(t-\tau(t))-x_l(t-\tau(t)))+\\&(f(x_k(t))-f(x_l(t)))^{\mathrm{T}}(Q_2+\eta^2Q_3)\cdot\\&(f(x_k(t))-f(x_l(t)))-\\&(1-\mu)(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t))))^{\mathrm{T}}\cdot\\&Q_2(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t))))-\\&[\int_{t-\sigma(t)}^{t}(f(x_k(s))-f(x_l(s)))\mathrm{d}s]^{\mathrm{T}}Q_3\cdot\\&[\int_{t-\sigma(t)}^{t}(f(x_k(s))-f(x_l(s)))\mathrm{d}s]+\\&2(x_k(t)-x_l(t))^{\mathrm{T}}[P_iA_i(f(x_k(t))-f(x_l(t)))+\\&P_iB_i(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t))))+\\&mh_{kl}(i)P_i\varGamma_2(x_k(t-\tau(t))-x_l(t-\tau(t))))+\\&P_iC_i\int_{t-\sigma(t)}^{t}(f(x_k(s))-f(x_l(s)))\mathrm{d}s]\},\end{aligned}\tag{10}$$

根据假设4, 得

$$\begin{aligned}&(f(x_k(t))-f(x_l(t))-D_1(x_k(t)-x_l(t)))^{\mathrm{T}}\cdot\\&(f(x_k(t))-f(x_l(t))-D_2(x_k(t)-x_l(t)))=\\&\begin{bmatrix}x_k(t)-x_l(t)\\f(x_k(t))-f(x_l(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{D}_1&-\bar{D}_2\\-\bar{D}_2^{\mathrm{T}}&I\end{bmatrix}\cdot\\&\begin{bmatrix}x_k(t)-x_l(t)\\f(x_k(t))-f(x_l(t))\end{bmatrix}\leqslant0,\end{aligned}\tag{11}$$

$$\begin{aligned}&(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t)))-\\&Z_1((x_k(t-\tau(t)))-x_l(t-\tau(t))))^{\mathrm{T}}\cdot\\&(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t)))-\\&Z_2((x_k(t-\tau(t)))-x_l(t-\tau(t))))=\\&\begin{bmatrix}x_k(t-\tau(t))-x_l(t-\tau(t))\\f(x_k(t-\tau(t)))-f(x_l(t-\tau(t)))\end{bmatrix}^{\mathrm{T}}\cdot\\&\begin{bmatrix}\bar{Z}_1&-\bar{Z}_2\\-\bar{Z}_2^{\mathrm{T}}&I\end{bmatrix}\cdot\\&\begin{bmatrix}x_k(t-\tau(t))-x_l(t-\tau(t))\\f(x_k(t-\tau(t)))-f(x_l(t-\tau(t)))\end{bmatrix}\leqslant0,\end{aligned}\tag{12}$$

设$\beta_1>0,\beta_2>0$, 由式(11) − (12)得

$$\begin{bmatrix}x_k(t)-x_l(t)\\f(x_k(t))-f(x_l(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-\beta_1\bar{D}_1&\beta_1\bar{D}_2\\\beta_1\bar{D}_2^{\mathrm{T}}&-\beta_1I\end{bmatrix}\cdot$$

$$\begin{bmatrix} x_k(t)-x_l(t) \\ f(x_k(t))-f(x_l(t)) \end{bmatrix} \geqslant 0, \tag{13}$$

$$\begin{bmatrix} x_k(t-\tau(t))-x_l(t-\tau(t)) \\ f(x_k(t-\tau(t)))-f(x_l(t-\tau(t))) \end{bmatrix}^{\mathrm T}\cdot \begin{bmatrix} -\beta_2\bar Z_1 & \beta_2\bar Z_2 \\ \beta_2\bar Z_2^{\mathrm T} & -\beta_2 I \end{bmatrix}\cdot \begin{bmatrix} x_k(t-\tau(t))-x_l(t-\tau(t)) \\ f(x_k(t-\tau(t)))-f(x_l(t-\tau(t))) \end{bmatrix} \geqslant 0, \tag{14}$$

由式(10)(13)–(14), 得

$$LV(x(t),t,i)\leqslant \sum_{1\leqslant k<l\leqslant m}\xi^{\mathrm T}(t)\varPhi_1\xi(t)<0, \tag{15}$$

其中

$$\begin{aligned}\xi(t)=[&(x_k(t)-x_l(t))^{\mathrm T}(x_k(t-\tau(t))-\\ &x_l(t-\tau(t)))^{\mathrm T}(f(x_k(t))-f(x_l(t)))^{\mathrm T}\cdot\\ &(f(x_k(t-\tau(t)))-f(x_l(t-\tau(t))))^{\mathrm T}\cdot\\ &(\int_{t-\sigma(t)}^{t}(f(x_k(s))-f(x_l(s)))\mathrm ds)^{\mathrm T}]^{\mathrm T},\end{aligned}$$

$$\varPhi_1=\begin{bmatrix} \varPhi_{11} & mh_{kl}(i)P_i\varGamma_2 & P_iA_i+\beta_1\bar D_2 & P_iB_i & P_iC_i \\ * & (\mu-1)Q_1-\beta_2\bar Z_1 & 0 & \beta_2\bar Z_2 & 0 \\ * & * & Q_2+\eta^2Q_3-\beta_1 I & 0 & 0 \\ * & * & * & (\mu-1)Q_2-\beta_2 I & 0 \\ * & * & * & * & -Q_3 \end{bmatrix}<0,$$

$$\varPhi_{11}=\varXi+\sum_{j=1}^{s}\pi_{ij}P_j,$$

$$\varXi=Q_1+mg_{kl}(i)P_i\varGamma_1+mg_{kl}(i)\varGamma_1^{\mathrm T}P_i^{\mathrm T}-\beta_1\bar D_1.$$

下面分3种情况讨论:

1) 若$i\in S_k^i$, $S_{uk}^i\neq\varnothing$, 设$S_k^i=\{k_1^i,k_2^i,\cdots,k_s^i\}$, 则$\exists\gamma\in S_{uk}^i,\forall j\in S_{uk}^i$, 满足$P_\gamma-P_j\geqslant 0$. 令

$$\varPi_i\triangleq\varXi+\sum_{j\in S_k^i}\pi_{ij}P_j+\sum_{j\in S_{uk}^i}\pi_{ij}P_j.$$

因为 $\pi_{ii}=-\sum\limits_{j=1,j\neq i}^{s}\pi_{ij}\leqslant 0,\ \pi_{ii}\in S_k^i,$

$$\sum_{j\in S_{uk}^i}\pi_{ij}=-\sum_{j\in S_k^i}\pi_{ij},$$

以及假设1, 得

$$\begin{aligned}\varPi_i\leqslant\ &\varXi+\sum_{j\in S_k^i}\pi_{ij}P_j+\sum_{j\in S_{uk}^i}\pi_{ij}P_\gamma=\\ &\varXi+\sum_{j\in S_k^i}\pi_{ij}P_j+(-\sum_{j\in S_k^i}\pi_{ij}P_\gamma)=\\ &\varXi+\sum_{j\in S_k^i}\pi_{ij}(P_j-P_\gamma)=\\ &\varXi+\sum_{j\in S_k^i}(\hat\pi_{ij}+\delta_{ij})(P_j-P_\gamma),\end{aligned}$$

利用引理4, 得

$$\begin{aligned}&\sum_{j\in S_k^i}\varDelta_{ij}(P_j-P_\gamma)=\\ &\sum_{j\in S_k^i}[\frac12\varDelta_{ij}(P_j-P_\gamma)+\frac12\varDelta_{ij}(P_j-P_\gamma)]\leqslant\\ &\sum_{j\in S_k^i}[\frac14\delta_{ij}^2W_{ij\gamma}+(P_j-P_\gamma)W_{ij\gamma}^{-1}(P_j-P_\gamma)^{\mathrm T}].\end{aligned}$$

所以

$$\begin{aligned}\varPi_i\leqslant\ &\varXi+\sum_{j\in S_k^i}\hat\pi_{ij}(P_j-P_\gamma)+\sum_{j\in S_k^i}\frac14\delta_{ij}^2W_{ij\gamma}+\\ &\sum_{j\in S_k^i}[(P_j-P_\gamma)W_{ij\gamma}^{-1}(P_j-P_\gamma)^{\mathrm T}].\end{aligned}$$

只要

$$\begin{aligned}&\varXi+\sum_{j\in S_k^i}\hat\pi_{ij}(P_j-P_\gamma)+\sum_{j\in S_k^i}\frac14\delta_{ij}^2W_{ij\gamma}+\\ &\sum_{j\in S_k^i}[(P_j-P_\gamma)W_{ij\gamma}^{-1}(P_j-P_\gamma)^{\mathrm T}]<0,\end{aligned}$$

则$\varPi_i<0$.

由引理5得, 式(6)成立.

2) 若$i\notin S_k^i$, 设$S_k^i=\{k_1^i,k_2^i,\cdots,k_m^i\}$. 由假设2,

$$\begin{aligned}\varOmega_i\triangleq\ &\varXi+\sum_{j\in S_k^i}\pi_{ij}P_j+\sum_{j\in S_{uk}^i,j\neq i}\pi_{ij}P_j+\pi_{ii}P_i\leqslant\\ &\varXi+\sum_{j\in S_k^i}\pi_{ij}P_j-(\pi_{ii}+\sum_{j\in S_k^i}\pi_{ij})P_i+\pi_{ii}P_i=\\ &\varXi+\sum_{j\in S_k^i}\pi_{ij}(P_j-P_i)=\\ &\varXi+\sum_{j\in S_k^i}(\hat\pi_{ij}+\varDelta_{ij})(P_j-P_i).\end{aligned}$$

由引理4, 容易得到

$$\begin{aligned}\varOmega_i\leqslant\ &\varXi+\sum_{j\in S_k^i}\hat\pi_{ij}(P_j-P_i)+\sum_{j\in S_k^i}\frac14\delta_{ij}^2T_{ij}+\\ &\sum_{j\in S_k^i}((P_j-P_i)T_{ij}^{-1}(P_j-P_i)^{\mathrm T}).\end{aligned}$$

由引理5, 式(7)成立.

3) 若$S_k^i=S$, $S_{uk}^i=\varnothing$,

$$\begin{aligned}\varPsi_i\triangleq\ &\varXi+\sum_{j=1}^{s}\pi_{ij}P_j=\\ &\varXi+\sum_{j=1,j\neq i}^{s}\pi_{ij}P_j+\pi_{ii}P_i=\\ &\varXi+\sum_{j=1,j\neq i}^{s}\pi_{ij}P_j+(-\sum_{j=1,j\neq i}^{s}\pi_{ij})P_i=\\ &\varXi+\sum_{j=1,j\neq i}^{s}\pi_{ij}(P_j-P_i)=\\ &\varXi+\sum_{j=1,j\neq i}^{s}\hat\pi_{ij}(P_j-P_i)+\end{aligned}$$

$$\sum_{j=1,j\neq i}^{s} \varDelta_{ij}(P_j - P_i).$$

另外, 由引理4, 得到

$$\Psi_i \leqslant \varXi + \sum_{j=1,j\neq i}^{s} \hat{\pi}_{ij}(P_j - P_i) + \sum_{j\in S_k^i} \frac{1}{4}\delta_{ij}^2 R_{ij} + \sum_{j=1,j\neq i}^{s} (P_j - P_i)R_{ij}^{-1}(P_j - P_i)^{\mathrm{T}}.$$

由引理5, 式(8)成立. 由式(15)得

$$\mathrm{E}\{LV(x(t),t,i)\} \leqslant \sum_{j\leqslant k<l\leqslant m} \mathrm{E}\{\xi^{\mathrm{T}}(t)\varPhi_1\xi(t)\} < 0.$$

证毕.

**注 1**　若$\forall i \in S, \forall j \in S_k^i, \delta_{ij} = 0$, 则转移概率矩阵(3)就退化为部分不确知的情况, 此时转移概率矩阵为

$$\begin{pmatrix} \pi_{11} & ? & ? & \cdots & ? \\ ? & ? & ? & \cdots & \pi_{2s} \\ \vdots & \vdots & \vdots & & \vdots \\ ? & \pi_{s2} & ? & \vdots & \pi_{ss} \end{pmatrix},$$

因此, 定理1也可用在转移概率矩阵为部分不确知的模型中.

## 4　数值算例(Numerical example)

给出一个数值仿真例子来说明本文所提方法的有效性. 设系统参数为

$$A_1 = \begin{bmatrix} 1.4 & -0.9 \\ -0.2 & 2.4 \end{bmatrix}, A_2 = \begin{bmatrix} 2.4 & -0.5 \\ -0.4 & 3.2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 2.5 & -0.5 \\ -3 & 5 \end{bmatrix}, B_1 = \begin{bmatrix} 5 & -1.5 \\ -1.3 & 3 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 2.3 & -2.5 \\ -3 & 4.5 \end{bmatrix}, B_3 = \begin{bmatrix} 2.7 & -1.5 \\ -0.3 & 3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1.6 & -0.9 \\ -0.3 & 3.4 \end{bmatrix}, C_2 = \begin{bmatrix} 4.4 & -1.5 \\ -0.4 & 3.2 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 2.6 & -0.9 \\ -1.3 & 6.4 \end{bmatrix}, \varGamma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix},$$

$$\varGamma_2 = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}, G_1 = \begin{bmatrix} -3.1 & 1.2 & 1.9 \\ 1.2 & -2 & 0.8 \\ 1.9 & 0.8 & -2.7 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} -1 & 0.7 & 0.3 \\ 0.7 & -1.3 & 0.6 \\ 0.3 & 0.6 & -0.9 \end{bmatrix},$$

$$G_3 = \begin{bmatrix} -0.3 & 0.1 & 0.2 \\ 0.1 & -1.9 & 1.8 \\ 0.2 & 1.8 & -2 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -0.8 & 0.4 & 0.4 \\ 0.4 & -0.7 & 0.3 \\ 0.4 & 0.3 & -0.7 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} -1.4 & 0.6 & 0.8 \\ 0.6 & -0.7 & 0.1 \\ 0.8 & 0.1 & -0.9 \end{bmatrix},$$

$$H_3 = \begin{bmatrix} -1.8 & 1.2 & 0.6 \\ 1.2 & -0.7 & 0.5 \\ 0.6 & 0.5 & -1.1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -1.2 & 0 & 1 \\ -1.5 & -0.2 & 0 \\ -1 & 0 & -1.2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.03 & -0.2 & 0 \\ -0.05 & 0 & -0.02 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} -0.1 & 0 & 0.09 \\ -0.32 & -0.2 & 0 \\ -0.06 & 0 & -0.12 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 0.05 & 0 & 0.1 \\ -0.42 & -0.2 & 0 \\ -1 & 0 & -0.6 \end{bmatrix},$$

$$f(x_k(t)) = \begin{bmatrix} x_{k2}(t) + 0.9\cos x_{k1}(t) \\ 1 - 0.2x_{k2}(t) - \sin x_{k1}(t) \end{bmatrix},$$

$\tau = 1, \sigma = 2, \mu = 0.85.$

设一般不确知转移概率矩阵为

$$\varPi = \begin{bmatrix} -2.5 + \varDelta_{11} & 2 + \varDelta_{12} & 0.5 + \varDelta_{13} \\ 0.9 + \varDelta_{21} & ? & ? \\ ? & ? & -3 + \varDelta_{33} \end{bmatrix},$$

其中:

$$\varDelta_{11} \in [-0.1, 0.1], \varDelta_{12} \in [-0.05, 0.05], \varDelta_{13} \in [-0.12, 0.12], \varDelta_{21} \in [-0.1, 0.1], \varDelta_{33} \in [-0.02, 0.02].$$

“?”表示完全未知部分.

利用LMI工具箱, 解得$\beta_1 = 0.6, \beta_2 = 2.5$,

$$Q_1 = \begin{bmatrix} 31.0149 & 0 & 0 \\ 0 & 23.0276 & 0 \\ 0 & 0 & 45.0188 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 32.9767 & 0 & 0 \\ 0 & 41.9733 & 0 \\ 0 & 0 & 62.9757 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 73.0048 & 0 & 0 \\ 0 & 26.9943 & 0 \\ 0 & 0 & 47.0043 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 8.9456 & -4.1428 & -3.7468 \\ -4.1428 & 3.8450 & 0.4178 \\ -3.7468 & 0.4178 & 3.3650 \end{bmatrix},$$

$$P_2=\begin{bmatrix}112.5717 & 0.0001 & 0.0003\\ 0.0001 & 102.9816 & 0.0000\\ 0.0003 & 0.0000 & 156.4386\end{bmatrix},$$

$$P_3=\begin{bmatrix}6.5534 & -2.8779 & -2.6195\\ -2.8779 & 2.7458 & 0.2521\\ -2.6195 & 0.2521 & 2.4035\end{bmatrix},$$

$$R_{12}=\begin{bmatrix}35.3879 & -12.6461 & -2.5914\\ -12.6461 & 72.9618 & -0.0435\\ -2.5914 & -0.0435 & 23.7031\end{bmatrix},$$

$$R_{13}=\begin{bmatrix}11.2851 & -10.4605 & -0.4512\\ -0.4512 & 73.3404 & -0.0072\\ -0.4512 & -0.0072 & 62.6431\end{bmatrix},$$

$$T_{21}=\begin{bmatrix}19.7591 & -0.0003 & 0.0001\\ -0.0003 & 41.7621 & -0.0014\\ 0.0001 & -0.0014 & 7.7595\end{bmatrix},$$

$$W_{332}=\begin{bmatrix}18.7690 & -2.9725 & -2.8421\\ -2.9725 & 20.1565 & -0.0836\\ -2.8421 & -0.0836 & 45.1172\end{bmatrix}.$$

设$e_{kl}=x_k-x_l$, 图1说明系统在不同模式之间切换, 从图2–4看出系统状态达到了渐近同步.

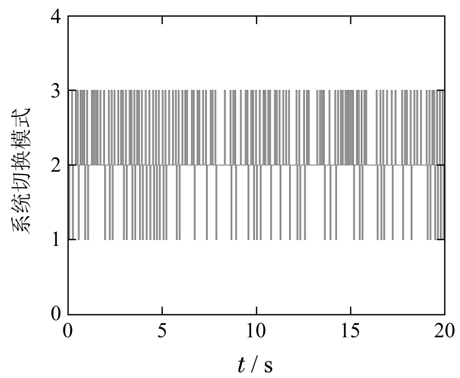


图 1 系统的切换模式

Fig. 1 System switching mode

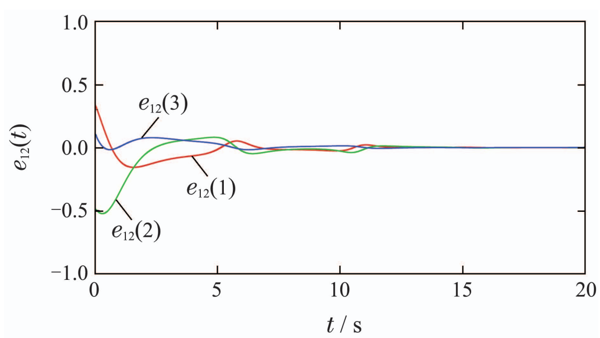


图 2 同步误差$e_{12}(t)$的状态轨迹

Fig. 2 State trajectory of the synchronization error of $e_{12}(t)$

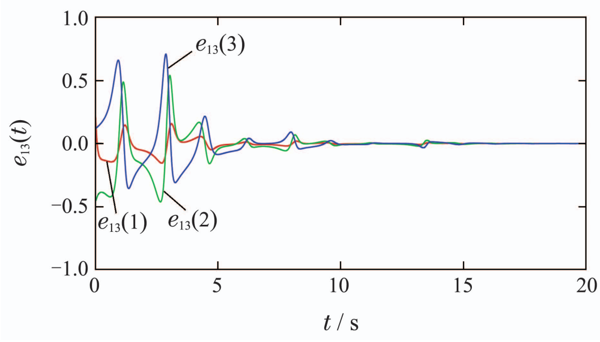


图 3 同步误差$e_{13}(t)$的状态轨迹

Fig. 3 State trajectory of the synchronization error of $e_{13}(t)$

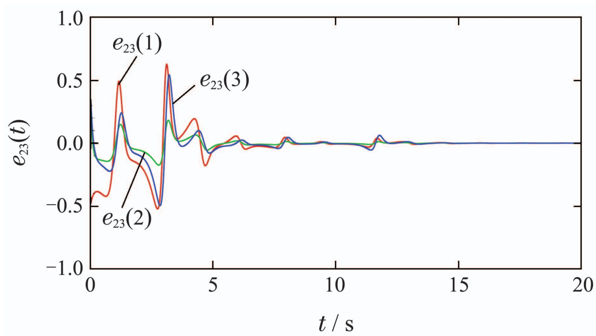


图 4 同步误差$e_{23}(t)$的状态轨迹

Fig. 4 State trajectory of the synchronization error of $e_{23}(t)$

## 5 结论(Conclusions)

本文主要讨论了具有一般不确知转移概率的时滞Markov跳变神经网络的均方渐近同步问题. 针对转移概率估计值和估计误差界是否已知分成3种情况进行了讨论, 并利用线性矩阵不等式给出了系统均方渐近同步的充分条件. 由于本文讨论的转移概率矩阵包含了有界不确知和部分不确知的情形, 因此具有更广泛的应用. 最后, 数值仿真例子说明本文结果的可行性和有效性.

## 参考文献(References):

[1] KAO Y G, WANG C, ZHANG L. Delay-dependent exponential stability of impulsive markovian jumping cohen-grossberg neural networks with reaction-diffusion and mixed delays [J]. *Neural Processing Letters*, 2013, 38(3): 321 – 346.

[2] WU Z, PARK J H, SU H Y, et al. Stochastic stability analysis for discrete-time singular Markov jump systems with time-varying delay and piecewise-constant transition probabilities [J]. *Journal of the Franklin Institute*, 2012, 349(9): 2889 – 2902.

[3] LIU Y R, WANG Z D, LIU X H. Exponential synchronization of complex networks with Markovian jump and mixed delays [J]. *Physics Letters A*, 2008, 372(22): 3986 – 3998.

[4] KAO Y G, GUO J F, WANG C H, et al. Delay-dependent robust exponential stability of Markovian jumping reaction-diffusion Cohen-Grossberg neural networks with mixed delays [J]. *Journal of the Franklin Institute*, 2012, 349 (6): 1972 – 1988.

[5] YANG X S, CAO J D, LU J Q. Synchronization of randomly coupled neural networks with Markovian jumping and time-delay [J]. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2013, 60 (2): 363 – 376.

[6] ZHANG H G, WANG Y C. Stability analysis of Markovian jumping stochastic Cohen-Grossberg neural networks with mixed time delays [J]. *IEEE Transactions on Neural Network*, 2008, 19(2): 366 – 370.

[7] BOUKAS E K. *Stochastic Switching Systems: Analysis and Design* [M]. Berlin: Birkhauser Boston Inc, 2005.

[8] WU Z G, SHI P, SU H, et al. Passivity analysis for discrete-time stochastic Markovian jump neural networks with mixed time delays [J]. *IEEE Transactions on Neural Networks*, 2011, 22 (10): 1566 – 1575.

[9] 田恩刚, 岳东, 杨继全. 具有随机非线性和部分转移概率未知的马尔科夫系统的$H_\infty$控制 [J]. 控制理论与应用, 2014, 31(3): 392 – 396.
(TIAN Engang, YUE Dong, YANG Jiquan. H-infinity control for Markovian jump systems with incomplete transition probabilities and probabilistic nonlinearities [J]. *Control Theory & Applications*, 2014, 31(3): 392 – 396.)

[10] 邱丽, 胥布工, 黎善斌. 具有数据包丢失及转移概率部分未知的网络控制系统$H_\infty$控制 [J]. 控制理论与应用, 2011, 28(8): 1105 – 1112.
(QIU Li, XU Bugong, LI Shanbin. H-infinity control for networked control systems with data packet dropouts and partly unknown transition probabilities [J]. *Control Theory & Applications*, 2011, 28(8): 1105 – 1112.)

[11] 盛立, 高明. 转移概率部分未知的随机Markov跳变系统的镇定控制 [J]. 控制与决策, 2011, 26(11): 1716 – 1720.
(SHENG Li, GAO Ming. Stabilization control of stochastic Markov jump systems with partly unknown transition probabilities [J]. *Control and Decision*, 2011, 26(11): 1716 – 1720.)

[12] MA Q, XU S Y, ZOU Y. Stability and synchronization for Markovian jump neural networks with partly unknown transition probabilities [J]. *Neurocomputing*, 2011, 74(17): 3403 – 3411.

[13] XIONG J L, LAM J. Robust control of Markovian jump systems with uncertain switching probabilities [J]. *International Journal of Systems Science*. 2009, 40(3): 255 – 265.

[14] GUO Y F, WANG Z J. Stability of Markovian jump systems with generally uncertain transition rates [J]. *Journal of the Franklin Institute*, 2013, 350(9): 2826 – 2836.

[15] CAO J D, LI P, WANG WW. Global synchronization in arrays of delayed neural networks with constant and delayed coupling [J]. *Physics Letters A*, 2006, 353(4): 318 – 325.

[16] ZHANG Y J, XU S Y, CHU Y M, et al. Robust global synchronization of complex networks with neutral-type delayed nodes [J]. *Applied Mathematics and computation*, 2010, 216(3): 768 – 778.

[17] LIU Y, WANG Z, LIANG J, et al. Stability and synchronization of discrete-time Markovian jumping neural networks with mixed mode-dependent time delays [J]. *IEEE Transactions on Neural Networks*, 2009, 20(7): 1102 – 1116.

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