线性系统的事件触发输出反馈有限时间有界控制

杨中林, 魏自航, 林相泽†

(南京农业大学 人工智能学院, 江苏 南京 210031)

摘要: 本文研究了线性系统的事件触发输出反馈有限时间有界控制问题. 与渐近稳定只定性地要求系统在采样间隔有界不同, 有限时间有界需要估计系统轨迹的上界以保证满足动态系统的定量要求. 本文基于类李雅普诺夫函数给出了保证闭环系统的有限时间有界性和避免芝诺现象的充分条件. 这些充分条件可以转化为线性矩阵不等式, 便于验证和实际应用. 此外, 为了节约资源, 提出了一种可变参数的事件触发规则, 提高了设计灵活性. 仿真结果验证了本文的主要结论.

关键词: 线性系统; 有限时间有界; 事件触发; 输出反馈; 非周期采样

引用格式: 杨中林, 魏自航, 林相泽. 线性系统的事件触发输出反馈有限时间有界控制. 控制理论与应用, 2022, 39(8): 1497 – 1505

DOI: 10.7641/CTA.2022.10865

Event-triggered output feedback finite-time bounded control of linear systems

YANG Zhong-lin, WEI Zi-hang, LIN Xiang-ze†

(College of Artificial Intelligence, Nanjing Agricultural University, Nanjing Jiangsu 210031, China)

Abstract: In this paper, event-triggered output feedback finite-time bounded control of linear systems is investigated. Unlike asymptotic stability which only requires qualitatively that the system is bounded during inter-sample, for finite-time boundedness, it is necessary to estimate the upper bound of the trajectory to ensure that the system meets the quantitative requirements. Based on Lyapunov-like function, sufficient conditions are given to ensure finite-time boundedness of the resultant closed-loop system and to avoid Zeno behavior in an emulation context. The conditions can be transformed into linear matrix inequalities such that they are easy to be verified and used in practice. Moreover, an event-triggered rule with variable parameters, which can improve the flexibility of design, is also proposed, aiming at conserving sources. Simulation results are employed to verify main results proposed in this paper.

Key words: linear systems; finite-time boundedness; event-triggered control; output feedback; aperiodic sampling

Citation: YANG Zhonglin, WEI Zihang, LIN Xiangze. Event-triggered output feedback finite-time bounded control of linear systems. *Control Theory & Applications*, 2022, 39(8): 1497 – 1505

1 Introduction

With the development of wire and wireless technologies, modern control systems have been operated by digital communication networks. In standard control books such as [1–2], periodic control is presented as the only choice for implementing feedback control laws on digital platforms. However, classical sampled-data control is based on performing sensing and actuation periodically. For a type of large-scale resource-constrained wireless embedded control systems, it is desirable to limit the sensor and control computation and communication to instances that the system needs attention. Hence, an aperiodic digital control called event-triggered control (ETC) [3–8] has been proposed to avoid the waste of sources and the congestion of computing devices or communication networks.

Up to present, for research on event-triggered control, most existing literatures focus on asymptotical stability (AS) [9–14]. It is well known that asymptotical stability of a system is a qualitative analysis over an infinite time interval. However, in many practical applications, such as missile systems, analog computer systems, active and parametric networks, the main concern is the quantitative behavior of the system over a fixed

Received 13 September 2021; Accepted 18 February 2022.
†Corresponding author. E-mail: xzlin@njau.edu.cn; Tel.: +86 13813936819.
Recommended by Associate Editor: LONG Li-jun.
Supported by National Natural Science Foundation of China (62173185, 61773216)

finite-time interval [15–19]. In these cases, finite-time stability, which focuses its attention on the system behavior over a finite-time interval, is more practical than asymptotical stability analysis of a system. In recent years, finite-time stability was extended to finite-time boundedness (FTB) [16, 18]. Most of existing results focus on conventional feedback control or classical sampled-data control and FTB of dynamical systems via event-triggered control is still in its embryonic stage.

ETC scheme, which is reactive and generates sensor sampling and control actuation when the triggering condition based on current measurements is violated [6], has been widely applied to deal with many control issues, such as state or output feedback control [5, 11–13, 20], because it can efficiently reduce the update number of control signal so that communication resources can be saved significantly. The first important issue is that ETC schemes should guarantee a positive minimum inter-event time (MIET) which is the minimal time distance between any two consecutive event time instants. Otherwise, Zeno behaviour occurs. It was proven that for state feedback control of linear systems without external disturbances the positive MIET is guaranteed to exist [5]. While, for an output feedback controller in a similar setup, the positive MIET may not exist [21]. Moreover, it was theoretically proven that with arbitrary small external disturbances, the positive MIET possibly becomes zero even though it can be ensured in the absence of external disturbances [22]. Hence, it is not an easy task to ensure that finite-time boundedness of the closed-loop systems in ETC output feedback scheme to avoid Zeno behaviour because for FTB, the presence of external disturbances is a indispensable part of the definition, which motivates this study.

For ETC schemes, the joint design of the feedback controller and the triggering condition is another important issue. In asymptotical stability of the closed-loop system via event-triggered control, emulation-based approaches are the common choice in practice [5, 9–10, 14]. Hence, in this paper, an emulation-based approach is also used to consider the triggering condition with a priori that the output feedback controller is given to make the closed-loop system finite-time bounded. However, as everyone knows, the disadvantage of such an emulation-based approach is that it is impossible to obtain an optimal joint design of the feedback controller and the triggering condition [10–11]. Hence, in this paper, to overcome this disadvantage, a free parameter is introduced in the design process of the triggering condition to minimize the update number of control signals while finite-time boundedness of the resultant closed-loop system is still guaranteed.

Main contribution and innovation of this note are listed as follows: 1) Event-triggered output feedback finite-time bounded control for linear systems is discussed, and the ETC strategies, including the triggering condition which can be tuned by regulating a free parameter, is proposed for the first time, to the best of authors' knowledge. 2) Sufficient conditions for finite-time boundedness of linear systems under event-triggered control are presented by virtue of Lyapunov-like function. The conditions are transformed into linear matrix inequalities such that it is easy to be verified and applied.

Notations: In this paper, $\mathbb{N}$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$ respectively represent the sets of natural numbers, n-dimensional vectors and $n\times n$ matrices. The superscript T stands for matrix transposition. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a real symmetric matrix.

2 Problem formulation and preliminaries

Consider the following linear system:

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t)+Dw(t),\\ y(t)=Cx(t),\end{cases}\tag{1}$$

where $x(t)=[x_1(t)\ \ x_2(t)\ \ \cdots\ \ x_n(t)]^{\mathrm{T}}\in\mathbb{R}^n$ is the state, $u(t)\in\mathbb{R}^m$ is the control input, $w(t)\in\mathbb{R}^q$ is the disturbance input, $y(t)\in\mathbb{R}^p$ is the output signal. The matrices A, B, C, and D are system matrices with appropriate dimensions.

An observer-based state feedback controller is as follows:

$$\begin{cases}\dot{\hat{x}}(t)=A\hat{x}(t)+Bu(t)-L(y(t)-\hat{y}(t)),\\ \hat{y}(t)=C\hat{x}(t),\\ u(t)=K\hat{x}(t),\end{cases}\tag{2}$$

where $\hat{x}(t)\in\mathbb{R}^n$ and $\hat{y}(t)\in\mathbb{R}^p$ are the state and the output of the observer, $L\in\mathbb{R}^{n\times p}$ and $K\in\mathbb{R}^{m\times n}$ are the observer and controller gains respectively.

Let $e(t)=x(t)-\hat{x}(t)$. Then, $\dot{e}(t)=(A+LC)\cdot e(t)+Dw(t)$. On the other hand, the dynamics of the observer with the state feedback controller $u(t)=K\hat{x}(t)$ is given by $\dot{\hat{x}}(t)=(A+BK)\hat{x}(t)-Le_y(t)$, where $e_y(t)=Ce(t)$ is the output estimation error.

In this paper, the control input u is updated at certain instants $\{t_k\}_{k\in\mathbb{N}}$ instead of continuing implementation. Thus, the closed-loop system can be represented by

$$\begin{cases}\dot{x}(t)=Ax(t)+Bu(t_k)+Dw(t),\\ \dot{\hat{x}}(t)=A\hat{x}(t)+Bu(t_k)-Le_y(t),\\ u(t_k)=K\hat{x}(t_k),\ \forall t\in[t_k,t_{k+1}).\end{cases}\tag{3}$$

Let $\delta(t)=u(t_k)-u(t)=K(\hat{x}(t_k)-\hat{x}(t))$, which can be viewed as a measure of the difference between sampled control input and continuous-time ones. Thus, the closed-loop system (3) can be expressed in terms of $\hat{x}$ and e as follows:

$$\begin{cases}\dot{x}(t)=Ax(t)+B\delta(t)+BK\hat{x}(t)+Dw(t),\\ \dot{\hat{x}}(t)=(A+BK)\hat{x}(t)+B\delta(t)-Le_y(t).\end{cases} \tag{4}$$

In order to discuss the event-triggered output feedback finite-time bounded control of linear systems, the following standard assumptions and definitions of finite-time boundedness, which are cited from [18–19, 23–24], are listed as follows.

Assumption 1 [18–19, 23–24] External disturbances $w(t)$ is time-varying and satisfies $\int_0^{T_f} w^{\mathrm{T}}(t)w(t)\mathrm{d}t \leqslant d_w, d_w \geqslant 0, [0,T_f]$ is the fixed time interval.

Definition 1 [18–19, 23–24] Given positive constants c_1, c_2, T_f with $c_1 < c_2$ and a positive-definite matrix R. If $\epsilon^{\mathrm{T}}(t_0)R\epsilon(t_0) \leqslant c_1$, we have $\epsilon^{\mathrm{T}}(t)R\epsilon(t) < c_2, \forall t \in [0,T_f]$, then linear system (1) is said to be finite-time bounded with respect to (c_1, c_2, T_f, d_w, R), where $\epsilon(t) = (x^{\mathrm{T}}(t)\ \ \hat{x}^{\mathrm{T}}(t))^{\mathrm{T}}$.

3 Main results

In this section, finite-time boundedness of linear system (1) via event-triggered output feedback control is to be discussed.

Theorem 1 For linear system (4), given controller and observer gains K and L, and a positive scalar T. If there are matrices $P_1>0$, $P_2>0$, $Q_\delta>0$, $S_1>0$, $S_2>0$, $Q_\varepsilon>0$, constants $\alpha_2 \geqslant \alpha_1 \geqslant 0$, $\beta_2 \geqslant \beta_1 \geqslant 0$ and the event-triggered sampling rule with a parameter η defined by $t_{k+1}=\min\{t \geqslant t_k+T$, s.t. $f(\delta(t),(\hat{x}(t)\ \ e_y(t))^{\mathrm{T}}) \geqslant 0\}$, such that

$$\phi=\begin{pmatrix}\varphi_{11} & \varphi_{12} & \varphi_{13} & \varphi_{14}\\ * & \varphi_{22} & \varphi_{23} & \varphi_{24}\\ * & * & \varphi_{33} & \varphi_{34}\\ * & * & * & \varphi_{44}\end{pmatrix}<0,$$

$$\varPhi=\begin{pmatrix}\varphi_{11}^* & \varphi_{12}^* & \varphi_{13}^* & \varphi_{14}^*\\ * & \varphi_{22}^* & \varphi_{23}^* & \varphi_{24}^*\\ * & * & \varphi_{33}^* & \varphi_{34}^*\\ * & * & * & \varphi_{44}^*\end{pmatrix}<0,$$

$$\begin{aligned}&(\lambda_2+\lambda_3)c_1+(\lambda_4\beta_1+\lambda_5\beta_2)d_w+\\ &\int_0^{T_f}\tilde{f}(\delta(s),(\hat{x}(s)\ \ e_y(s))^{\mathrm{T}})\mathrm{d}s<\\ &\lambda_1 c_2 \mathrm{e}^{-(\alpha_1+\alpha_2)T_f},\end{aligned} \tag{5}$$

then linear system (4) is finite-time bounded with respect to (c_1, c_2, T_f, d_w, R), where

$$\begin{aligned}&\varphi_{11}=A^{\mathrm{T}}P_1+P_1A-\alpha_1P_1,\\ &\varphi_{12}=P_1BK-C^{\mathrm{T}}L^{\mathrm{T}}P_2,\\ &\varphi_{13}=P_1B,\ \varphi_{14}=P_1D,\ \varphi_{23}=P_2B,\ \varphi_{24}=0,\\ &\varphi_{22}=(A+BK+LC)^{\mathrm{T}}P_2+\\ &\quad P_2(A+BK+LC)-\alpha_1P_2,\\ &\varphi_{33}=0,\ \varphi_{34}=0,\ \varphi_{44}=-\beta_1S_1,\\ &\varphi_{11}^*=A^{\mathrm{T}}P_1+P_1A-\alpha_2P_1+C^{\mathrm{T}}Q_{\varepsilon 3}C,\\ &\varphi_{12}^*=P_1BK-C^{\mathrm{T}}L^{\mathrm{T}}P_2-C^{\mathrm{T}}Q_{\varepsilon 3}C+C^{\mathrm{T}}Q_{\varepsilon 2}^{\mathrm{T}},\\ &\varphi_{13}^*=P_1B,\ \varphi_{14}^*=P_1D,\\ &\varphi_{22}^*=(A+BK+LC)^{\mathrm{T}}P_2+\\ &\quad P_2(A+BK+LC)-\alpha_2P_2-\\ &\quad Q_{\varepsilon 2}C-C^{\mathrm{T}}Q_{\varepsilon 2}^{\mathrm{T}}+C^{\mathrm{T}}Q_{\varepsilon 3}C+Q_{\varepsilon 1},\\ &\varphi_{23}^*=P_2B,\ \varphi_{24}^*=0,\ \varphi_{33}=-Q_\delta,\\ &\varphi_{34}^*=0,\ \varphi_{44}^*=-\beta_2S_2,\\ &f(\delta(t),(\hat{x}(t)\ \ e_y(t))^{\mathrm{T}})=\\ &\delta^{\mathrm{T}}(t)Q_\delta\delta(t)-(\hat{x}^{\mathrm{T}}(t)\ \ e_y^{\mathrm{T}}(t))Q_\varepsilon(\hat{x}(t)\ \ e_y(t))^{\mathrm{T}}-\eta,\\ &\tilde{f}(\delta(t),(\hat{x}(t)\ \ e_y(t))^{\mathrm{T}})=\\ &\delta^{\mathrm{T}}(t)Q_\delta\delta(t)-(\hat{x}^{\mathrm{T}}(t)\ \ e_y^{\mathrm{T}}(t))Q_\varepsilon(\hat{x}(t)\ \ e_y(t))^{\mathrm{T}},\\ &Q_\varepsilon=\begin{pmatrix}Q_{\varepsilon 1} & Q_{\varepsilon 2}\\ * & Q_{\varepsilon 3}\end{pmatrix},\ P=\begin{pmatrix}P_1 & 0\\ * & P_2\end{pmatrix},\\ &\lambda_1=\frac{\lambda_{\min}(P)}{\lambda_{\max}(R)},\ \lambda_2=\frac{\lambda_{\max}(P_1)}{\lambda_{\min}(R)},\ \lambda_3=\frac{\lambda_{\max}(P_2)}{\lambda_{\min}(R)},\\ &\lambda_4=\lambda_{\max}(S_1),\ \lambda_5=\lambda_{\max}(S_2).\end{aligned}$$

Proof The proof procedure can be divided into two parts:

Part I Upper-bound estimation of Lyapunov-like function.

Choose a Lyapunov-like function

$$V(x(t),\hat{x}(t))=\begin{pmatrix}x(t)\\ \hat{x}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}P_1 & 0\\ 0 & P_2\end{pmatrix}\begin{pmatrix}x(t)\\ \hat{x}(t)\end{pmatrix}. \tag{6}$$

Taking derivative of $V(x(t),\hat{x}(t))$ yields

$$\begin{aligned}&\dot{V}=\\ &\dot{x}^{\mathrm{T}}(t)P_1x(t)+x^{\mathrm{T}}(t)P_1\dot{x}(t)+\\ &\dot{\hat{x}}^{\mathrm{T}}(t)P_2\hat{x}(t)+\hat{x}^{\mathrm{T}}(t)P_2\dot{\hat{x}}(t)=\\ &x^{\mathrm{T}}(t)[A^{\mathrm{T}}P_1+P_1A]x(t)+w^{\mathrm{T}}(t)D^{\mathrm{T}}P_1x(t)+\\ &x^{\mathrm{T}}(t)[P_1BK-C^{\mathrm{T}}L^{\mathrm{T}}P_2]\hat{x}(t)+\\ &x^{\mathrm{T}}(t)P_1B\delta(t)+x^{\mathrm{T}}(t)P_1Dw(t)+\\ &\hat{x}^{\mathrm{T}}(t)[K^{\mathrm{T}}B^{\mathrm{T}}P_1-P_2LC]x(t)+\\ &\hat{x}^{\mathrm{T}}(t)[(A+BK+LC)^{\mathrm{T}}P_2+\\ &P_2(A+BK+LC)]\hat{x}(t)+\\ &\hat{x}^{\mathrm{T}}(t)P_2B\delta(t)+\delta^{\mathrm{T}}(t)B^{\mathrm{T}}P_1x(t)+\\ &\delta^{\mathrm{T}}(t)B^{\mathrm{T}}P_2\hat{x}(t).\end{aligned} \tag{7}$$

For ETC design, the event-triggered time is separated at least by T which is used to avoid Zeno phenomenon. During $[t_k, t_k+T)$, the control is not updated. After t_k+T, event condition determines when the control should be updated guaranteeing system performance. Hence, upper-bound estimation in $[t_k, t_k+T)$ and $[t_k+T, t_{k+1})$ should be done respectively.

Step 1 Upper-bound estimation in $[t_k, t_k+T)$.

Based on (6) and (7), it yields

$$\begin{aligned}&\dot V-\alpha_1 V<\\&x^{\mathrm T}(t)[A^{\mathrm T}P_1+P_1A-\alpha_1P_1]x(t)+\\&w^{\mathrm T}(t)D^{\mathrm T}P_1x(t)+x^{\mathrm T}(t)[P_1BK-\\&C^{\mathrm T}L^{\mathrm T}P_2]\hat x(t)+x^{\mathrm T}(t)P_1B\delta(t)+\\&x^{\mathrm T}(t)P_1Dw(t)+\hat x^{\mathrm T}(t)[K^{\mathrm T}B^{\mathrm T}P_1-\\&P_2LC]x(t)+\hat x^{\mathrm T}(t)[(A+BK+LC)^{\mathrm T}P_2+\\&P_2(A+BK+LC)-\alpha_1P_2]\hat x(t)+\\&\hat x^{\mathrm T}(t)P_2B\delta(t)+\delta^{\mathrm T}(t)B^{\mathrm T}P_1x(t)+\\&\delta^{\mathrm T}(t)B^{\mathrm T}P_2\hat x(t)+\beta_1w^{\mathrm T}(t)S_1w(t)-\\&\beta_1w^{\mathrm T}(t)S_1w(t)=\\&\zeta^{\mathrm T}(t)\phi\zeta(t)+\beta_1w^{\mathrm T}(t)S_1w(t),\end{aligned}\tag{8}$$

where $\zeta(t)=(x^{\mathrm T}(t)\ \ \hat x^{\mathrm T}(t)\ \ \delta^{\mathrm T}(t)\ \ w^{\mathrm T}(t))^{\mathrm T}$.

Thus, we have $\dot V-\alpha_1V\leqslant\beta_1w^{\mathrm T}(t)S_1w(t)$. Integrating both sides of this inequality for $t\in[t_k,t_k+T)$,

$$V(t)\leqslant \mathrm e^{\alpha_1(t-t_k)}V(t_k)+\int_{t_k}^{t}\beta_1w^{\mathrm T}(s)S_1w(s)\mathrm ds.\tag{9}$$

Step 2 Upper-bound estimation in $[t_k+T, t_{k+1})$.

As that of Case 1, it is not difficult to get that

$$\begin{aligned}&\dot V-\alpha_2V-f(\delta(t),(\hat x(t)\ \ e_y(t))^{\mathrm T})<\\&x^{\mathrm T}(t)[A^{\mathrm T}P_1+P_1A-\alpha_2P_1+C^{\mathrm T}Q_{\varepsilon3}C]x(t)+\\&w^{\mathrm T}(t)D^{\mathrm T}P_1x(t)+x^{\mathrm T}(t)[P_1BK-C^{\mathrm T}L^{\mathrm T}P_2+\\&C^{\mathrm T}Q_{\varepsilon2}^{\mathrm T}-C^{\mathrm T}Q_{\varepsilon3}C]\hat x(t)+x^{\mathrm T}(t)P_1B\delta(t)+\\&x^{\mathrm T}(t)P_1Dw(t)+\hat x^{\mathrm T}(t)[K^{\mathrm T}B^{\mathrm T}P_1-P_2LC+\\&Q_{\varepsilon2}C-C^{\mathrm T}Q_{\varepsilon3}C]x(t)+\\&\hat x^{\mathrm T}(t)[(A+BK+LC)^{\mathrm T}P_2+\\&P_2(A+BK+LC)-\alpha_2P_2-Q_{\varepsilon2}C-\\&C^{\mathrm T}Q_{\varepsilon2}^{\mathrm T}+C^{\mathrm T}Q_{\varepsilon3}C+Q_{\varepsilon1}]\hat x(t)+\\&\hat x^{\mathrm T}(t)P_2B\delta(t)+\delta^{\mathrm T}(t)B^{\mathrm T}P_1x(t)+\\&\delta^{\mathrm T}(t)B^{\mathrm T}P_2\hat x(t)-\delta^{\mathrm T}(t)Q_\delta\delta(t)+\\&\beta_2w^{\mathrm T}(t)S_2w(t)-\beta_2w^{\mathrm T}(t)S_2w(t)+\eta=\\&\zeta^{\mathrm T}(t)\varPhi\zeta(t)+\beta_2w^{\mathrm T}(t)S_2w(t)+\eta.\end{aligned}\tag{10}$$

Hence, one obtains $\dot V-\alpha_2V-f(\delta(t),(\hat x(t)\cdot e_y(t))^{\mathrm T})\leqslant\beta_2w^{\mathrm T}(t)S_2w(t)+\eta$. Integrating both sides of this inequality for $t\in[t_k+T,t_{k+1})$, it holds that

$$\begin{aligned}V(t)\leqslant{}&\mathrm e^{\alpha_2(t-(t_k+T))}V(t_k+T)+\\&\int_{t_k+T}^{t}[\tilde f(\delta(t),(\hat x(t)\ \ e_y(t))^{\mathrm T})+\\&\beta_2w^{\mathrm T}(t)S_2w(t)]\mathrm ds.\end{aligned}\tag{11}$$

Part II Finite-time boundedness analysis.

Using the iterative method, for $t\in(0,T_f)$ and $t>t_k+T$, based on (9) and (11), it yields

$$\begin{aligned}&V(t)\leqslant\\&\mathrm e^{\alpha_2(t-(t_k+T))}V(t_k+T)+\\&\int_{t_k+T}^{t}[\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})+\\&\beta_2w^{\mathrm T}(s)S_2w(s)]\mathrm ds\leqslant\\&\mathrm e^{\alpha_2(t-(t_k+T))}\mathrm e^{\alpha_1T}V(t_k)+\\&\mathrm e^{\alpha_2(t-(t_k+T))}\int_{t_k}^{t_k+T}\beta_1w^{\mathrm T}(s)S_1w(s)\mathrm ds+\\&\int_{t_k+T}^{t}[\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})+\\&\beta_2w^{\mathrm T}(s)S_2w(s)]\mathrm ds\leqslant\\&\mathrm e^{\alpha_2(t-(t_k+T))}\mathrm e^{\alpha_1T}\mathrm e^{\alpha_2(t_k-(t_{k-1}+T))}V(t_{k-1}+T)+\\&\mathrm e^{\alpha_2(t-(t_k+T))}\mathrm e^{\alpha_1T}\times\\&\int_{t_{k-1}+T}^{t_k}[\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})+\\&\beta_2w^{\mathrm T}(s)S_2w(s)]\mathrm ds+\\&\mathrm e^{\alpha_2(t-(t_k+T))}\int_{t_k}^{t_k+T}\beta_1w^{\mathrm T}(s)S_1w(s)\mathrm ds+\\&\int_{t_k+T}^{t}[\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})+\\&\beta_2w^{\mathrm T}(s)S_2w(s)]\mathrm ds\leqslant\\&\mathrm e^{\alpha_1N_\eta^{\mathrm T}T}\mathrm e^{\alpha_2(t-N_\eta^{\mathrm T}T)}[V(t_0)+\\&\beta_1\lambda_{\max}(S_1)\int_0^{T_f}w^{\mathrm T}(s)w(s)\mathrm ds+\\&\beta_2\lambda_{\max}(S_2)\int_0^{T_f}w^{\mathrm T}(s)w(s)\mathrm ds+\\&\int_0^{T_f}\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})\mathrm ds]\leqslant\\&\mathrm e^{(\alpha_1+\alpha_2)T_f}[V(t_0)+(\beta_1\lambda_4+\beta_2\lambda_5)d_w+\\&\int_0^{T_f}\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})\mathrm ds],\end{aligned}$$

or for $t\in(0,T_f)$ and $t<t_k+T$,

$$\begin{aligned}V(t)\leqslant{}&\mathrm e^{(\alpha_1+\alpha_2)T_f}[V(t_0)+(\beta_1\lambda_4+\beta_2\lambda_5)d_w+\\&\int_0^{T_f}\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})\mathrm ds],\end{aligned}$$

where $N_\eta^{\mathrm T}$ denotes the maximum number of events triggered with respect to parameters T and η.

In summary, one obtains $V(t)\leqslant\mathrm e^{(\alpha_1+\alpha_2)T_f}[V(t_0)+(\beta_1\lambda_4+\beta_2\lambda_5)d_w+\int_0^{T_f}\tilde f(\delta(s),(\hat x(s)\ \ e_y(s))^{\mathrm T})\mathrm ds]$.

Meanwhile,

$$V(t)=x^{\mathrm T}(t)P_1x(t)+\hat x^{\mathrm T}(t)P_2\hat x(t)\geqslant$$

$$\frac{\lambda_{\min}(P)}{\lambda_{\max}(R)}\epsilon^{\mathrm{T}}(t)R\epsilon(t)=\lambda_1\epsilon^{\mathrm{T}}(t)R\epsilon(t), \qquad (12)$$

and

$$V(t_0)\leqslant\frac{\lambda_{\max}(P_1)}{\lambda_{\min}(R)}[x^{\mathrm{T}}(t_0)Rx(t_0)+\hat{x}^{\mathrm{T}}(t_0)R\hat{x}(t_0)]+ \frac{\lambda_{\max}(P_2)}{\lambda_{\min}(R)}[x^{\mathrm{T}}(t_0)Rx(t_0)+\hat{x}^{\mathrm{T}}(t_0)R\hat{x}(t_0)]\leqslant (\lambda_2+\lambda_3)c_1, \qquad (13)$$

where $\bar{R}=\begin{pmatrix}R & 0\\ * & R\end{pmatrix}$.

Thus, it yields

$$\epsilon^{\mathrm{T}}(t)R\epsilon(t)\leqslant\frac{V(t)}{\lambda_1}< \frac{V(t_0)+(\beta_1\lambda_4+\beta_2\lambda_5)d_w+\int_0^{T_f}\tilde{f}(\delta(s),(\hat{x}(s)\ \ e_y(s))^{\mathrm{T}})\mathrm{d}s}{\lambda_1}\cdot \mathrm{e}^{(\alpha_1+\alpha_2)T_f}.$$

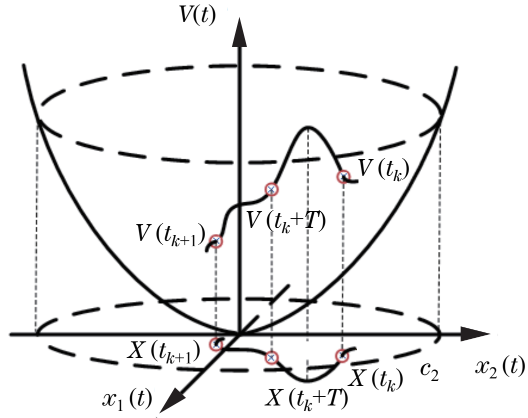
By virtue of equation (5), it is not difficult to get

$$\epsilon^{\mathrm{T}}(t)R\epsilon(t)\leqslant\frac{\lambda_1 c_2}{\lambda_1}\times\mathrm{e}^{-(\alpha_1+\alpha_2)T_f}\times\mathrm{e}^{(\alpha_1+\alpha_2)T_f}\leqslant c_2.$$

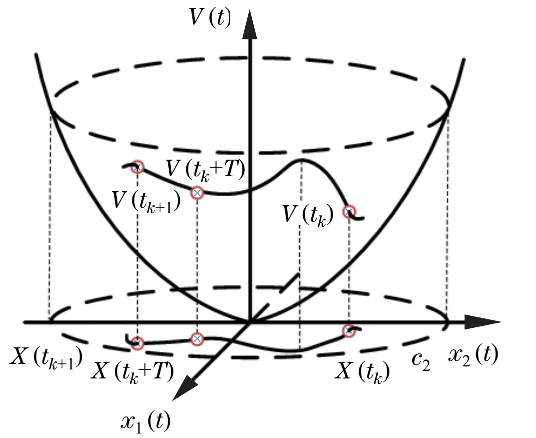
Thus, Theorem 1 is proved. □

Remark 1 It is well known that asymptotic stability is a qualitative analysis but finite-time boundedness is a quantitative one, and AS of the system does not mean FTB of the system and vice visa [15, 18, 23]. But, in the framework of event-triggered control, the difference of AS and FTB should be noted extraordinarily. For AS, the overall energy trend needs to be downward to guarantee asymptotical stability of the system, for example, during the period between two updated time such as $[t_k, t_{k+1})$ the energy $V(x(t))$ of the system is required to decline [5] or be bounded during $[t_k, t_k+T)$ and comes down during $[t_k+T, t_{k+1})$ [11]. Fig. 1(a) shows the case as that in [11]. During $[t_k, t_k+T)$, the energy $V(x(t))$ of the system is only required to be bounded but not declined, the system is AS but not FTB. However, for FTB, the energy $V(x(t))$ of the system does not need to decline, but the upper-bound estimation of the Lyapunov-like function $V(x(t))$ should be known to guarantee finite-time boundedness of the closed-loop system, Fig. 1(b).

Remark 2 From Theorem 1, it is not difficult to see that the event-triggered sampling rule designed in this paper includes two parts, time condition and event condition. Time condition can effectively avoid Zeno phenomenon for the sampling time separated at least by T. Event condition is proposed to coordinately determine the next update time of control signal. It is worth mentioning that the free parameter η in event condition, which is used to coordinately regulate the number of updates of the controller, is put forward for the first time, to the best of our knowledge.



(a) Asymptotic stable



(b) Finite-time bounded

Fig. 1 Difference between asymptotic stable and finite-time bounded

Remark 3 For update ways of control signal, it is mainly affected by the adjustment of two free parameters in the event triggered sampling rules, which are T in time condition and η in event condition, respectively. In terms of the update forms of control signal, it can be divided into periodic update and aperiodic update, e.g. see Fig. 5(a): aperiodicity and Fig. 5(c): periodicity. On the other hand, according to the update influence of control signal on the finite-time boundedness of system, it can be regarded as effective control signal and invalid control signal. In Fig. 3, for Case 1 and Case 2, the system is Un-FTB (UFTB), so the corresponding updated control signal is invalid control signal. However, for Case 3, the system is FTB, so the updated control signal at this time is the effective control signal.

4 Illustrative example

In this section, simulation results are presented to show the validity of the method proposed in this paper.

Example 1 Consider the following linear system with an event-triggered output feedback controller:

$$\begin{cases} \dot{x}(t) = \begin{pmatrix} 0 & 0.08 \\ -2 & -1 \end{pmatrix} x(t) + \\ \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t_k) + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} w(t), \\ y(t) = (1\ \ 0)x(t), \end{cases} \tag{14}$$

where $u(t_k) = K\hat{x}(t_k)$, $\hat{x}(0) = 0$ and $w(t) = (0.35\sin t\ \ 0.25\cos t)^{\mathrm{T}}$.

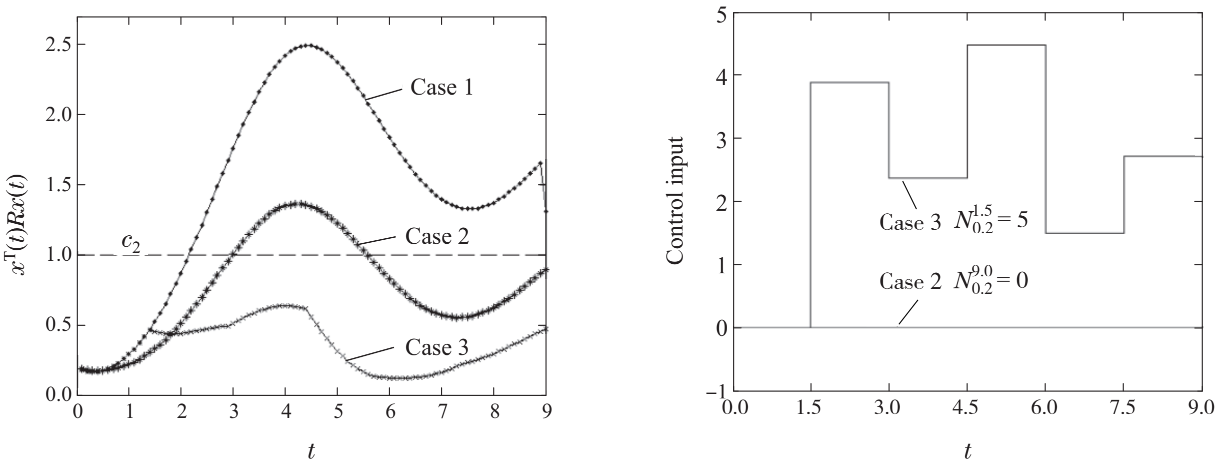
Let $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $\beta_1 = 0.2$, $\beta_2 = 0.6$, $d_w = 1.0$, $T_f = 9$ s, $R = I$, $c_1 = 0.2$, $c_2 = 1.0$.

As will be readily seen from Fig. 2(a), Case 1, linear system (14) is AS, but it is not FTB. To make linear system (14) FTB, let $K = (3\ \ -6)$ and $L = (-0.5\ \ -6)^{\mathrm{T}}$. With $\eta = 0.2$, statuses of control signal are exhibited in Table 1, and responses of $x^{\mathrm{T}}(t)Rx(t)$ are shown in Fig. 2. From Fig. 2(a), Case 2, it can be seen that with the initial control values un-updated, linear system (14) is still not finite-time bounded, which means that control input needs more updation. If control signal updates according to an appropriate event-triggered condition, the closed-loop linear system is finite-time bounded, see it in Fig. 2(a), Case 3. So, the effectiveness of the proposed method is proved.

Table 1 Control signal with $\eta = 0.2$

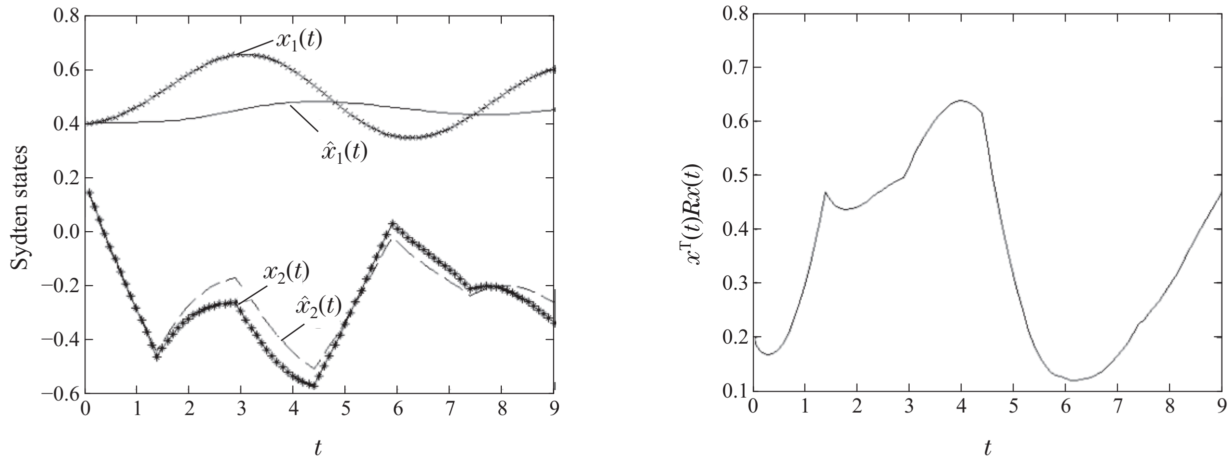
	K	T/s	Control signal
Case 1	$(3\ \ -6)$	9	$u(t) \equiv 0$
Case 2	$(3\ \ -6)$	9	$u(t) \equiv K\hat{x}(t_0)$
Case 3	$(3\ \ -6)$	1.5	$u(t) = K\hat{x}(t_k),\ k = 1, \cdots, 6$

Moreover, based on numerical calculation, maximum effective values of T and η, $\bar{T}_{\max}$ and $\bar{\eta}_{\max}$, can be found. For linear system (14) with $K = (3\ \ -6)$ and $L = (-0.5\ \ -6)^{\mathrm{T}}$, $\bar{T}_{\max} = 2.2$ s and $\bar{\eta}_{\max} = 17.1$. Linear system (14) is FTB as long as $0 < T \leqslant 2.2$ s and $\eta \leqslant 17.1$. And in Fig. 3, $T = 1.5$ s, $\eta = 0.18$ and $T = 2.0$ s, $\eta = 10$ are respectively chosen.

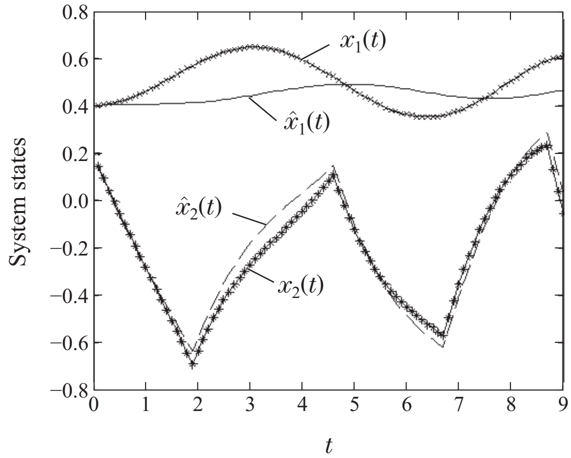


(a) Three cases of responses of $x^{\mathrm{T}}(t)Rx(t)$ (b) Three cases of control signals

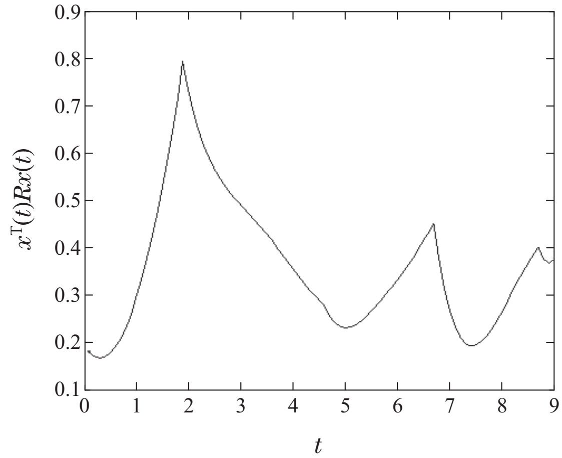
Fig. 2 State response and control input



(a) Response of states ($T = 1.5$ s, $\eta = 0.18$) (b) Response of $x^{\mathrm{T}}(t)Rx(t)$ ($T = 1.5$ s, $\eta = 0.18$)



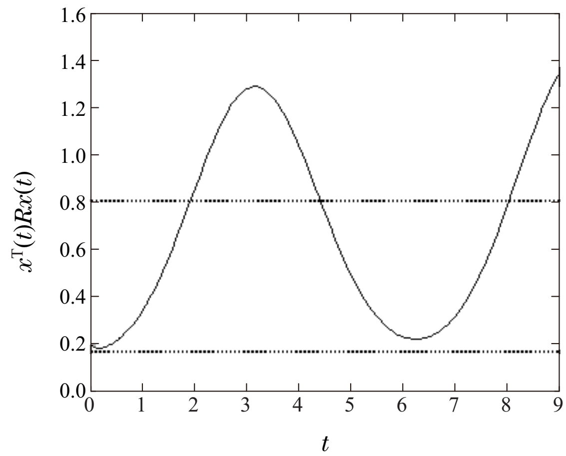
(c) Response of states ($T = 2.0$ s, $\eta = 10$)



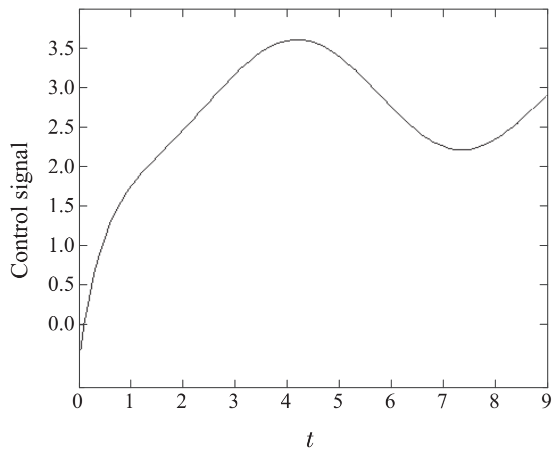
(d) Response of $x^{\mathrm{T}}(t)Rx(t)$ ($T = 2.0$ s, $\eta = 10$)

Fig. 3 System response with different parameters

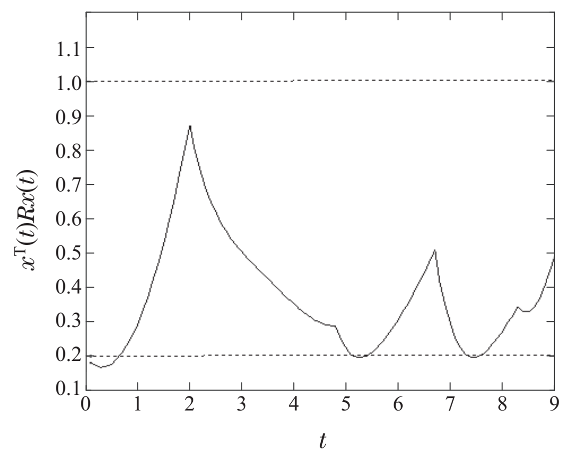
For linear system (14) with $K = (3 \quad -6)$ and $L = (-0.5 \quad -6)^{\mathrm{T}}$, in case of continuous control, the system is not FTB, as shown in Fig. 4(a) and Fig. 4(c), but with an appropriate event-triggered control strategy, the system may achieve FTB as shown in Fig. 4(b) and Fig. 4(d) with $T = 1.0$ s, $\eta = 15$. On the other hand, for linear system (14), and the parameters c_1, c_2 and K changed to be $c_1 = 0.04$, $c_2 = 1.0$. and $K = (0.1 \quad -10)$, then the system is FTB with continuous control, as shown in Fig. 5(a) and Fig. 5(c), but for inappropriate ETC, the closed-loop system is not FTB, as shown in Fig. 5(b) and Fig. 5(d).



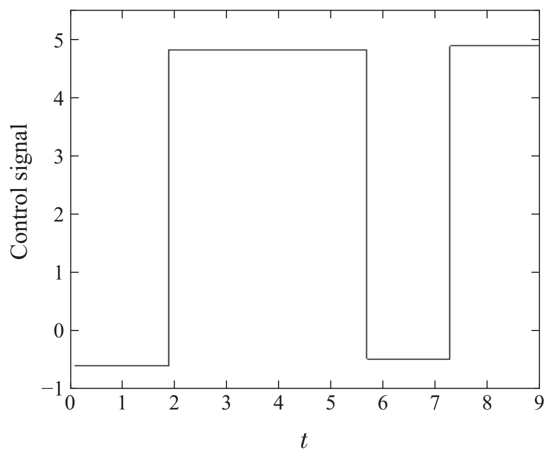
(a) Response of $x^{\mathrm{T}}(t)Rx(t)$ (Continuous control)



(b) Control signals (Continuous control)

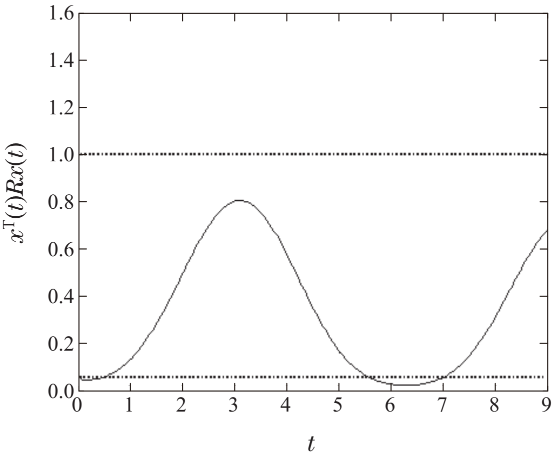


(c) Response of $x^{\mathrm{T}}(t)Rx(t)$ (Appropriate ETC strategy)

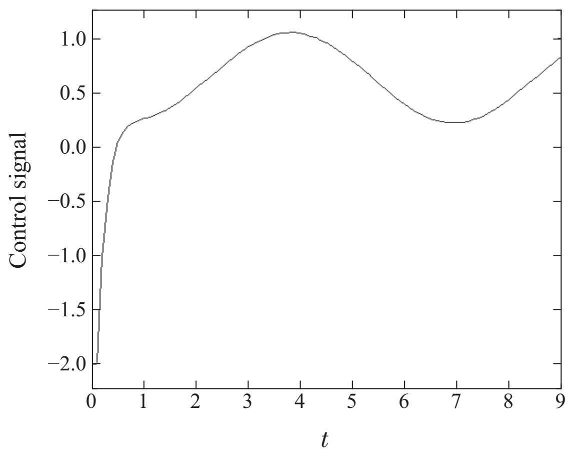


(d) Control signals (Appropriate ETC strategy)

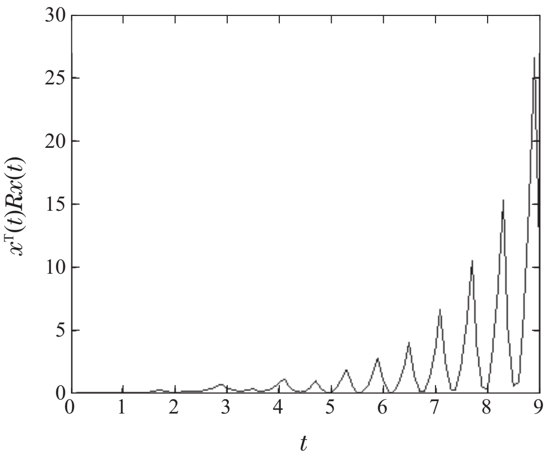
Fig. 4 State response and control input with continuous and appropriate ETC strategies



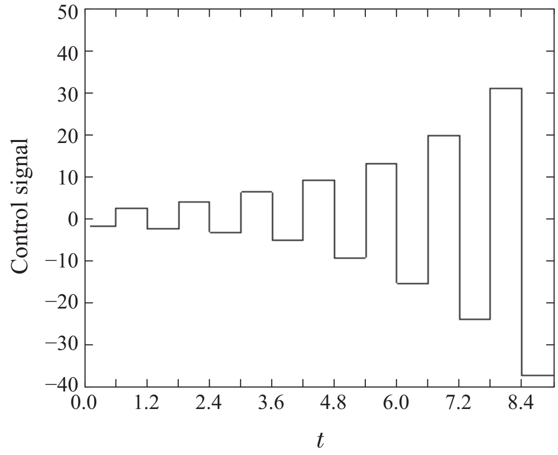
(a) Response of $x^{\mathrm{T}}(t)Rx(t)$ (Continuous control)



(b) Control signals (Continuous control)



(c) Response of $x^{\mathrm{T}}(t)Rx(t)$ (Inappropriate ETC strategy)



(d) Control signals (Inappropriate ETC strategy)

Fig. 5 State response and control input with continuous and inappropriate ETC strategies

5 Conclusion

Finite-time boundedness of linear systems via event-triggered output feedback control have been addressed. Based on Lyapunov-like function, sufficient conditions have been presented to ensure FTB of the resultant closed-loop system via ETC. Moreover, an appropriate event-triggered rule with tunable parameters has been proposed to avoid Zeno behavior and to guarantee system performance. How to extend the proposed method to switched linear systems needs further research.

References:

[1] ASTROM K J, WITTENMARK B. *Computer-Controlled Systems: Theory and Design, 2nd ed*. New Jersey: Prentice Hall, 1990.

[2] FRANKLIN G. *Feedback Control of Dynamic Systems*. New Jersey: Prentice Hall, 2007.

[3] ASTROM K J, BERNHARDSSON B. Comparison of periodic and event based sampling for first-order stochastic systems. *IFAC Proceedings Volumes*, 1999, 32(2): 5006 – 5011.

[4] ARZEN A. A simple event-based pid controller. *IFAC Proceedings Volumes*, 1999, 32(2): 8687 – 8692.

[5] TABUADA P. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 2007, 52(9): 1680 – 1685.

[6] HEEMELS W P M H, JOHANSSON K H, TABUADA P. An introduction to event-triggered and self-triggered control, *2012 the 51st IEEE Conference on Decision and Control (CDC)*. Maui, HI, USA: IEEE, 2012: 3270 – 3285.

[7] HETEL L, FITER C, OMRAN H, et al. Recent developments on the stability of systems with aperiodic sampling: An overview. *Automatica*, 2017, 76: 309 – 335.

[8] DOLK V S, BORGERS D P, HEEMELS W P M H. Output-based and decentralized dynamic event-triggered control with guaranteed $\mathcal{L}_p$-gain performance and zeno-freeness. *IEEE Transactions on Automatic Control*, 2017, 62(1): 34 – 49.

[9] TALLAPRAGADA P, CHOPRA N. On event triggered tracking for nonlinear systems. *IEEE Transactions on Automatic Control*, 2013, 58(9): 2343 – 2348.

[10] GOMMANS T M P, ANTUNES D, DONKERS T, et al. Heemels. Self-triggered linear quadratic control. *Automatica*, 2014, 50(4): 1279 – 1287.

[11] TARBOURIECH S, SEURET A, SILVA J M G D, et al. Observer-based event-triggered control co-design for linear systems. *IET Control Theory and Applications*, 2016, 10(18): 2466 – 2473.

[12] PERALEZ J, ANDRIEU V, NADRI M, et al. Event-triggered output feedback stabilization via dynamic high-gain scaling. *IEEE Transactions on Automatic Control*, 2018, 63(8): 2537 – 2549.

[13] FERDINANDO M D, PEPE P. Sampled-data emulation of dynamic output feedback controllers for nonlinear time-delay systems. *Automatica*, 2019, 99: 120 – 131.

[14] BRUNNER F D, HEEMELS W P M H, ALLGWER F. Event-triggered and self-triggered control for linear systems based on reachable sets. *Automatica*, 2019, 101: 15 – 26.

[15] DORATO P. *Short-time stability in linear time-varying systems, rel. tec*. New York: Polytechnic Institute of Brooklyn, 1961.

[16] AMATO F, ARIOLA M, DORATO P. Technical communique: Finite-time control of linear systems subject to parametric uncertainties and disturbances. *Automatica*, 2001, 37(9): 1459 – 1463.

[17] BAYAT F, MOBAYEN S, JAVADI S. Finite-time tracking control of nth-order chained-form non-holonomic systems in the presence of disturbances. *ISA Transactions*, 2016, 63: 78 – 83.

[18] AMATO F, ARIOLA M, COSENTINO C. Technical communique: Finite-time stabilization via dynamic output feedback. *Automatica*, 2006, 42(2): 337 – 342.

[19] AMATO F, AMBROSINO R, ARIOLA M, et al. On the finite-time boundedness of linear systems. *Automatica*, 2019, 107: 454 – 466.

[20] KOIKE R, ENDO T, MATSUNO F. Output-based dynamic event-triggered consensus control for linear multiagent systems. *Automatica*, 2021, 133: 109863.

[21] DONKERS M C F, HEEMELS W P M H. Output-based event-triggered control with guaranteed-gain and improved and decentralized event-triggering. *IEEE Transactions on Automatic Control*, 2012, 57(6): 1362 – 1376.

[22] BORGERS D P, HEEMELS W P M H. Event-separation properties of event-triggered control systems. *IEEE Transactions on Automatic Control*, 2014, 59(10): 2644 – 2656.

[23] AMATO F, AMBROSINO R, ARIOLA M, et al. Finite-time stability and control. *Lecture Notes in Control & Information Ences*. London: Springer Verlag, 2014.

[24] LIN X Z, LI X L, LI S H, et al. Finite-time boundedness for switched systems with sector bounded nonlinearity and constant time delay. *Applied Mathematics and Computation*, 2016, 274: 25 – 40.

作者简介:

杨中林 硕士研究生, 研究方向为切换系统的有限时间稳定性, E-mail: zly_626539@163.com;

魏自航 硕士研究生, 研究方向为受约束非线性切换系统反馈镇定问题, E-mail: zh_wei5944@163.com;

林相泽 教授, 博士生导师, 研究方向为控制理论与控制技术、机器人与智能控制、深度学习算法及其应用等, E-mail: xzlin@njau.edu.cn.