惯性忆阻神经网络固定时间抗干扰聚类同步控制

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摘要: 本文研究了具有外部干扰的惯性忆阻神经网络固定时间聚类同步控制问题. 通过设计一类经济型的固定时间控制器, 保证了耦合网络系统在固定时间内以较少的能量消耗实现聚类同步. 为提高网络的普适性, 文中神经网络的激活函数为非连续函数. 基于Filippov解理论以及固定时间控制理论, 获得了耦合网络的同步性判据, 给出了收敛时间的具体上界, 验证了网络抗干扰能力. 最后, 通过数值仿真说明了理论结果的有效性.

关键词: 惯性忆阻神经网络; 聚类同步; 固定时间; 外部干扰

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Fixed-time anti-disturbance cluster synchronization of inertial memristive neural networks

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Abstract: This paper investigates the fixed-time cluster synchronization problems of inertial memristive neural networks (IMNNs) with disturbances. By designing an economical fixed-time controller, the cluster synchronization of the IMNNs is guaranteed in a fixed time with less energy consumption. The discontinuous functions are chosen as the activations to improve the universality of networks. Several sufficient conditions for the coupled network are obtained in the sense of Filippov solution and fixed-time control theory. Meanwhile, the upper-bounds of settling times are obtained and the robustness of the networks is proved. Finally, some numerical simulations are provided to illustrate the feasibility and practicality of the theoretical results.

Key words: inertial memristive neural networks; cluster synchronization; fixed time; disturbances

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1 引言

忆阻器作为连接磁通量和电荷的基本电路元件之一, 是蔡少棠在1971年首次提出, 并由惠普实验室在2008年成功制造[1]. 由于忆阻器具有记忆性、低功耗、高密度和可扩展性等特点, 其已广泛应用于非易失性存储技术和神经形态计算机中. 近年来, 忆阻神经网络(memristive neural networks, MNNs)受到研究人员的广泛关注, 并取得了丰硕的研究成果[2–5]. 考虑到电路中电感的影响, 人们将惯性项引入神经网络的研究中. 但是与传统的神经网络不同, 惯性项导致神经网络具有更复杂的动力学行为, 也是非线性系统产生分岔和混沌的关键. 近年来, 结合MNNs与惯性项的各自优势, 研究人员构造了惯性忆阻神经网络(inertial memristive neural networks, IMNNs)并对其动力学行为展开了研究[6–9].

同步作为网络系统最重要的动力学行为之一, 按照同步时间的不同可划分为无限时间同步与有限时间同步. 文献[6]设计了一类具有线性扩散项和不连续项的控制器, 基于Lyapunov稳定性理论和不等式技巧, 得到了IMNNs全局指数同步的充分性条件. 文

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献[7]在状态反馈控制器的基础上, 提出了静态和动态事件触发控制条件, 减少了控制器的更新次数, 降低了IMNNs同步的控制成本. 然而, 在实际工程中机器的寿命往往是有限的, 因此人们开始研究收敛速度更快, 抗干扰能力更强的有限时间同步. 文献[8–9]基于有限时间控制理论分别设计了采样控制器和时滞控制器, 确保了驱动响应系统的有限时间同步. 随着研究的深入, 人们发现有限时间同步依赖于系统初始值, 当系统初值难以计算或不可测时, 将无法得到有限时间同步的收敛上界. 鉴于此, Polyakov提出了固定时间同步, 其收敛时间上界只与控制器参数有关而与初值无关[10]. 文献[11]设计了4类固定时间反馈控制器, 实现了IMNNs的固定时间同步, 并根据系统和控制器参数估计了收敛时间上界. 文献[12]构造了固定时间和有限时间的统一框架, 通过调整参数得出了IMNNs固定时间和有限时间同步的充分判据.

值得注意的是, 上述关于IMNNs同步的研究成果都是完全同步. 在现实生活中, 一些大型网络往往会被划分为多个集群, 每个集群具有各自的同步目标, 即每个集群内的节点同步, 不同集群之间的节点不同步, 这通常称为聚类同步[13–15]. 文献[13–14]研究了牵制控制下的复杂网络有限时间和固定时间聚类同步问题. 文献[15]基于微分包含和集值映射理论研究了耦合模糊细胞神经网络有限时间聚类同步. 目前, 关于IMNNs固定时间聚类同步的相关结果较少, 这激发了本文的研究动力.

此外, 由于系统在运行过程中往往会受到不确定因素的影响, 而有限时间和固定时间控制具有一定的抗干扰能力. 文献[16–17]提出了一个统一的理论框架来研究具有外部干扰的复杂网络有限时间和固定时间同步问题. 但是目前关于IMNNs有限时间或固定时间控制的研究, 其激活函数大多为连续型, 在实际运用中存在一定的局限性, 因此具有不连续激活函数的神经网络获得了广泛的关注[15].

基于以上讨论, 本文主要研究了具有外部干扰和不连续激活函数的惯性忆阻神经网络固定时间聚类同步问题. 本文主要创新如下:

1) 与完全同步[6–9,11–12]不同, 本文通过设计耦合控制协议, 研究了IMNNs的聚类行为, 实现了IMNNs的聚类同步;

2) 与常见的固定时间控制协议[11–15]相比, 本文设计了一类经济型的固定时间控制器, 利用更少的控制项, 保证了IMNNs以较少的控制消耗达到固定时间聚类同步;

3) 在文献[6–9]的基础上, 本文将激活函数从连续推广到不连续的情形, 利用微分包含和集值映射理论来处理不连续系统, 并验证了系统的抗干扰能力.

2　模型描述与主要引理

本文考虑一类由N个节点耦合而成的具有不连续激活函数与外部干扰的时滞惯性忆阻神经网络模型, 其第i个节点($i\in\mathcal{N}=\{1,2,\cdots,N\}$)的动力学方程表述如下:

$$\begin{aligned}\frac{\mathrm{d}^2x_i(t)}{\mathrm{d}t^2}=&-a_i\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}-b_ix_i(t)+\\&\sum_{h=1}^{n}c_{ih}(x_i(t))f_h(x_h(t))+\\&\sum_{h=1}^{n}d_{ih}(x_i(t))f_h(x_h(t-\tau(t)))+\\&I_i+W_i(t)+U_i(t),\end{aligned}\tag{1}$$

其中: $x_i(t)$是t时刻第i个节点的状态向量; a_i, b_i为正常数; $f_h(\cdot)$是具有可数跳跃点的不连续激活函数; $\tau(t)$是时变时滞; $W_i(t)$是外部干扰, 满足$|W_i(t)|\leqslant W_i$, 且$W_i>0$; I_i是外部输入; $U_i(t)$是控制器; $c_{ih}(x_i(t))$和$d_{ih}(x_i(t))$是网络中忆阻器的连接权重, 满足

$$c_{ih}(x_i(t))=\begin{cases}\hat{c}_{ih},\ g_i(x_i(t))\leqslant T_i,\\ \check{c}_{ih},\ g_i(x_i(t))>T_i,\end{cases}\tag{2}$$

$$d_{ih}(x_i(t))=\begin{cases}\hat{d}_{ih},\ g_i(x_i(t))\leqslant T_i,\\ \check{d}_{ih},\ g_i(x_i(t))>T_i,\end{cases}\tag{3}$$

其中: $T_i>0$为切换边界; $\hat{c}_{ih}$, $\check{c}_{ih}$, $\hat{d}_{ih}$和$\check{d}_{ih}$为已知常数, 且$i,j\in\mathcal{N}$, $g_i(x_i(t))$为阈值函数.

本文考虑的复杂网络, 其图为$G=\{V,E,A\}$, 其中V表示$\{1,\cdots,N\}$的顶点集, E表示$\langle i,j\rangle\in E$的边集, 当且仅当顶点j到顶点i存在一条边. $A=(a_{ij})_{N\times N}$表示节点间的邻接矩阵, 其中$a_{ij}\neq 0$是节点v_i与v_j连接边的权值. 本文不考虑自环, 即$a_{ij}=0$. 对于无向图, 其邻接矩阵A是对称的, 即$a_{ij}=a_{ji}$. 图G的Laplacian矩阵定义为$\varGamma=(\gamma_{ij})_{N\times N}$, 其中当$i=j$时, $\gamma_{ij}=\sum\limits_{j=1}^{N}a_{ij}$; 当$i\neq j$时, $\gamma_{ij}=-a_{ij}$.

为了研究聚类同步, 假设网络中的节点集可以划分为m个簇, $\{1,\cdots,N\}=C_1\cup C_2\cup\cdots\cup C_m$, 其中

$$\begin{cases}C_1=\{1,\cdots,r_1\},\\ C_2=\{r_1+1,\cdots,r_2\},\\ \quad\vdots\\ C_k=\{r_{k-1}+1,\cdots,r_k\},\\ \quad\vdots\\ C_m=\{r_{m-1}+1,\cdots,N\}.\end{cases}\tag{4}$$

定义 1[18]　若对称矩阵$\varGamma\in R^{N\times N}$满足

$$\varGamma=\begin{pmatrix}\varGamma_{11}&\varGamma_{12}&\dots&\varGamma_{1m}\\ \varGamma_{21}&\varGamma_{22}&\dots&\varGamma_{2m}\\ \vdots&\vdots&&\vdots\\ \varGamma_{n1}&\varGamma_{n2}&\dots&\varGamma_{mm}\end{pmatrix},$$

其中: $\Gamma_{uv}=(\gamma_{ij})\in R^{(r_u-r_{u-1})\times(r_v-r_{v-1})}$是零行和矩阵, 且$\sum\limits_{j\in C_v}\gamma_{ij}=0\ (i\in C_u)$; 同时, 每一个对角块$\Gamma_{uu}=(\gamma_{ij})\in R^{k_u-k_{u-1}}\ (k\in\{1,\cdots,m\})$满足$\gamma_{ij}\geqslant 0\ (i\neq j)$及$\gamma_{ij}=-\sum\limits_{j\in C_v}\gamma_{ij}$, 则称其属于$\mathcal{L}$类矩阵, 记作$\Gamma\in\mathcal{L}$.

定义 2[13] 对于具有拓扑结构(4)的复杂网络, 若存在一个时间参数$T>0$, 使得$i,j\in C_k, k=1,2,\cdots,m$时, 有

$$\lim_{t\to T}||x_i(t)-x_j(t)||=0,$$

且对任意$t>T$, 有$x_i(t)\equiv x_j(t)$, 而当$i\in C_{k_1}, j\in C_{k_2}$, $k_1\neq k_2$时, $x_i(t)\neq x_j(t)$, 则称该复杂网络达到固定时间聚类同步.

假设簇C_k中的目标轨道$s_i(t)$为

$$\begin{aligned}\frac{\mathrm{d}^2s_i(t)}{\mathrm{d}t^2}=&-a_i\frac{\mathrm{d}s_i(t)}{\mathrm{d}t}-b_is_i(t)+\\&\sum_{j=1}^{n}c_{ih}(s_i(t))f_h(s_h(t))+I_i+\\&\sum_{j=1}^{n}d_{ih}(s_i(t))f_h(s_h(t-\tau(t))),\end{aligned}\tag{5}$$

且不同簇之间的目标轨道不同, 即$s_{k_i}(t)\neq s_{k_j}(t)$, $k_i\neq k_j\in\{1,\cdots,m\}$.

引入变量替换$y_i(t)=\dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t}+\xi_ix_i(t)$和$z_i(t)=\dfrac{\mathrm{d}s_i(t)}{\mathrm{d}t}+\xi_is_i(t)$, 则系统(1)和(5)分别更新为

$$\begin{cases}\dfrac{\mathrm{d}x_i(t)}{\mathrm{d}t}=-\xi_ix_i(t)+y_i(t),\\ \dfrac{\mathrm{d}y_i(t)}{\mathrm{d}t}=\alpha_ix_i(t)+\beta_iy_i(t)+\\ \quad\sum\limits_{h=1}^{n}c_{ih}(x_i(t))f_h(x_h(t))+\\ \quad\sum\limits_{h=1}^{n}d_{ih}(x_i(t))f_h(x_h(t-\tau(t)))+\\ \quad I_i+W_i(t)+U_i(t)\end{cases}\tag{6}$$

和

$$\begin{cases}\dfrac{\mathrm{d}s_i(t)}{\mathrm{d}t}=-\xi_is_i(t)+z_i(t),\\ \dfrac{\mathrm{d}z_i(t)}{\mathrm{d}t}=\alpha_is_i(t)+\beta_iz_i(t)+\\ \quad\sum\limits_{h=1}^{n}c_{ih}(s_i(t))f_h(s_h(t))+I_i+\\ \quad\sum\limits_{h=1}^{n}d_{ih}(s_i(t))f_h(s_h(t-\tau(t))),\end{cases}\tag{7}$$

其中: ξ_i为正常数, $\alpha_i=-\xi_i^2+a_i\xi_i-b_i$, $\beta_i=\xi_i-a_i$.

定义系统的聚类同步误差为$e_{1i}(t)=x_i(t)-s_i(t)$和$e_{2i}(t)=y_i(t)-z_i(t)$, 则误差系统为

$$\begin{cases}\dfrac{\mathrm{d}e_{1i}(t)}{\mathrm{d}t}=-\xi_ie_{1i}(t)+e_{2i}(t),\\ \dfrac{\mathrm{d}e_{2i}(t)}{\mathrm{d}t}=\alpha_ie_{1i}(t)+\beta_ie_{2i}(t)+\\ \quad\sum\limits_{h=1}^{n}c_{ih}(x_i(t))f_h(x_h(t))+\\ \quad\sum\limits_{h=1}^{n}d_{ih}(x_i(t))f_h(x_h(t-\tau(t)))-\\ \quad\sum\limits_{h=1}^{n}c_{ih}(s_i(t))f_h(s_h(t))-\\ \quad\sum\limits_{h=1}^{n}d_{ih}(s_i(t))f_h(s_h(t-\tau(t)))+\\ \quad W_i(t)+U_i(t).\end{cases}\tag{8}$$

为确保不连续系统(8)解的存在性, 根据Filippov正则化[19], 将系统(8)的Filippov解定义为满足以下微分包含的形式:

$$\begin{cases}\dfrac{\mathrm{d}e_{1i}(t)}{\mathrm{d}t}=-\xi_ie_{1i}(t)+e_{2i}(t),\\ \dfrac{\mathrm{d}e_{2i}(t)}{\mathrm{d}t}\subseteq\alpha_ie_{1i}(t)+\beta_ie_{2i}(t)+\\ \quad\sum\limits_{h=1}^{n}c_{ih}(x_i(t))K[F_h(x_h(t))]-\\ \quad\sum\limits_{h=1}^{n}c_{ih}(s_i(t))K[F_h(s_h(t))]+\\ \quad\sum\limits_{h=1}^{n}d_{ih}(x_i(t))K[F_h(x_h(t-\tau(t)))]-\\ \quad\sum\limits_{h=1}^{n}d_{ih}(s_i(t))K[F_h(s_h(t-\tau(t)))]+\\ \quad W_i(t)+U_i(t).\end{cases}\tag{9}$$

根据可测选择定理[19], 存在

$$\begin{aligned}&\tilde{f}_h(x_h(t))\in K[F_h(x_h(t))],\\&\tilde{f}_h(s_h(t))\in K[F_h(s_h(t))],\\&\tilde{f}_h(x_h(t-\tau(t)))\in K[F_h(x_h(t-\tau(t)))],\\&\tilde{f}_h(s_h(t-\tau(t)))\in K[F_h(s_h(t-\tau(t)))].\end{aligned}$$

使得

$$\begin{cases}\dfrac{\mathrm{d}e_{1i}(t)}{\mathrm{d}t}=-\xi_ie_{1i}(t)+e_{2i}(t),\\ \dfrac{\mathrm{d}e_{2i}(t)}{\mathrm{d}t}=\alpha_ie_{1i}(t)+\beta_ie_{2i}(t)+\\ \quad\sum\limits_{h=1}^{n}c_{ih}(x_i(t))\tilde{f}_h(x_h(t))-\\ \quad\sum\limits_{h=1}^{n}c_{ih}(s_i(t))\tilde{f}_h(s_h(t))+\\ \quad\sum\limits_{h=1}^{n}d_{ih}(x_i(t))\tilde{f}_h(x_h(t-\tau(t)))-\\ \quad\sum\limits_{h=1}^{n}d_{ih}(s_i(t))\tilde{f}_h(s_h(t-\tau(t)))+\\ \quad W_i(t)+U_i(t),\end{cases}\tag{10}$$

该误差系统Filippov解的存在性可以根据Gronwall不等式和Leray-Schauder选择定理得出(具体可参见文献[20–22]).

假设 1　非连续激活函数$f_i(\cdot)$满足以下条件:

1. 存在正常数M_i, 使得$|f_i(z)| \leqslant M_i, \forall z \in \mathbb{R}$, $i \in \mathcal{N}$;

2. 存在非负常数l_i和ϵ_i $(i \in \mathcal{N})$使得

$$\sup |p_i - q_i| \leqslant l_i|x-y| + \epsilon_i,\ x,\ y \in \mathbb{R},$$

其中: $p_i \in K[f_i(x)]$, $q_i \in K[f_i(y)]$, $K[f_i] = [\min\{f_i^-, f_i^+\}, \max\{f_i^-, f_i^+\}]$.

引理 1[23]　对于系统$\dot{x}(t) = f(x(t))$, 如果存在一个连续、正定且径向无界的函数$V(x(t)) : \mathbb{R}^n \to \mathbb{R}$, 使得其任意解$x(t)$满足不等式

$$\frac{\mathrm{d}}{\mathrm{d}t}V(x(t)) \leqslant \begin{cases} \lambda V(x(t)) - \mu_1(V(x(t)))^p,\ 0 \leqslant V(x(t)) < 1, \\ \lambda V(x(t)) - \mu_2(V(x(t)))^p,\ V(x(t)) \geqslant 1, \end{cases}$$

其中: $p = \theta + \mathrm{sgn}(V(x(t)) - 1)$, $\lambda < \min\{\mu_1, \mu_2\}$, $\mu_1 > 0$, $\mu_2 > 0$, $1 \leqslant \theta < 2$, 则该系统是全局固定时间稳定的, 且收敛时间的上界表示为

$$\begin{cases} T_1 = \dfrac{1}{\lambda(2-\theta)}\ln\dfrac{\mu_1}{\mu_1 - \lambda} + \dfrac{1}{\theta(\mu_2 - \lambda)}, & \lambda > 0, \\ T_2 = \dfrac{1}{\mu_1(2-\theta)} + \dfrac{1}{\mu_2\theta}, & \lambda = 0, \\ T_3 = \dfrac{1}{\lambda(2-\theta)}\ln\dfrac{\mu_1}{\mu_1 - \lambda} + \dfrac{1}{\lambda\theta}\ln\dfrac{\mu_2}{\mu_2 - \lambda}, & \lambda < 0. \end{cases}$$

引理 2[24]　若$x_1,\ x_2, \cdots,\ x_n \geqslant 0$, $0 < p \leqslant 1$和$q > 1$, 则

$$\sum_{i=1}^{n} x_i^p \geqslant (\sum_{i=1}^{n} x_i)^p,\ \sum_{i=1}^{n} x_i^q \geqslant n^{1-q}(\sum_{i=1}^{n} x_i)^q.$$

3　主要结论

设计控制器$U_i(t) = U_{1i}(t) + U_{2i}(t)$, 其中:

$$\begin{aligned} U_{1i}(t) = &-\phi_{1i}\mathrm{sgn}(e_{2i}(t))|e_{1i}(t)| - \\ &\phi_{2i}e_{2i}(t) - \eta_i\mathrm{sgn}(e_{2i}(t)) - \\ &\psi_i\mathrm{sgn}(e_{2i}(t))|e_{1i}(t-\tau(t))|; \end{aligned} \tag{11}$$

$$\begin{aligned} U_{2i}(t) = &\rho\sum_{j \in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t) - e_{1i}(t)) + \\ &\sum_{k' \neq k}\mathrm{sig}^p(\sum_{j \in C_{k'}}\gamma_{ij}e_{1j}(t)) + \\ &\delta\sum_{j \in C_k}\gamma_{ij}\mathrm{sig}^p(e_{2j}(t) - e_{2i}(t)) + \\ &\sum_{k' \neq k}\mathrm{sig}^p(\sum_{j \in C_{k'}}\gamma_{ij}e_{2j}(t)); \end{aligned} \tag{12}$$

$\phi_{1i}, \phi_{2i}, \psi_i, \eta_i, \rho$和$\delta$为控制器增益; 函数$\mathrm{sig}^p(x) = (\mathrm{sgn}(x^1)|x^1|^p\ \ \mathrm{sgn}(x^2)|x^2|^p\ \cdots\ \mathrm{sgn}(x^n)|x^n|^p)^{\mathrm{T}}$, 其中: $p = \theta + \mathrm{sgn}(||e(t)||_1 - 1)$, $1 \leqslant \theta < 2$.

记 $\bar{\gamma}_{ij} = \gamma_{ij}^{\frac{2}{p}}$, $\tilde{\gamma} = \max\limits_{i \in C_k, j \in C_{k'}, k \neq k'}\{|\gamma_{ij}|\}$,

$$\begin{aligned} &\bar{\varGamma}_{kk} = (\bar{\gamma}_{ij})_{(r_k - r_{k-1}) \times (r_k - r_{k-1})}, \\ &\bar{r} = \max_{k=1,\cdots,m}\{N - (r_k - r_{k-1})\}, \\ &\bar{N} = n\sum_{k=1}^{m}\frac{(r_k - r_{k-1})(r_k - r_{k-1} - 1)}{2}, \\ &\sigma_1 = \tilde{\gamma}^p\bar{r},\ \sigma_2 = \tilde{\gamma}^p\bar{r}(Nn)^{1-p}, \\ &\bar{\rho}_1 = 2^{\frac{p}{2}}\rho(Nn)^{-1}\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))^{\frac{p}{2}}, \\ &\bar{\rho}_2 = 2^{\frac{p}{2}}\rho\bar{N}^{1-\frac{p}{2}}(Nn)^{-1}\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))^{\frac{p}{2}}, \\ &\bar{\delta}_1 = 2^{\frac{p}{2}}\delta(Nn)^{-1}\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))^{\frac{p}{2}}, \\ &\bar{\delta}_2 = 2^{\frac{p}{2}}\delta\bar{N}^{1-\frac{p}{2}}(Nn)^{-1}\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))^{\frac{p}{2}}, \\ &\pi_1 = \min\{\bar{\rho}_1 - \sigma_1, \bar{\delta}_1 - \sigma_1\}, \\ &\pi_2 = \min\{\bar{\rho}_2 - \sigma_2, \bar{\delta}_2 - \sigma_2\}, \\ &\bar{c}_{ih} = \max\{\hat{c}_{ih}, \check{c}_{ih}\},\ \underline{c}_{ih} = \min\{\hat{c}_{ih}, \check{c}_{ih}\}, \\ &\tilde{c}_{ih} = \max\{|\hat{c}_{ih}|, |\check{c}_{ih}|\},\ \bar{d}_{ih} = \max\{\hat{d}_{ih}, \check{d}_{ih}\}, \\ &\underline{d}_{ih} = \min\{\hat{d}_{ih}, \check{d}_{ih}\},\ \tilde{d}_{ih} = \max\{|\hat{d}_{ih}|, |\check{d}_{ih}|\}. \end{aligned}$$

定理 1　当网络拓扑的Laplacian矩阵$\varGamma = (\gamma_{ij})_{N \times N} \in \mathcal{L}$时, 惯性忆阻神经网络(6)在控制器$U_i(t)$下, 将会达到固定时间聚类同步, 如果$\pi_1 > 0$, $\pi_2 > 0$, $\lambda < \min\{\pi_1, \pi_2\}$, 并且对于任意的$i \in \mathcal{N}$, 满足以下不等式:

$$\psi_i \geqslant \sum_{h=1}^{n} l_i\tilde{d}_{ih}, \tag{13}$$

$$\eta_i > \varTheta_i, \tag{14}$$

其中: $\lambda = \min\limits_{i \in \mathcal{N}}\{-\xi_i + |\alpha_i| + \sum\limits_{h=1}^{n} l_i\tilde{c}_{ih} + \phi_{1i},\ 1 + \beta_i + \phi_{2i}\}$, $\varTheta_i = \sum\limits_{h=1}^{n}(\tilde{c}_{ih}\epsilon_i + \tilde{d}_{ih}\epsilon_i) + \sum\limits_{h=1}^{n}(|\bar{c}_{ih} - \underline{c}_{ih}| + |\bar{d}_{ih} - \underline{d}_{ih}|)M_h + W_i$.

证　将式(10)的右端记为$\zeta(t)$, 构造如下Lyapunov泛函:

$$V(t) = ||e(t)||_1 = \sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}|e_{1i}(t)| + \sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}|e_{2i}(t)|. \tag{15}$$

沿误差系统(10)对$V(t)$关于时间t求集值李导数, 可得

$$\begin{aligned} &L_\zeta V(t) = \\ &[\partial V(t)]^{\mathrm{T}} \cdot F[\zeta(t)] \subseteq \\ &\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm{T}}(e_{1i}(t))\dot{e}_{1i}(t) + \\ &\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\dot{e}_{2i}(t) = \end{aligned}$$

$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm T}(e_{1i}(t))[-\xi_i e_{1i}(t)+e_{2i}(t)]+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))[\alpha_i e_{1i}(t)+$$
$$\beta_i e_{2i}(t)+\sum_{h=1}^{n}c_{ih}(x_i(t))\tilde f_h(x_h(t))+$$
$$\sum_{h=1}^{n}d_{ih}(x_i(t))\tilde f_h(s_h(t-\tau(t)))-$$
$$\sum_{h=1}^{n}c_{ih}(s_i(t))\tilde f_h(s_h(t))-$$
$$\sum_{h=1}^{n}d_{ih}(s_i(t))\tilde f_h(x_h(t-\tau(t)))+$$
$$W_i(t)+\tilde U_i(t)]. \tag{16}$$

其中

$$\begin{aligned}\tilde U_i(t)=&-\phi_{1i}\mathrm{SGN}(e_{2i}(t))|e_{1i}(t)|-\phi_{2i}e_{2i}(t)-\\&\psi_i\mathrm{SGN}(e_{2i}(t))|e_{1i}(t-\tau(t))|-\\&\eta_i\mathrm{SGN}(e_{2i}(t))+\rho\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t)-\\&e_{1i}(t))+\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}(t))+\\&\delta\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{2j}(t)-e_{2i}(t))+\\&\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{2j}(t)),\end{aligned} \tag{17}$$

且集值函数SGN(x)定义为

$$\mathrm{SGN}(x)=\begin{cases}-1, & x<0,\\ [-1,1], & x=0,\\ 1, & x>0.\end{cases} \tag{18}$$

根据假设1可得

$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))[\sum_{h=1}^{n}c_{ih}(x_i(t))\times$$
$$\tilde f_h(x_h(t))-\sum_{h=1}^{n}c_{ih}(s_i(t))\tilde f_h(s_h(t))]=$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))[\sum_{h=1}^{n}c_{ih}(x_i(t))\times$$
$$[\tilde f_h(x_h(t))-\tilde f_h(s_h(t))]+\sum_{h=1}^{n}[c_{ih}(x_i(t))-$$
$$c_{ih}(s_i(t))]\tilde f_h(s_h(t))]\leqslant$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}[\sum_{h=1}^{n}l_i\tilde c_{ih}|e_{1i}(t)|+$$
$$\sum_{h=1}^{n}\tilde c_{ih}\epsilon_i+\sum_{h=1}^{n}|\bar c_{ih}-\underline{c}_{ih}|M_h]. \tag{19}$$

同理可得

$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))[\sum_{h=1}^{n}d_{ih}(x_i(t))\times$$
$$\tilde f_h(x_h(t-\tau(t)))-\sum_{h=1}^{n}d_{ih}(s_i(t))\times$$
$$\tilde f_h(s_h(t-\tau(t)))]\leqslant$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}[\sum_{h=1}^{n}l_i\tilde d_{ih}|e_{1i}(t-\tau(t))|+$$
$$\sum_{h=1}^{n}\tilde d_{ih}\epsilon_i+\sum_{h=1}^{n}|\bar d_{ih}-\underline{d}_{ih}|M_h]. \tag{20}$$

另一方面, 可以得到

$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm T}(e_{1i}(t))\tilde U_i(t)\leqslant$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\phi_{1i}|e_{1i}(t)|+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\phi_{2i}|e_{2i}(t)|+\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\eta_i+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\psi_i|e_{1i}(t-\tau(t))|+$$
$$\rho\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$
$$\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$
$$\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}(t))+$$
$$\delta\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$
$$\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{2j}(t)-e_{2i}(t))+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$
$$\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{2j}(t)). \tag{21}$$

结合式(16)–(21), 并根据条件(13)–(14), 计算可得

$$L_\zeta V(t)\leqslant$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}(-\xi_i+|\alpha_i|+\sum_{h=1}^{n}l_i\tilde c_{ih}+\phi_{1i})|e_{1i}(t)|+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}(1+\beta_i+\phi_{2i})|e_{2i}(t)|+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}(\sum_{h=1}^{n}l_i\tilde d_{ih}+\psi_i)|e_{1i}(t-\tau(t))|+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}[\sum_{j=1}^{n}(|\bar c_{ih}-\underline{c}_{ih}|+|\bar d_{ih}-\underline{d}_{ih}|)M_h+$$
$$\sum_{h=1}^{n}\tilde c_{ih}\epsilon_i+\sum_{h=1}^{n}\tilde d_{ih}\epsilon_i+W_i+\eta_i]+$$
$$\rho\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$
$$\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))+$$
$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$
$$\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}(t))+$$
$$\delta\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm T}(e_{2i}(t))\times$$

$$\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{2j}(t)-e_{2i}(t))+\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{2j}(t))\leqslant\lambda V(t)+\omega_1(t)+\omega_2(t),\tag{22}$$

其中

$$\begin{cases}\omega_1(t)=\sum\limits_{k=1}^{m}\sum\limits_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times\\ \quad[\rho\sum\limits_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))+\\ \quad\sum\limits_{k'\neq k}\mathrm{sig}^p(\sum\limits_{j\in C_{k'}}\gamma_{ij}e_{1j}(t))],\\ \omega_2(t)=\sum\limits_{k=1}^{m}\sum\limits_{i=r_{k-1}+1}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times\\ \quad[\delta\sum\limits_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{2j}(t)-e_{2i}(t))+\\ \quad\sum\limits_{k'\neq k}\mathrm{sig}^p(\sum\limits_{j\in C_{k'}}\gamma_{ij}e_{2j}(t))].\end{cases}\tag{23}$$

根据引理2, 对式(22)进行讨论.

情 形 1 当$0\leqslant V(t)<1$, $0\leqslant||e(t)||_1<1$, 且$p=\theta+(||e(t)||_1-1)=\theta-1<1$ 时, 可得

$$\begin{aligned}&\rho\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))=\\&\frac{\rho}{2}\sum_{k=1}^{m}\sum_{i,j\in C_k}\gamma_{ij}(\mathrm{SGN}(e_{2i}(t))-\mathrm{SGN}(e_{2j}(t)))^{\mathrm{T}}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))\leqslant\\&-\rho\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{i,j\in C_k}\gamma_{ij}|e_{1j}^l(t)-e_{1i}^l(t)|^p\leqslant\\&-\rho[\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{i,j\in C_k}\gamma_{ij}^{2/p}|e_{1j}^l(t)-e_{1i}^l(t)|^2]^{p/2}=\\&-\rho[\sum_{k=1}^{m}E_k^{\mathrm{T}}(t)[(-2\bar{\varGamma}_{kk})\otimes I]E_k(t)]^{p/2}\leqslant\\&-\rho[2\sum_{k=1}^{m}\lambda_2(-\bar{\varGamma}_{kk})E_k^{\mathrm{T}}(t)E_k(t))]^{p/2}\leqslant\\&-\rho[2\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))(\sum_{k=1}^{m}\sum_{i\in C_k}e_{1i}^2(t))]^{p/2}\leqslant\\&-2^{p/2}\rho(Nn)^{-1}\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))^{p/2}\times[\sum_{k=1}^{m}\sum_{i\in C_k}e_{1i}(t)]^p=\\&-\bar{\rho}_1[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p.\end{aligned}\tag{24}$$

并且,

$$\begin{aligned}&\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}(t))\leqslant\\&\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{k'\neq k}\sum_{i=r_{k-1}+1}^{r_k}|\mathrm{SGN}(e_{2i}^l(t))|\times|\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}^l(t)|^p\leqslant\\&\tilde{\gamma}^p\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{k'\neq k}\sum_{i\in C_k}\sum_{j\in C_{k'}}|e_{1j}^l(t)|^p=\\&\tilde{\gamma}^p\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}^l(t)|^p[N-(r_k-r_{k-1})]\leqslant\\&\tilde{\gamma}^p\max_{k=1,\cdots,m}[N-(r_k-r_{k-1})]\times\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{1i}^l(t)|^p\leqslant\\&\tilde{\gamma}^p\bar{r}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{1i}^l(t)|^p\leqslant\\&\sigma_1[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p.\end{aligned}\tag{25}$$

将式(24)与式(25)结合, 可得

$$\omega_1(t)\leqslant-\bar{\rho}_1[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p+\sigma_1[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p.\tag{26}$$

同理可得

$$\omega_2(t)\leqslant-\bar{\delta}_1[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{2i}(t)|]^p+\sigma_1[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{2i}(t)|]^p.\tag{27}$$

综上可得

$$\begin{aligned}&\omega_1(t)+\omega_2(t)\leqslant\\&-(\bar{\rho}_1-\sigma_1)[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p-\\&(\bar{\delta}_1-\sigma_1)[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{2i}(t)|]^p\leqslant\\&-\pi_1V^p(t).\end{aligned}\tag{28}$$

情形 2 当$V(t)\geqslant1$, $||e(t)||_1\geqslant1$且

$$p=\theta+\mathrm{sgn}(||e(t)||_1-1)\geqslant\gamma>1,$$

可得

$$\begin{aligned}&\rho\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times\sum_{j\in C_k}\gamma_{ij}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))=\\&\frac{\rho}{2}\sum_{k=1}^{m}\sum_{i,j\in C_k}\gamma_{ij}(\mathrm{SGN}(e_{2i}(t))-\mathrm{SGN}(e_{2j}(t)))^{\mathrm{T}}\mathrm{sig}^p(e_{1j}(t)-e_{1i}(t))\leqslant\\&-\rho\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{i,j\in C_k}\gamma_{ij}|e_{1j}^l(t)-e_{1i}^l(t)|^p\leqslant\\&-\rho\bar{N}^{1-p/2}[\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{i,j\in C_k}\gamma_{ij}^{2/p}|e_{1j}^l(t)-e_{1i}^l(t)|^2]^{p/2}=\end{aligned}$$

$$-\rho\bar{N}^{1-p/2}[\sum_{k=1}^{m}E^{\mathrm{T}}(t)[(-2\bar{\varGamma}_{kk})\otimes I]E(t)]^{p/2}\leqslant$$
$$-\rho\bar{N}^{1-p/2}[2\sum_{k=1}^{m}\lambda_2(-\bar{\varGamma}_{kk})E^{\mathrm{T}}(t)E(t)]^{p/2}\leqslant$$
$$-\rho\bar{N}^{1-p/2}[2\min_{k=1,\cdots,m}(-\lambda_2(\bar{\varGamma}_{kk}))\times$$
$$(\sum_{k=1}^{m}\sum_{i\in C_k}e_{1i}^2(t))]^{p/2}\leqslant$$
$$-2^{p/2}\rho\bar{N}^{1-p/2}(Nn)^{-1}\min_{k=1,\cdots,m}(-\lambda_2(\varGamma_{kk}))^{p/2}\times$$
$$[\sum_{k=1}^{m}\sum_{i\in C_k}e_{1i}(t)]^p=-\bar{\rho}_2[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p. \tag{29}$$

并且,

$$\sum_{k=1}^{m}\sum_{i=r_{k-1}+1}^{r_k}\mathrm{SGN}^{\mathrm{T}}(e_{2i}(t))\times$$
$$\sum_{k'\neq k}\mathrm{sig}^p(\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}(t))\leqslant$$
$$\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{k'\neq k}\sum_{i=r_{k-1}+1}^{r_k}|\mathrm{SGN}(e_{2i}^l(t))|\times$$
$$|\sum_{j\in C_{k'}}\gamma_{ij}e_{1j}^l(t)|^p\leqslant$$
$$\tilde{\gamma}^p\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{k'\neq k}\sum_{i\in C_k}\sum_{j\in C_{k'}}|e_{1j}^l(t)|^p=$$
$$\tilde{\gamma}^p\sum_{l=1}^{n}\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}^l(t)|^p[N-(r_k-r_{k-1})]\leqslant$$
$$\tilde{\gamma}^p\max_{k=1,\cdots,m}[N-(r_k-r_{k-1})]\times$$
$$\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{1i}^l(t)|^p\leqslant$$
$$\tilde{\gamma}^p\bar{r}\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{1i}^l(t)|^p\leqslant$$
$$\tilde{\gamma}^p\bar{r}(Nn)^{1-p}[\sum_{k=1}^{m}\sum_{i\in C_k}\sum_{l=1}^{n}|e_{1i}^l(t)|]^p\leqslant$$
$$\sigma_2[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p. \tag{30}$$

与情形1类似, 可得

$$\omega_1(t)+\omega_2(t)\leqslant$$
$$-\bar{\rho}_2[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p+\sigma_2[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p-$$
$$\bar{\delta}_2[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{2i}(t)|]^p+\sigma_2[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{2i}(t)|]^p\leqslant$$
$$-(\bar{\rho}_2-\sigma_2)[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{1i}(t)|]^p-$$
$$(\bar{\delta}_2-\sigma_2)[\sum_{k=1}^{m}\sum_{i\in C_k}|e_{2i}(t)|]^p\leqslant$$
$$-\pi_2V^p(t). \tag{31}$$

综上所述, 可得

$$L_\zeta V(t)\leqslant$$
$$\begin{cases}\lambda V(t)-\pi_1V(t)^{\theta+\mathrm{sgn}(V(t)-1)}, & 0\leqslant V(t)<1,\\ \lambda V(t)-\pi_2V(t)^{\theta+\mathrm{sgn}(V(t)-1)}, & V(t)\geqslant 1.\end{cases}$$

基于引理1, 可得惯性忆阻神经网络(6)–(7)将实现固定时间聚类同步, 相应的时间上界如下:

$$\begin{cases}T_1=\dfrac{1}{\lambda(2-\theta)}\ln\dfrac{\pi_1}{\pi_1-\lambda}+\dfrac{1}{\theta(\pi_2-\lambda)}, & \lambda>0,\\ T_2=\dfrac{1}{\pi_1(2-\theta)}+\dfrac{1}{\pi_2\theta}, & \lambda=0,\\ T_3=\dfrac{1}{\lambda(2-\theta)}\ln\dfrac{\pi_1}{\pi_1-\lambda}+\dfrac{1}{\lambda\theta}\ln\dfrac{\pi_2}{\pi_2-\lambda}, & \lambda<0.\end{cases}$$

证毕.

注 1 近年来, 关于IMNNs固定时间同步的研究越来越多[11–12], 其中大部分使用的固定时间引理为$\dot{V}(x(t))\leqslant -a\times V^p(x(t))-bV^q(x(t))(0<p<1,q>1)$. 因此, 控制器通常包括$\mathrm{sgn}(e_1(t))(|e_1^p|+|e_2^p|)$, $\mathrm{sgn}(e_2(t))(|e_1^q|+|e_2^q|)$两项, 分别在同步误差$|e_i(t)|$小于1和大于1时起作用. 由于这两项的控制过程会发生重叠, 从而导致部分能源浪费. 文献[23]将两项指数融合为$\theta+\mathrm{sgn}(V(x(t))-1)(1<\theta<2)$. 通过判断$V(t)\geqslant 1$和$0<V(t)<1$的两种情况进行智能切换, 从而实现固定时间同步. 受此启发, 本文设计了这类经济型固定时间聚类控制器$U_i(t)$, 在文献[13]的基础上进一步降低了固定时间聚类控制成本.

注 2 考虑到群体间的差异性, 本文研究了IMNNs的聚类同步, 即同一集群中的节点可以实现完全同步, 而不同集群的节点将不会发生同步. 当网络中所有节点都位于同一个集群时, 本文的聚类同步将退化为完全同步[6–9, 11–12].

注 3 目前大部分关于IMNNs同步研究的结果都是基于连续的激活函数, 而在实际工程中不连续激活函数是普遍存在的. 因此, 本文利用微分包含和集值映射理论研究了非连续的网络系统. 此外, 定理1以及接下来的数值仿真也验证了固定时间控制是处理外部干扰的有效工具, 其可以提高神经网络的鲁棒性, 增强系统的实用价值.

4 数值仿真

为验证本文所提出方法的有效性, 给出如下仿真.

考虑由5个一维神经元组成的惯性忆阻神经网络(6), 将其划分为两个簇$C_1=\{1,2\}$和$C_2=\{3,4,5\}$. 相应的耦合矩阵为

$$\varGamma=\begin{bmatrix}\varGamma_{11}&\varGamma_{12}\\ \varGamma_{21}&\varGamma_{22}\end{bmatrix},\ \varGamma_{11}=\begin{bmatrix}1&-1\\1&-1\end{bmatrix},\ \varGamma_{21}=\varGamma_{12}^{\mathrm{T}},$$

$$\varGamma_{12}=\begin{bmatrix}-0.1&0.3&-0.2\\0.1&-0.3&0.2\end{bmatrix},\ \varGamma_{22}=\begin{bmatrix}-2&1&1\\1&-2&1\\1&1&-2\end{bmatrix}.$$

激活函数$f_h(x_i(t))=0.05\times(\mathrm{sgn}(x_i(t))+0.5)$, 模型参数$a_i=3, b_i=1, \xi_i=2, \alpha_i=1, \beta_i=-1$, 时滞$\tau=3\times\cos(x_i(t))$, 输入$I_i=0$, 外部干扰$W_i(t)=\sin(x_i(t))$, 假设1中$M_i=0.1$, $l_i=0.1$, 忆阻权重如下:

$$c_{1i}=\begin{cases}0.4, & |x_i(t)|\leqslant 0.05,\\ 0.5, & |x_i(t)|>0.05,\end{cases}$$

$$d_{1i}=\begin{cases}0.6, & |x_i(t)|\leqslant 0.05,\\ 0.8, & |x_i(t)|>0.05.\end{cases}$$

选取初值为$x_1(0)=0.8$, $x_2(0)=1.5$, $x_3(0)=-1.5$, $x_4(0)=-1$, $x_5(0)=0.4$和$y_1(0)=0.8$, $y_2(0)=1.5$, $y_3(0)=-1.5$, $y_4(0)=-1$, $y_5(0)=0.4$. 目标系统(7)的初值为$s_1(0)=0.5$, $s_2(0)=-0.5$和$z_1(0)=0.5$, $z_2(0)=-0.5$. 选择控制器参数为$\psi_i=0.1$, $\eta_i=1.5$, $\rho=8$, $\delta=8$, $\theta=1.5$.

当参数$\phi_{1i}=2$, $\phi_{2i}=2$ 时, $\lambda>0$, 惯性忆阻神经网络(6)将在固定时间$T_1=5.48$内聚类同步到目标轨道(7), 系统状态轨迹如图1–2所示. 当$\phi_{1i}=1$, $\phi_{2i}=0$时, $\lambda=0$, 从图3–4可以看出, 系统(6)的状态$x_i(t)$和$y_i(t)$均能够在$T_2=4.25$内同步到目标状态$s_i(t)$和$z_i(t)$. 图5–6为当$\phi_{1i}=0$, $\phi_{2i}=0$, $\lambda<0$时系统(6)–(7)的状态轨迹, 收敛时间为$T_3=3.63$.

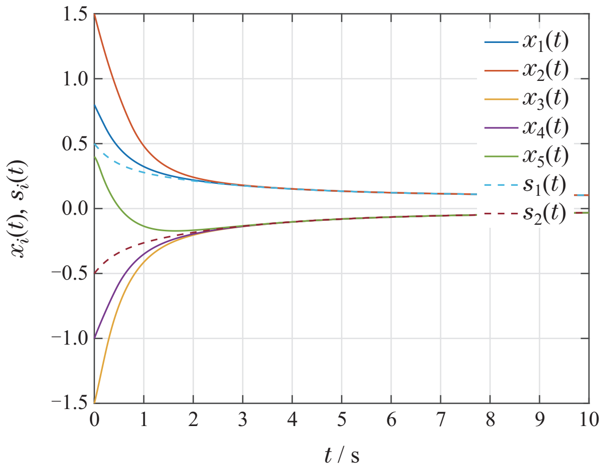

图 3　系统(6)的$x_i(t)$与目标系统(7)的$s_i(t)$的运动轨迹($\lambda=0$ 时)

Fig. 3　Trajectories of $x_i(t)$ for system (6) and $s_i(t)$ for target system (7) ($\lambda=0$)

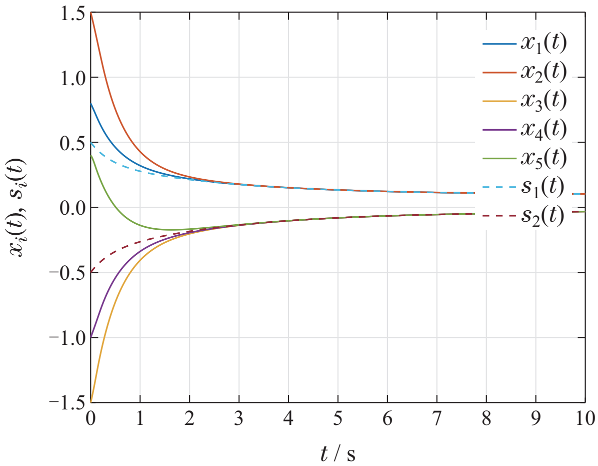

图 1　系统(6)的$x_i(t)$与目标系统(7)的$s_i(t)$的运动轨迹($\lambda>0$时)

Fig. 1　Trajectories of $x_i(t)$ for system (6) and $s_i(t)$ for target system (7) ($\lambda>0$)

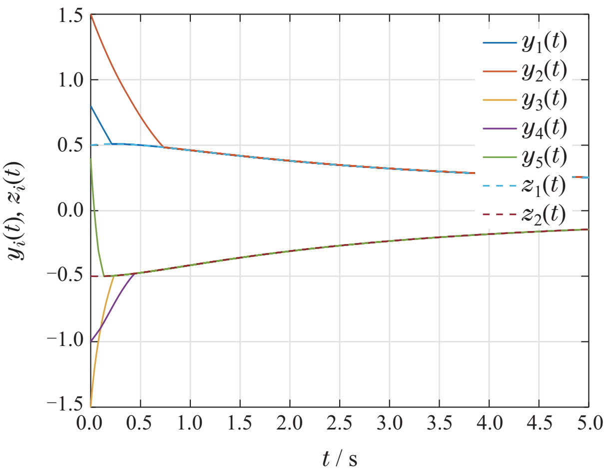

图 4　系统(6)的$y_i(t)$与目标系统(7)的$z_i(t)$的运动轨迹($\lambda=0$ 时)

Fig. 4　Trajectories of $y_i(t)$ for system (6) and $z_i(t)$ for target system (7) ($\lambda=0$)

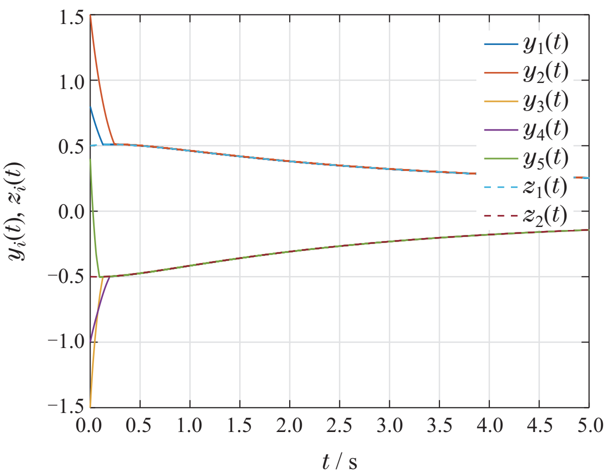

图 2　系统(6)的$y_i(t)$与目标系统(7)的$z_i(t)$的运动轨迹($\lambda>0$ 时)

Fig. 2　Trajectories of $y_i(t)$ for system (6) and $z_i(t)$ for target system (7) ($\lambda>0$)

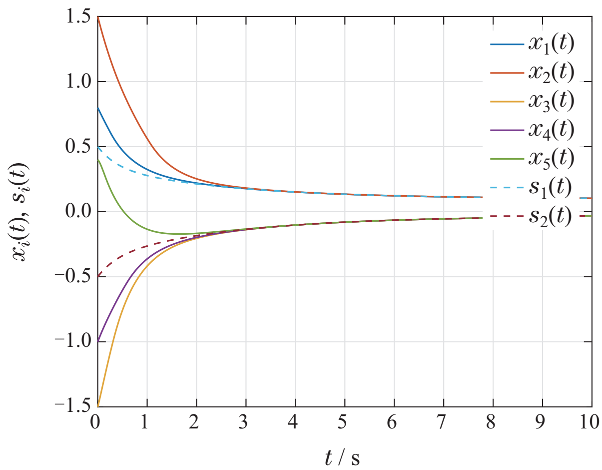

图 5　系统(6)的$x_i(t)$与目标系统(7)的$s_i(t)$的运动轨迹($\lambda<0$ 时)

Fig. 5　Trajectories of $x_i(t)$ for system (6) and $s_i(t)$ for target system (7) ($\lambda<0$)

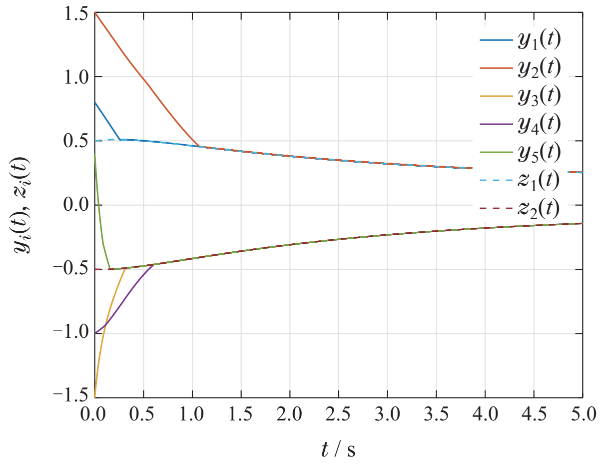

图 6 系统(6)的$y_i(t)$与目标系统(7)的$z_i(t)$的运动轨迹($\lambda < 0$ 时)

Fig. 6 Trajectories of $y_i(t)$ for system (6) and $z_i(t)$ for target system (7) ($\lambda < 0$)

5 结论

本文对一类具有外部干扰的惯性忆阻神经网络固定时间聚类同步问题进行了研究. 利用较少的控制项, 设计了经济型抗干扰聚类同步控制器, 降低了网络的能量消耗. 通过构造合适的Lyapunov泛函, 获得IMNNs固定时间聚类同步的充分判据. 此外, 利用微分包含和集值映射理论, 成功地将网络激活函数推广至不连续情形, 增强了网络实际的应用范围. 最后通过仿真算例验证了本文理论结果的有效性.

在未来的工作中, 将考虑二分同步问题, 并结合不同控制策略, 如牵制控制、间歇控制或脉冲控制进行研究. 目前, 复值神经网络和四元数神经网络也是值得深入探索的领域[25].

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