带随机量测时滞和随机丢包的改进无迹卡尔曼滤波

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摘要: 经典卡尔曼滤波要求量测值可实时获取, 且仅适用于线性系统. 然而, 在工程实际应用中, 系统多为非线性系统, 量测值也会发生滞后或者丢失等现象, 此时经典卡尔曼滤波已不适用. 因此, 本文针对一类带有随机量测一步时滞和随机丢包的非线性离散系统的状态估计问题, 用两个满足伯努利分布的独立随机变量来描述随机量测一步滞后和随机丢包的现象. 当量测丢失时, 用量测值的一步预测值来代替零输入进行补偿. 在此基础上应用正交投影理论和无迹变换的方法提出了一种改进的无迹卡尔曼滤波算法. 最后, 通过仿真例子验证在考虑随机量测一步时滞和随机丢包的情况下, 所提出的改进算法相比于经典无迹卡尔曼滤波算法具有更高的精度.

关键词: 无迹卡尔曼滤波; 正交投影; 随机时滞; 随机丢包

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An improved unscented Kalman filter with randomly delayed measurements and randomly missing measurements

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Abstract: The classical Kalman filter requires that the measurements can be obtained in real time, and it is only suitable for linear systems. However, in practical engineering applications, most of the systems are nonlinear systems, and the measurements are sometimes delayed or lost, the classical Kalman filter is no longer applicable in this case. Therefore, in this paper, the problem of state estimation for nonlinear discrete-time systems with randomly one-step delayed measurements and missing measurements is studied. The phenomena of randomly one-step delayed measurements and missing measurements are described by two independent random variables satisfying the Bernoulli distribution. When the measurement is missing, the one-step prediction value of the measurement is used to replace the zero input for compensation. On this basis, an improved unscented Kalman filter is proposed by using the orthogonal projection theory and unscented transformation method. Finally, a simulation example is given to illustrate that the improved algorithm has higher accuracy than the classical unscented Kalman filter in the case of considering randomly one-step delayed measurements and missing measurements.

Key words: unscented Kalman filter; orthogonal projections; random delay; random packet losses

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1　引言

在许多复杂的工程应用场景中, 如目标跟踪系统[1]、图像处理[2]、汽车导航[3]以及工业生产[4]等, 要使得系统达到预期的控制效果, 需要利用传感器实时获得系统的状态信息, 但由于网络故障、环境噪声和量测噪声等因素的干扰, 观测值往往不能准确的反映系统的状态信息. 因此, 系统的状态估计问题在近几十年来得到了广泛的研究. 文献[5]以线性离散系统为

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研究对象, 基于最小均方误差和正交投影理论, 提出了著名的卡尔曼滤波器. 基于这一开创性的设计, 众多学者在系统状态估计领域开展了相关研究, 并取得了丰硕的研究成果[6–9]. 在实际工程应用场景中, 非线性系统随处可见, 经典卡尔曼滤波却不能对非线性系统进行状态估计. 为了解决这个问题, 文献[10]运用泰勒展开的方法, 将非线性系统中的非线性函数进行线性化, 再利用经典卡尔曼滤波对系统进行状态估计. 文献[11]采用无迹变换的方法来近似求解系统状态的均值和协方差, 使其满足最小均方误差的原则, 从而为非线性离散系统设计了无迹卡尔曼滤波算法. 无迹卡尔曼滤波相比于拓展卡尔曼滤波具有估计精度高且计算量较小的优点, 因此在包括系统状态估计在内的许多应用领域中, 无迹卡尔曼滤波器是拓展卡尔曼滤波的一种优越的替代方案.

无论是在线性离散系统还是非线性离散系统中, 系统的状态估计问题通常假设系统的测量值是实时可用的. 然而, 在许多实际的应用场景下, 测量值可能会受到随机时延的影响, 尤其是在具有远程传感器的通信网络中, 传感器的传输机制随机失效、通信网络拥堵或者传感器没有及时测得系统的状态信息等这些原因[12]都有可能导致量测时滞的发生, 针对随机量测时滞问题, 一般可分为确定性时滞[13]和随机时滞[14]. 文献[15]将随机量测一步时滞现象用一组服从伯努利分布的随机变量来描述并建立了一种新模型, 线性滤波器的解由黎卡提差分方程和李雅普诺夫差分方程给出. 文献[16]运用去随机化方法和状态增广方法将具有随机一步量测时滞的线性系统转化为具有虚拟白噪声的线性系统, 设计了相应的卡尔曼滤波器. 考虑到实际应用过程中一般无法满足延迟时间是采样间隔时间的整数倍的假设. 文献[17]提出了一种新型卡尔曼滤波算法, 解决了系统在量测微小时延情况下的状态估计问题.

同样, 由于长距离的传输和通信网络不可靠等原因, 系统的传感器测量值有可能会在传输过程中丢失. 针对随机丢包问题, 文献[18]研究了时变复杂网络在具有量测随机丢失情况下的状态估计问题, 同样采用一组服从伯努利分布的随机变量来描述随机丢包. 文献[19]基于信息熵理论进行随机丢包情况的建模, 在此模型上提出了序贯拓展卡尔曼滤波算法来对系统进行状态估计. 文献[20]针对一类非线性随机系统, 将事件触发传输和随机丢包的情况结合起来, 构造了一种时变滤波器, 推导了滤波误差协方差的上界, 并通过适当调整滤波器增益使其最小化. 结合事件触发传输和无迹卡尔曼滤波算法, 文献[21]研究了与事件触发阈值相关的事件触发无迹卡尔曼滤波算法. 对于具有多个传感器的无线传感器网络, 文献[22]在分布式卡尔曼滤波算法和一致性滤波的基础上, 提出了有偏和无偏两种补偿策略来提高无线传感器网络随机丢包问题的跟踪精度和可靠性. 而针对同时具有随机量测时滞和随机丢包现象的系统, 文献[23]通过应用重组信息分析方法, 基于线性最小均方误差得出最优线性滤波算法. 文献[24]将具有多重随机时滞和随机丢包的系统转化为具有多个随机延迟状态和噪声的系统, 利用正交投影方法设计了一种最优线性滤波器. 文献[25]基于事件触发传输机制研究了一类带有随机丢包的随机时滞非线性系统的递归滤波算法, 其算法参数主要借助于求解两个类黎卡提方程. 文献[26]考虑了一类不满足高斯分布的非线性离散随机系统, 基于贝叶斯滤波框架研究了具有随机量测时滞和随机丢包的粒子滤波算法.

虽然, 针对随机量测时滞和随机丢包现象下的系统状态估计研究已经取得了丰硕的成果, 然而在同时具有随机量测时滞和随机丢包现象下的状态估计的研究成果还不够完善, 且现有的研究成果主要基于线性离散随机系统, 鲜有针对随机量测时滞和随机丢包共同作用下的非线性离散随机系统的状态估计问题的研究. 受文献[26]的启发, 本文基于无迹卡尔曼滤波算法研究了同时具有随机量测一步时滞和随机丢包现象的非线性系统状态估计问题, 提出了一种改进的无迹卡尔曼滤波算法, 主要贡献如下:

1) 考虑了同时具有随机量测一步时滞和随机丢包现象下的非线性系统状态估计问题.

2) 当发生丢包时, 采用量测值的一步预测值代替零输入进行补偿.

3) 给出了一种新的递归算法, 得到非线性系统在同时具有随机量测一步时滞和随机丢包现象下的实时状态估计值.

符号说明: $\mathbb{R}^n$表示n维欧几里得空间, A^{T}表示矩阵A的转置, $\mathrm{E}\{x\}$表示随机变量x的期望值.

2 问题描述

带随机量测一步时滞和随机丢包的非线性离散随机系统模型如下:

$$x_{k+1} = f_k(x_k) + w_k, \tag{1}$$

$$z_k = h_k(x_k) + v_k, \tag{2}$$

$$\begin{aligned} y_k = {} & \lambda_k z_k + (1-\lambda_k)\xi_k z_{k-1} + \\ & (1-\lambda_k)(1-\xi_k)\hat{z}_{k|k-1}, \end{aligned} \tag{3}$$

其中: $x_{k+1} \in \mathbb{R}^n$表示系统在$k+1$时刻的状态值; $z_k \in \mathbb{R}^m$表示传感器在没有发生量测时滞和丢包情况下的理想测量值; $y_k \in \mathbb{R}^m$表示滤波器在k时刻接收到的实际测量值; $f(\cdot)$和$h(\cdot)$ 分别表示系统的状态方程和量测方程; w_k和v_k分别表示系统的过程噪声和量测噪声; $\hat{z}_{k|k-1}$表示量测的一步预测值; λ_k和ξ_k则是为了描述系统随机量测时滞和随机丢包现象所引入的

随机变量. 针对上述模型, 做出以下假设.

假设 1 　w_k和v_k均为独立的零均值高斯白噪声, 其方差分别为Q_k和R_k, 即满足

$$\mathrm{E}\{\begin{bmatrix} w_k \\ v_k \end{bmatrix}[w_l^{\mathrm{T}} \;\; v_l^{\mathrm{T}}]\} = \begin{bmatrix} Q_k & 0 \\ 0 & R_k \end{bmatrix}\delta_{kl}, \tag{4}$$

其中δ_{kl}表示克罗内克函数.

假设 2 　λ_k和ξ_k均表示取值为0或1的满足伯努利分布的独立随机变量, 即

$$\text{Prob}\,(\lambda_k = 1) = \mathrm{E}\{\lambda_k\} = \alpha, \tag{5}$$

$$\text{Prob}\,(\lambda_k = 0) = 1 - \mathrm{E}\{\lambda_k\} = 1 - \alpha, \tag{6}$$

$$\text{Prob}\,(\xi_k = 1) = \mathrm{E}\{\xi_k\} = \beta, \tag{7}$$

$$\text{Prob}\,(\xi_k = 0) = 1 - \mathrm{E}\{\xi_k\} = 1 - \beta, \tag{8}$$

其中: Prob表示概率; α和β表示可取值为0到1之间的标量.

假设 3 　系统的初始状态x_0服从正态分布, 且与w_k, v_k均互不相关, 初始状态的均值和协方差分别为

$$\hat{x}_0 = \mathrm{E}\{x_0\}, \tag{9}$$

$$P_0 = \mathrm{Cov}(x_0, x_0^{\mathrm{T}}) = \mathrm{E}\{(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^{\mathrm{T}}\}. \tag{10}$$

注 1 　在式(3)中, 如果$\lambda_k = 1$, 则$y_k = z_k$, 表明滤波器成功接收到当前时刻的量测数据; 如果$\lambda_k = 0$且$\xi_k = 1$, 则$y_k = z_{k-1}$, 表明量测数据发生了一步时滞, 滤波器接收到的数据为上一时刻的量测数据; 如果$\lambda_k = 0$且$\xi_k = 0$, 则$y_k = \hat{z}_{k|k-1}$, 表明系统发生了丢包, 滤波器采用的量测值由量测一步预测值代替.

本文的目的是基于正交投影理论和无迹变换方法, 根据量测值序列$Y_{k+1} = \{y_1, y_2, \cdots, y_{k+1}\}$, 为同时具有随机量测一步时滞和随机丢包的非线性系统(1)–(3)设计一种改进的无迹卡尔曼滤波算法.

3　改进无迹卡尔曼滤波算法

由线性最小均方误差理论可知基于量测值y对系统状态x进行估计时, 其最小均方误差估计值$\hat{x}(y)$即为x在y上的正交投影, 记为$\hat{x}(y) = \hat{\mathrm{E}}\{x|y\}$. 在进行滤波算法推导之前, 先给出下文需要使用到的正交投影理论如下[27]:

引理 1 　设x和y为两个随机向量, 且均具有二阶矩, 则x在y上的正交投影$\hat{x}$等于基于y的线性最小均方误差估计值, 即

$$\hat{\mathrm{E}}\{x|y\} = \mathrm{E}\{x\} + \mathrm{Cov}(x, y)[\mathrm{Var}(y)]^{-1}[y - \mathrm{E}\{y\}]. \tag{11}$$

引理 2 　设x_1, x_2和y为3个随机向量, 且均具有二阶矩, A和B为已知的系数矩阵, 则

$$\hat{\mathrm{E}}\{(Ax_1 + Bx_2)|y\} = A\hat{\mathrm{E}}\{x_1|y\} + B\hat{\mathrm{E}}\{x_2|y\}. \tag{12}$$

引理 3 　设x, y_1和y_2为3个随机向量, 且均具有二阶矩, $y = [y_1 \;\; y_2]$, 则

$$\hat{\mathrm{E}}\{x|y\} = \hat{\mathrm{E}}\{x|y_1\} + \hat{\mathrm{E}}\{\tilde{x}|\tilde{y}_2\} = \hat{\mathrm{E}}\{x|y_1\} + \mathrm{E}\{\tilde{x}\tilde{y}_2^{\mathrm{T}}\}[\mathrm{E}\{\tilde{y}_2\tilde{y}_2^{\mathrm{T}}\}]^{-1}\tilde{y}_2, \tag{13}$$

其中: $\tilde{x} = x - \hat{\mathrm{E}}\{x|y_1\}$, $\tilde{y}_2 = y_2 - \hat{\mathrm{E}}\{y_2|y_1\}$.

所设计的改进无迹卡尔曼滤波算法由状态估计和算法实现两部分构成.

3.1　状态估计

针对带有随机量测一步时滞和随机丢包的非线性系统的状态估计问题, 给出以下定理.

定理 1 　给定系统在k时刻的状态估计值$\hat{x}_{k|k}$和估计误差协方差矩阵$P_{k|k}$, 以及$k-1$时刻的状态估计值$\hat{x}_{k-1|k-1}$和估计误差协方差矩阵$P_{k-1|k-1}$, 基于量测值序列$Y_{k+1} = \{y_1, y_2, \cdots, y_{k+1}\}$, 在假设1、假设2和假设3条件下, 系统在$k+1$时刻的状态估计值$\hat{x}_{k+1|k+1}$、滤波器增益$K_{k+1}$以及估计误差协方差矩阵$P_{k+1|k+1}$为

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}(y_{k+1} - \hat{y}_{k+1|k}), \tag{14}$$

$$\hat{x}_{k+1|k} = f_k(\hat{x}_{k|k}), \tag{15}$$

$$\hat{y}_{k+1|k} = \alpha h_{k+1}(\hat{x}_{k+1|k}) + (1-\alpha)\beta h_k(\hat{x}_{k|k-1}) + (1-\alpha)(1-\beta)h_{k+1}(\hat{x}_{k+1|k}), \tag{16}$$

$$\begin{aligned} K_{k+1} = {}& [\alpha P_{x_{k|k}y_{k+1|k}} + (\beta - \alpha\beta)P_{x_{k|k}y_{k|k-1}}] \cdot \\ & [\alpha P_{y_{k+1|k}y_{k+1|k}} - \alpha\beta(1-\alpha)(\varTheta_{y_{k+1|k}\tilde{y}} + \\ & \varTheta_{\tilde{y}y_{k+1|k}}) - (\alpha^2\beta^2 + \alpha\beta - 2\alpha\beta^2)(\varTheta_{y_{k|k-1}\tilde{y}} + \\ & \varTheta_{\tilde{y}y_{k|k-1}}) + \beta(1-\alpha)P_{y_{k|k-1}y_{k|k-1}} + (\beta^2\alpha + \beta - \\ & \beta^2 - \alpha\beta)\varTheta_{\tilde{y}\tilde{y}} + \alpha R_{k+1} + \beta(1-\alpha)R_k]^{-1}, \end{aligned} \tag{17}$$

$$P_{k+1|k} = \mathrm{E}\{[f_k(x_k) - f_k(\hat{x}_{k|k})][f_k(x_k) - f_k(\hat{x}_{k|k})]^{\mathrm{T}}\} + Q_k, \tag{18}$$

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1}[\alpha P_{x_{k|k}y_{k+1|k}} + (\beta - \alpha\beta)P_{x_{k|k}y_{k|k-1}}]^{\mathrm{T}}, \tag{19}$$

式中:

$$P_{x_{k|k}y_{k+1|k}} = \mathrm{E}\{[f_k(x_k) - f_k(\hat{x}_{k|k})] \cdot [h_{k+1}(x_{k+1}) - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{20}$$

$$P_{x_{k|k}y_{k|k-1}} = \mathrm{E}\{[f_k(x_k) - f_k(\hat{x}_{k|k})] \cdot [h_k(x_k) - h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}, \tag{21}$$

$$P_{y_{k+1|k}y_{k+1|k}} = \mathrm{E}\{[h_{k+1}(x_{k+1}) - h_{k+1}(\hat{x}_{k+1|k})] \cdot [h_{k+1}(x_{k+1}) - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{22}$$

$$\varTheta_{y_{k+1|k}\tilde{y}} = \mathrm{E}\{[h_{k+1}(x_{k+1}) - h_{k+1}(\hat{x}_{k+1|k})] \cdot$$

$$[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\},\tag{23}$$

$$\Theta_{\tilde{y}y_{k+1|k}}=\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\},\tag{24}$$

$$\Theta_{y_{k|k-1}\tilde{y}}=\mathrm{E}\{[h_k(x_k)-h_k(\hat{x}_{k|k-1})]\cdot[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\},\tag{25}$$

$$\Theta_{\tilde{y}y_{k|k-1}}=\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\},\tag{26}$$

$$P_{y_{k|k-1}y_{k|k-1}}=\mathrm{E}\{[h_k(x_k)-h_k(\hat{x}_{k|k-1})]\cdot[h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\},\tag{27}$$

$$\Theta_{\tilde{y}\tilde{y}}=\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\},\tag{28}$$

其中: $P_{k+1|k}$表示系统的状态一步预测误差协方差矩阵; $\hat{x}_{k+1|k}$和$\hat{y}_{k+1|k}$分别表示系统状态值和量测值的一步预测值; Q_k和R_k分别表示过程噪声w_k和量测噪声v_k的协方差矩阵.

证 首先, 已知w_k和v_k均为独立的零均值高斯白噪声, 可得

$$\mathrm{Cov}(w_k,Y_k)=0,\tag{29}$$

$$\mathrm{Cov}(v_k,Y_k)=0,\tag{30}$$

则根据引理1得

$$\hat{\mathrm{E}}\{w_k|Y_k\}=\mathrm{E}\{w_k\}=0,\tag{31}$$

$$\hat{\mathrm{E}}\{v_k|Y_k\}=\mathrm{E}\{v_k\}=0.\tag{32}$$

由假设2可知, 对随机变量λ_k和ξ_k分别取均值得$\mathrm{E}\{\lambda_k\}=\alpha$和$\mathrm{E}\{\xi_k\}=\beta$. 结合式(1)(3)(31)和式(32), 根据引理2可得$k+1$时刻系统的状态一步预测值和量测一步预测值分别为

$$\hat{x}_{k+1|k}=\hat{\mathrm{E}}\{x_{k+1}|Y_k\}=\hat{\mathrm{E}}\{f_k(x_k)|Y_k\},\tag{33}$$

$$\begin{aligned}\hat{y}_{k+1|k}&=\hat{\mathrm{E}}\{y_{k+1}|Y_k\}=\\&\alpha\hat{\mathrm{E}}\{h_{k+1}(x_{k+1})|Y_k\}+(1-\alpha)\beta\hat{\mathrm{E}}\{h_k(x_k)|Y_k\}+\\&(1-\alpha)(1-\beta)\hat{\mathrm{E}}\{h_{k+1}(x_{k+1})|Y_k\},\end{aligned}\tag{34}$$

其中: $\hat{\mathrm{E}}\{f_k(x_k)|Y_k\}$表示$k$时刻的状态估计值$\hat{x}_{k|k}$经非线性状态函数$f_k(\cdot)$传递之后的后验均值; $\hat{\mathrm{E}}\{h_{k+1}(x_{k+1})|Y_k\}$表示$k+1$时刻系统状态的一步预测值$\hat{x}_{k+1|k}$经非线性量测方程$h_{k+1}(\cdot)$传递之后的后验均值, $\hat{\mathrm{E}}\{h_k(x_k)|Y_k\}$同理.

然而, 不同于线性系统, 在非线性系统中求解$\hat{\mathrm{E}}\{f_k(x_k)|Y_k\}$和$\hat{\mathrm{E}}\{h_{k+1}(x_{k+1})|Y_k\}$的值将变得异常困难, 因此可以通过无迹变换等方法来近似求解. 令

$$f_k(\hat{x}_{k|k})=\hat{\mathrm{E}}\{f_k(x_k)|Y_k\},\tag{35}$$

$$h_{k+1}(\hat{x}_{k+1|k})=\hat{\mathrm{E}}\{h_{k+1}(x_{k+1})|Y_k\},\tag{36}$$

$$h_k(\hat{x}_{kk-1})=\hat{\mathrm{E}}\{h_k(x_k)|Y_k\},\tag{37}$$

于是式(33)–(34)可分别改写成

$$\hat{x}_{k+1|k}=f_k(\hat{x}_{k|k}),\tag{38}$$

$$\begin{aligned}\hat{y}_{k+1|k}=&\alpha h_{k+1}(\hat{x}_{k+1|k})+(1-\alpha)\beta h_k(\hat{x}_{k|k-1})+\\&(1-\alpha)(1-\beta)h_{k+1}(\hat{x}_{k+1|k}),\end{aligned}\tag{39}$$

由式(38)–(39)可知式(15)–(16)得证.

系统状态一步预测误差由式(1)和式(38)可得

$$\begin{aligned}\tilde{x}_{k+1|k}&=x_{k+1}-\hat{x}_{k+1|k}=\\&f_k(x_k)-f_k(\hat{x}_{k|k})+w_k,\end{aligned}\tag{40}$$

则系统的状态一步预测误差协方差矩阵为

$$\begin{aligned}P_{k+1|k}&=\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{x}^{\mathrm{T}}_{k+1|k}\}=\\&\mathrm{E}\{[f_k(x_k)-f_k(\hat{x}_{k|k})+w_k]\cdot\\&[f_k(x_k)-f_k(\hat{x}_{k|k})+w_k]^{\mathrm{T}}\}=\\&\mathrm{E}\{[f_k(x_k)-f_k(\hat{x}_{k|k})]\cdot\\&[f_k(x_k)-f_k(\hat{x}_{k|k})]^{\mathrm{T}}\}+Q_k,\end{aligned}\tag{41}$$

式(18)得证. 同理, 系统的量测一步预测误差由式(3)和式(39)可得

$$\begin{aligned}&\tilde{y}_{k+1|k}=y_{k+1}-\hat{y}_{k+1|k}=\\&\lambda_kz_k+(1-\lambda_k)\xi_kz_{k-1}+(1-\lambda_k)(1-\xi_k)\hat{z}_{k|k-1}-\\&\alpha h_{k+1}(\hat{x}_{k+1|k})+(1-\alpha)\beta h_k(\hat{x}_{k|k-1})+\\&(1-\alpha)(1-\beta)h_{k+1}(\hat{x}_{k+1|k})=\\&\lambda_{k+1}[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]+(\xi_{k+1}-\\&\xi_{k+1}\lambda_{k+1})[h_k(x_k)-h_k(\hat{x}_{k|k-1})]-(\beta-\alpha\beta-\\&\xi_{k+1}+\xi_{k+1}\lambda_{k+1})[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]+\\&\lambda_{k+1}v_{k+1}+(\xi_{k+1}-\xi_{k+1}\lambda_{k+1})v_k,\end{aligned}\tag{42}$$

则系统的量测一步预测误差协方差矩阵为

$$\begin{aligned}&P_{\tilde{y}_{k+1|k}}=\mathrm{E}\{\tilde{y}_{k+1|k}\tilde{y}^{\mathrm{T}}_{k+1|k}\}=\\&\alpha\mathrm{E}\{[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})][h_{k+1}(x_{k+1})-\\&h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}-\alpha\beta(1-\alpha)\mathrm{E}\{[h_{k+1}(x_{k+1})-\\&h_{k+1}(\hat{x}_{k+1|k})][h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}+\\&\beta(1-\alpha)\mathrm{E}\{[h_k(x_k)-h_k(\hat{x}_{k|k-1})][h_k(x_k)-\\&h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}-(\alpha^2\beta^2+\alpha\beta-2\alpha\beta^2)\mathrm{E}\{[h_k(x_k)-\\&h_k(\hat{x}_{k|k-1})][h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}-\\&\alpha\beta(1-\alpha)\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot\\&[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}-\\&(\alpha^2\beta^2+\alpha\beta-2\alpha\beta^2)\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-\\&h_{k+1}(\hat{x}_{k+1|k})][h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}+\\&(\beta^2\alpha+\beta-\beta^2-\alpha\beta)\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-\\&h_{k+1}(\hat{x}_{k+1|k})][h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}+\\&\alpha R_{k+1}+\beta(1-\alpha)R_k=\end{aligned}$$

$$\alpha P_{y_{k+1|k}y_{k+1|k}}-\alpha\beta(1-\alpha)(\Theta_{y_{k+1|k}\tilde{y}}+\Theta_{\tilde{y}y_{k+1|k}})-(\alpha^2\beta^2+\alpha\beta-2\alpha\beta^2)(\Theta_{y_{k|k-1}\tilde{y}}+\Theta_{\tilde{y}y_{k|k-1}})+\beta(1-\alpha)P_{y_{k|k-1}y_{k|k-1}}+(\beta^2\alpha+\beta-\beta^2-\alpha\beta)\cdot\Theta_{\tilde{y}\tilde{y}}+\alpha R_{k+1}+\beta(1-\alpha)R_k, \tag{43}$$

式中:

$$P_{y_{k+1|k}y_{k+1|k}}=\mathrm{E}\{[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{44}$$

$$\Theta_{y_{k+1|k}\tilde{y}}=\mathrm{E}\{[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{45}$$

$$\Theta_{\tilde{y}y_{k+1|k}}=\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{46}$$

$$\Theta_{y_{k|k-1}\tilde{y}}=\mathrm{E}\{[h_k(x_k)-h_k(\hat{x}_{k|k-1})]\cdot[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{47}$$

$$\Theta_{\tilde{y}y_{k|k-1}}=\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}, \tag{48}$$

$$P_{y_{k|k-1}y_{k|k-1}}=\mathrm{E}\{[h_k(x_k)-h_k(\hat{x}_{k|k-1})]\cdot[h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}, \tag{49}$$

$$\Theta_{\tilde{y}\tilde{y}}=\mathrm{E}\{[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]\cdot[h_k(\hat{x}_{k|k-1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{50}$$

由式(44)–(50)可知式(22)–(28)得证.

由式(40)和式(42)进一步可得

$$P_{\tilde{x}_{k+1|k}\tilde{y}_{k+1|k}}=\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\}=\alpha\mathrm{E}\{[f_k(x_k)-f_k(\hat{x}_{k|k})]\cdot[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}+(\beta-\alpha\beta)\mathrm{E}\{[f_k(x_k)-f_k(\hat{x}_{k|k})]\cdot[h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}=\alpha P_{x_{k|k}y_{k+1|k}}+(\beta-\alpha\beta)P_{x_{k|k}y_{k|k-1}}, \tag{51}$$

式中:

$$P_{x_{k|k}y_{k+1|k}}=\mathrm{E}\{[f_k(x_k)-f_k(\hat{x}_{k|k})]\cdot[h_{k+1}(x_{k+1})-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}\}, \tag{52}$$

$$P_{x_{k|k}y_{k|k-1}}=\mathrm{E}\{[f_k(x_k)-f_k(\hat{x}_{k|k})]\cdot[h_k(x_k)-h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}\}, \tag{53}$$

由式(52)和式(53)可知式(20)和式(21)得证.

接下来, 求解$k+1$时刻系统的状态估计值, 已知$Y_{k+1}=\{Y_k,y_{k+1}\}$, 根据引理3可得

$$\hat{x}_{k+1|k+1}=\hat{\mathrm{E}}\{x_{k+1}|Y_{k+1}\}=\hat{\mathrm{E}}\{x_{k+1}|Y_k\}+\hat{\mathrm{E}}\{\tilde{x}_{k+1|k}|\tilde{y}_{k+1|k}\}=\hat{x}_{k+1|k}+\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\}\cdot(\mathrm{E}\{\tilde{y}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\})^{-1}\tilde{y}_{k+1|k}. \tag{54}$$

令滤波器增益K_{k+1}为

$$K_{k+1}=\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\}\cdot(\mathrm{E}\{\tilde{y}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\})^{-1}, \tag{55}$$

于是式(54)改写为

$$\hat{x}_{k+1|k+1}=\hat{x}_{k+1|k}+K_{k+1}(y_{k+1}-\hat{y}_{k+1|k}). \tag{56}$$

将式(43)和式(51)代入式(55)可得

$$K_{k+1}=[\alpha P_{x_{k|k}y_{k+1|k}}+(\beta-\alpha\beta)P_{x_{k|k}y_{k|k-1}}]\cdot[\alpha P_{y_{k+1|k}y_{k+1|k}}-\alpha\beta(1-\alpha)(\Theta_{y_{k+1|k}\tilde{y}}+\Theta_{\tilde{y}y_{k+1|k}})-(\alpha^2\beta^2+\alpha\beta-2\alpha\beta^2)(\Theta_{y_{k|k-1}\tilde{y}}+\Theta_{\tilde{y}y_{k|k-1}})+\beta(1-\alpha)P_{y_{k|k-1}y_{k|k-1}}+(\beta^2\alpha+\beta-\beta^2-\alpha\beta)\Theta_{\tilde{y}\tilde{y}}+\alpha R_{k+1}+\beta(1-\alpha)R_k]^{-1}, \tag{57}$$

由式(56)和式(57)可知式(14)和式(17)得证.

最后, 系统的状态估计误差为

$$\tilde{x}_{k+1|k+1}=x_{k+1}-\hat{x}_{k+1|k+1}=x_{k+1}-\hat{x}_{k+1|k}-K_{k+1}\tilde{y}_{k+1|k}=\tilde{x}_{k+1|k}-K_{k+1}\tilde{y}_{k+1|k}, \tag{58}$$

则系统的状态估计误差协方差矩阵为

$$P_{k+1|k+1}=\mathrm{E}\{\tilde{x}_{k+1|k+1}\tilde{x}_{k+1|k+1}^{\mathrm{T}}\}=\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{x}_{k+1|k}^{\mathrm{T}}\}-\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\}K_{k+1}^{\mathrm{T}}-K_{k+1}\mathrm{E}\{\tilde{y}_{k+1|k}\tilde{x}_{k+1|k}^{\mathrm{T}}\}+K_{k+1}\mathrm{E}\{\tilde{y}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\}K_{k+1}^{\mathrm{T}}=\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{x}_{k+1|k}^{\mathrm{T}}\}-K_{k+1}\mathrm{E}\{\tilde{x}_{k+1|k}\tilde{y}_{k+1|k}^{\mathrm{T}}\}^{\mathrm{T}}. \tag{59}$$

将式(41)和式(51)代入式(59)可得

$$P_{k+1|k+1}=P_{k+1|k}-K_{k+1}[\alpha P_{x_{k|k}y_{k+1|k}}+(\beta-\alpha\beta)P_{x_{k|k}y_{k|k-1}}]^{\mathrm{T}}, \tag{60}$$

即式(19)得证.　　证毕.

注 2　由于长距离的传输和通信网络不可靠等原因, 系统的传感器测量值有可能会在传输过程中发生滞后或者丢失的现象, 且当量测值丢失时, 用量测值的一步预测值来代替零输入进行补偿. 所计算的$P_{x_{k|k}y_{k+1|k}}$、$P_{y_{k+1|k}y_{k+1|k}}$以及$P_{x_{k|k}y_{k|k-1}}$等参数的值, 即为了降低量测时滞和丢包补偿对系统状态估计精度的影响.

在定理1中给出了系统在$k+1$时刻的状态估计值$\hat{x}_{k+1|k+1}$、滤波器增益K_{k+1}以及估计误差协方差矩阵$P_{k+1|k+1}$的值, 但在系统为非线性系统时难以求解, 因此本文采用了无迹变换的方法进行近似求解.

3.2 算法实现

本节将运用无迹变换的方法, 求解定理1中给出的系统在$k+1$时刻的状态估计值$\hat{x}_{k+1|k+1}$、滤波器增益K_{k+1}以及估计误差协方差矩阵$P_{k+1|k+1}$的值, 步骤如下:

步骤 1 首先, 选取Sigma点集, 计算系统的一步预测值. 给定系统在k时刻和$k-1$时刻的状态估计值和估计误差协方差矩阵, 即$\hat{x}_{k|k}, \hat{x}_{k-1|k-1}, P_{k|k}$以及$P_{k-1|k-1}$已知, 则基于$\hat{x}_{k|k}$和$P_{k|k}$选取$2n+1$个点作为一组Sigma点集如下:

$$\begin{aligned}
&\chi^0_{k|k}=\hat{x}_{k|k},\\
&\chi^s_{k|k}=\hat{x}_{k|k}+(\sqrt{(n+\gamma)P_{k|k}})_s,\ s=1,\cdots,n,\\
&\chi^s_{k|k}=\hat{x}_{k|k}-(\sqrt{(n+\gamma)P_{k|k}})_{s-n},\ s=n+1,\cdots,2n.
\end{aligned}$$

同理, 基于$\hat{x}_{k-1|k-1}$和$P_{k-1|k-1}$选取$2n+1$个点作为一组Sigma点集如下:

$$\begin{aligned}
\chi^0_{k-1|k-1}&=\hat{x}_{k-1|k-1},\\
\chi^s_{k-1|k-1}&=\hat{x}_{k-1|k-1}+(\sqrt{(n+\gamma)P_{k-1|k-1}})_s,\\
&\quad s=1,\cdots,n,\\
\chi^s_{k-1|k-1}&=\hat{x}_{k-1|k-1}-(\sqrt{(n+\gamma)P_{k-1|k-1}})_{s-n},\\
&\quad s=n+1,\cdots,2n,
\end{aligned}$$

式中: $\gamma=\sigma^2(n+\kappa)-n, \sigma$为常数, 可通过改变$\sigma$的值来控制Sigma点的分布, κ表示比例因子, 通常取0, n表示系统的维数; $(\sqrt{(n+\gamma)P_{k|k}})_s$表示矩阵$(n+\gamma)P_{k|k}$的平方根的第$s$列. 给出均值和协方差的权值如下:

$$\begin{aligned}
w^m_0&=\frac{\gamma}{n+\gamma},\\
w^c_0&=\frac{\gamma}{n+\gamma}+(1-\sigma^2+\theta),\\
w^m_s&=w^c_s=\frac{1}{2(n+\gamma)},\ s=1,2,\cdots,2n,
\end{aligned}$$

其中$\theta\geqslant 0$为另一比例因子, 通常取2.

k时刻的Sigma点集经系统状态函数$f_k(\cdot)$传递之后的值为

$$\chi^s_{k+1|k}=f_k(\chi^s_{k|k}),\ s=0,1,\cdots,2n, \tag{61}$$

则k时刻系统的状态一步预测$\hat{x}_{k+1|k}$和一步预测误差协方差矩阵$P_{k+1|k}$的值为

$$\hat{x}_{k+1|k}=\sum_{s=0}^{2n}w^m_s\chi^s_{k+1|k}, \tag{62}$$

$$\begin{aligned}
P_{k+1|k}=\sum_{s=0}^{2n}&w^c_s(\chi^s_{k+1|k}-\hat{x}_{k+1|k})\cdot\\
&(\chi^s_{k+1|k}-\hat{x}_{k+1|k})^{\mathrm{T}}+Q_k,
\end{aligned} \tag{63}$$

然后, 基于$\hat{x}_{k+1|k}$和$P_{k+1|k}$的值重新选取$2n+1$个点作为一组Sigma点集如下:

$$\begin{aligned}
\eta^0_{k+1|k}&=\hat{x}_{k+1|k},\\
\eta^s_{k+1|k}&=\hat{x}_{k+1|k}+(\sqrt{(n+\gamma)P_{k+1|k}})_s,\\
&\quad s=1,\cdots,n,\\
\eta^s_{k+1|k}&=\hat{x}_{k+1|k}-(\sqrt{(n+\gamma)P_{k+1|k}})_{s-n},\\
&\quad s=n+1,\cdots,2n,
\end{aligned}$$

经系统量测函数$h_{k+1}(\cdot)$传递之后的值为

$$\delta^s_{k+1|k}=h_{k+1}(\eta^s_{k+1|k}),\ s=0,1,\cdots,2n, \tag{64}$$

则基于$\hat{x}_{k+1|k}$和$P_{k+1|k}$的系统量测一步预测为

$$h_{k+1}(\hat{x}_{k+1|k})=\sum_{s=0}^{2n}w^m_s\delta^s_{k+1|k}. \tag{65}$$

同理, $k-1$时刻的Sigma点集经系统状态函数$f_{k-1}(\cdot)$传递之后的值为

$$\chi^s_{k|k-1}=f_{k-1}(\chi^s_{k-1|k-1}),\ s=0,1,\cdots,2n, \tag{66}$$

则$k-1$时刻系统的状态一步预测$\hat{x}_{k|k-1}$和一步预测误差协方差矩阵$P_{k|k-1}$的值为

$$\hat{x}_{k|k-1}=\sum_{s=0}^{2n}w^m_s\chi^s_{k|k-1}, \tag{67}$$

$$\begin{aligned}
&P_{k|k-1}=\\
&\sum_{s=0}^{2n}w^c_s(\chi^s_{k|k-1}-\hat{x}_{k|k-1})\cdot\\
&(\chi^s_{k|k-1}-\hat{x}_{k|k-1})^{\mathrm{T}}+Q_{k-1},
\end{aligned} \tag{68}$$

然后, 基于$\hat{x}_{k|k-1}$和$P_{k|k-1}$的值重新选取$2n+1$个点作为一组Sigma点集如下:

$$\begin{aligned}
g^0_{k|k-1}&=\hat{x}_{k|k-1},\\
g^s_{k|k-1}&=\hat{x}_{k|k-1}+(\sqrt{(n+\gamma)P_{k|k-1}})_s,\\
&\quad s=1,\cdots,n,\\
g^s_{k|k-1}&=\hat{x}_{k|k-1}-(\sqrt{(n+\gamma)P_{k|k-1}})_{s-n},\\
&\quad s=n+1,\cdots,2n,
\end{aligned}$$

经系统量测函数$h_k(\cdot)$传递之后的值为

$$\zeta^s_{k|k-1}=h_k(g^s_{k|k-1}),\ s=0,1,\cdots,2n, \tag{69}$$

则基于$\hat{x}_{k|k-1}$和$P_{k|k-1}$的系统量测一步预测为

$$h_k(\hat{x}_{k|k-1})=\sum_{s=0}^{2n}w^m_s\zeta^s_{k|k-1}. \tag{70}$$

步骤 2 计算参数的值. 式(15)和式(18)可分别由式(62)和式(63)来计算, 式(16)可由式(65)和式(70)来计算, 式(20)–(28)的值可以计算如下:

$$\begin{aligned}
P_{x_{k|k}y_{k+1|k}}=\sum_{s=0}^{2n}&w^c_s(\chi^s_{k+1|k}-\hat{x}_{k+1|k})\cdot\\
&[\delta^s_{k+1|k}-h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}},
\end{aligned} \tag{71}$$

$$P_{x_{k|k}y_{k|k-1}}=\sum_{s=0}^{2n}w^c_s(\chi^s_{k+1|k}-\hat{x}_{k+1|k})\cdot$$

$$[\zeta^s_{k|k-1} - h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}, \tag{72}$$

$$P_{y_{k+1|k}y_{k+1|k}} = \sum_{s=0}^{2n} w^c_s[\delta^s_{k+1|k} - h_{k+1}(\hat{x}_{k+1|k})]\cdot [\delta^s_{k+1|k} - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}, \tag{73}$$

$$\Theta_{y_{k+1|k}\tilde{y}} = \sum_{s=0}^{2n} w^c_s[\delta^s_{k+1|k} - h_{k+1}(\hat{x}_{k+1|k})]\cdot [h_k(\hat{x}_{k|k-1}) - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}, \tag{74}$$

$$\Theta_{\tilde{y}y_{k+1|k}} = \sum_{s=0}^{2n} w^c_s[h_k(\hat{x}_{k|k-1}) - h_{k+1}(\hat{x}_{k+1|k})]\cdot [\delta^s_{k+1|k} - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}, \tag{75}$$

$$\Theta_{y_{k|k-1}\tilde{y}} = \sum_{s=0}^{2n} w^c_s[\zeta^s_{k|k-1} - h_k(\hat{x}_{k|k-1})]\cdot [h_k(\hat{x}_{k|k-1}) - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}, \tag{76}$$

$$\Theta_{\tilde{y}y_{k|k-1}} = \sum_{s=0}^{2n} w^c_s[h_k(\hat{x}_{k|k-1}) - h_{k+1}(\hat{x}_{k+1|k})]\cdot [\zeta^s_{k|k-1} - h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}, \tag{77}$$

$$P_{y_{k|k-1}y_{k|k-1}} = \sum_{s=0}^{2n} w^c_s[\zeta^s_{k|k-1} - h_k(\hat{x}_{k|k-1})]\cdot [\zeta^s_{k|k-1} - h_k(\hat{x}_{k|k-1})]^{\mathrm{T}}, \tag{78}$$

$$\Theta_{\tilde{y}\tilde{y}} = [h_k(\hat{x}_{k|k-1}) - h_{k+1}(\hat{x}_{k+1|k})]\cdot [h_k(\hat{x}_{k|k-1}) - h_{k+1}(\hat{x}_{k+1|k})]^{\mathrm{T}}. \tag{79}$$

步骤 3　计算系统在$k+1$时刻的状态估计值.

1) 将式(71)–(79)的值代入式(17)中得滤波器增益K_{k+1}的值.

2) 将式(15)–(16)和K_{k+1}的值代入式(14)得系统在$k+1$时刻的状态估计值$\hat{x}_{k+1|k+1}$.

3) 将式(18)(71)–(72)和K_{k+1}的值代入式(19)得系统$k+1$时刻的状态估计误差协方差矩阵$P_{k+1|k+1}$.

注 3　在算法的实现过程中, 为了进一步降低量测一步时滞和丢包补偿对系统状态估计精度的影响, 总共进行了4次Sigma点集的选取. 首先分别基于系统在k和$k-1$时刻的状态估计值和估计误差协方差矩阵选取2组Sigma点集, 将权值代入经系统的状态方程传递之后的Sigma点集, 求出状态一步预测值和误差协方差矩阵. 其次, 基于求出的状态一步预测值和误差协方差矩阵重新选取2个Sigma点集, 经系统的量测方程传递之后求出量测的一步预测值.

4　仿真分析

在这一小节中, 对所提出的改进无迹卡尔曼滤波算法进行验证, 并与经典无迹卡尔曼滤波算法[11]进行比较. 首先, 给出一个非线性离散系统模型如下:

$$x_{1,k} = x_{1,k-1} + Tx_{2,k-1} + w_{1,k-1},$$

$$x_{2,k} = -10T\sin x_{1,k-1} + (1-T)x_{2,k-1} + w_{2,k-1},$$

$$z_{1,k} = 2\sin\frac{x_{1,k}}{2} + \sin x_{1,k} + v_{1,k},$$

$$z_{2,k} = \frac{x_{1,k}}{2} + v_{2,k},$$

式中: T为滤波周期; w和v均为0均值的高斯白噪声且独立, 其协方差矩阵分别为

$$Q = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.001 \end{bmatrix},\ R = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

假设系统的初始值如下:

$$x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ \hat{x}_{0|0} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix},\ \hat{x}_{1|1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$P_{0|0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ P_{1|1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$T = 0.05$ s, σ取0.1, θ取2, κ取0, $\alpha = \beta = 0.8$.

用$\mathrm{RMSE}_{i,k}$来表示状态i在k时刻的均方根误差, 其定义如下:

$$\mathrm{RMSE}_{i,k} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x^{(n)}_{i,k} - \hat{x}^{(n)}_{i,k|k})^2},\ i = 1, 2,$$

式中: N表示进行多次仿真的次数, 此次仿真中令$N=100$, $x^{(n)}_{i,k}$和$\hat{x}^{(n)}_{i,k|k}$分别表示第n次仿真中k时刻系统的状态真实值和估计值. 在定理1中, 针对具有随机量测一步时滞和随机丢包的非线性系统, 基于正交投影理论和无迹变换方法, 给出了系统的状态估计值. 根据定理1中所述的改进算法, 递推地计算出系统在每一个时刻的估计值, 仿真结果如图1–6所示.

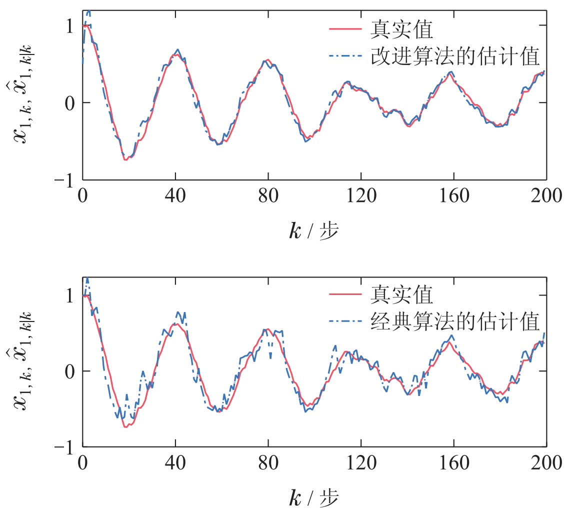

图 1　改进算法和经典算法下$x_{1,k}$和$\hat{x}_{1,k|k}$的轨迹对比

Fig. 1　Trajectory comparison between $x_{1,k}$ and $\hat{x}_{1,k|k}$ under improved algorithm and classical algorithm

图1–2表示当$\alpha = \beta = 0.8$时, 本文所提出的改进算法和文献[11]中的滤波算法的状态估计值比较, 从图中可以看出改进算法下的状态估计值更接近于真实值; 图3–4表示当$\alpha = \beta = 0.8$时, 本文所提出的改进算法和文献[11]中的滤波算法的均方根误差比较, 以均方根误差作为评价算法的性能指标, 可以看出改进算法的整体性能要优于文献[11]中的经典算法; 图5–6表示在本文所提出的改进无迹卡尔曼滤波算法下,

取不同的时滞率和丢包率时的均方根误差比较, 当量测值发生时滞或丢包时, 分别采用了上一步的传输值或量测一步预测值来进行补偿, 故可以看出当选取不同的时滞率和丢包率时改进算法均具有良好的估计性能, 但时滞率和丢包率较小时改进算法的整体估计性能要优于时滞率和丢包率较大时的整体估计性能.

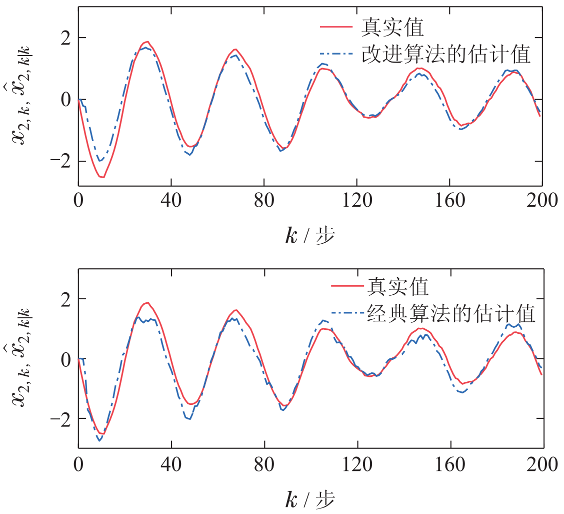

图 2 改进算法和经典算法下$x_{2,k}$和$\hat{x}_{2,k|k}$的轨迹对比

Fig. 2 Trajectory comparison between $x_{2,k}$ and $\hat{x}_{2,k|k}$ under improved algorithm and classical algorithm

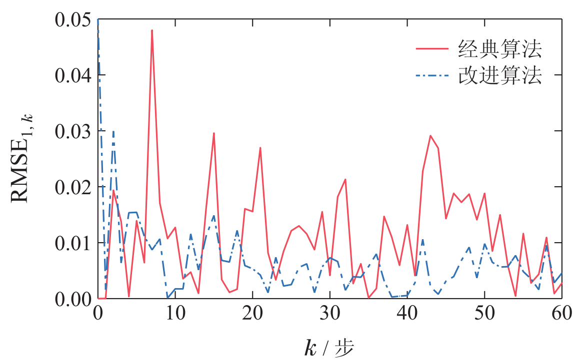

图 3 改进算法和经典算法下$\mathrm{RMSE}_{1,k}$的轨迹对比

Fig. 3 Trajectory comparison of $\mathrm{RMSE}_{1,k}$ under improved algorithm and classical algorithm

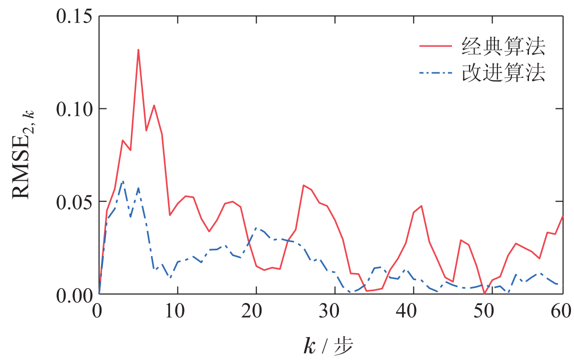

图 4 改进算法和经典算法下$\mathrm{RMSE}_{2,k}$的轨迹对比

Fig. 4 Trajectory comparison of $\mathrm{RMSE}_{2,k}$ under improved algorithm and classical algorithm

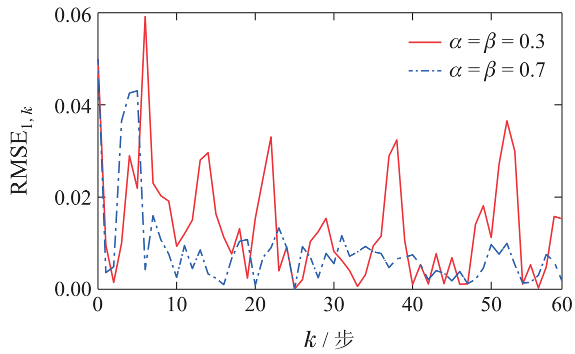

图 5 改进算法下$\alpha=\beta=0.3$和$\alpha=\beta=0.7$时$\mathrm{RMSE}_{1,k}$的轨迹对比

Fig. 5 Trajectory comparison of $\mathrm{RMSE}_{1,k}$ between $\alpha=\beta=0.3$ and $\alpha=\beta=0.7$ under improved algorithm

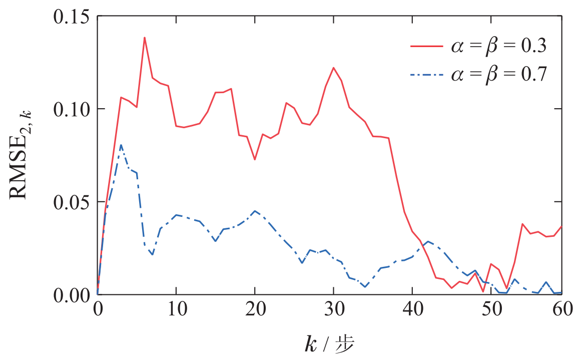

图 6 改进算法下$\alpha=\beta=0.3$和$\alpha=\beta=0.7$时$\mathrm{RMSE}_{2,k}$的轨迹对比

Fig. 6 Trajectory comparison of $\mathrm{RMSE}_{2,k}$ between $\alpha=\beta=0.3$ and $\alpha=\beta=0.7$ under improved algorithm

以上的仿真实例表明本文所提出的改进无迹卡尔曼滤波算法在系统发生随机一步时滞和随机丢包的情况下是可行有效的, 且其对系统的状态估计性能要优于经典的无迹卡尔曼滤波算法.

5 结论

本文研究了一类具有随机量测一步滞后和随机丢包的非线性系统的状态估计问题. 首先, 针对发生量测一步时滞的情况, 滤波器将采用上一时刻的传输值进行滤波; 针对发生丢包的情况, 则采取了量测的一步预测值作为补偿. 其次, 基于最小均方误差理论和正交投影理论, 通过重新计算滤波参数来降低量测时滞和丢包补偿对状态估计的影响, 得到了改进的无迹卡尔曼滤波算法. 最后, 基于无迹变换的方法求得各个滤波参数的值, 并通过仿真例子验证本文所提出的改进无迹卡尔曼滤波算法能更有效地对同时具有随机量测一步时滞和随机丢包现象的非线性系统进行状态估计.

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