第 39 卷第 10 期
2022 年 10 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 39 No. 10
Oct. 2022

基于迭代学习的具适多智能体系统分布式跟踪控制

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摘要: 文章考虑了具适多智能体系统的分布式跟踪控制问题. 通过设计带有初始学习机制的P型和PD$^\alpha$型迭代学习控制策略求解跟踪问题. 具适导数具有良好的性质且可以刻画不同步长的实际数据采样情况. 初始学习机制放松了初始值条件且提高了算法实现趋同跟踪的性能. 在可重复操作环境和有向通信拓扑的假设下, 提出了一种分布式迭代学习方案, 通过重复同一轨迹的控制尝试和用跟踪误差修正不满意的控制信号来实现有限时间趋同. 严格证明了随着迭代次数增加, 提出的P型和PD$^\alpha$ 型迭代学习控制策略使得所有智能体能渐近跟踪上参考轨迹. 两个代表性数值仿真验证了算法的有效性.

关键词: 迭代技术; 趋同跟踪控制; 具适导数; 多智能体系统

引用格式: 王小文, 刘帅, 王锦荣. 基于迭代学习的具适多智能体系统分布式跟踪控制. 控制理论与应用, 2022, 39(10): 1836 – 1844

DOI: 10.7641/CTA.2022.10937

Iterative learning-based consensus tracking control for conformable multi-agent systems

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Abstract: This paper considers the consensus tracking control problem for conformable multi-agent systems with linear and nonlinear dynamics by designing P-type and PD$^\alpha$-type iterative learning control law with initial learning mechanisms. Conformable derivative is well-behaved and can characterize a different step in real data sampling. The initial learning mechanism relaxes the initial value condition and improves the performance of the protocol to achieve consensus tracking. A distributed iterative learning scheme is proposed to realize the finite-time consensus by repeating the control attempt of the same trajectory and correcting the unsatisfactory control signal with the tracking error under the assumption of repeatable operation environments as well as a directed communication topology. The asymptotical convergence of the proposed P-type and the PD$^\alpha$-type distributed iterative learning protocol for all agents is strictly proved as the iteration number increases. Two numerical examples are simulated to verify the effectiveness of the protocols.

Key words: iterative techniques; consensus tracking control; conformable derivative; multi-agent systems

Citation: WANG Xiaowen, LIU Shuai, WANG Jinrong. Iterative learning-based consensus tracking control for conformable multi-agent systems. *Control Theory & Applications*, 2022, 39(10): 1836 – 1844

1 Introduction

Multi-agent systems are composed of a set of intelligent agents that through mutual communication, cooperation, and other ways to complete complex tasks that a single agent cannot realize. Distributed cooperative control of multi-agent systems include consensus, flocking, formation, swarming and rendezvous has been concerned by many researchers due to its wide applications in many areas such as physics [1], biology [2], satellites [3] and control engineering [4]. In particular, the consensus tracking control problem [5–7] is a kind of practical cooperation task that all agents are required to achieve specified value as desired. It can be applied in the vehicles and aerospace areas, exploration of unknown environments, navigation in harsh environments, cooperation on transportation tasks, helicopters and so on [8–10].

In recent years, researchers have proposed many approaches for the multi-agent system to realize desired consensus tracking from initial configuration [11–13]. For example, Liu et al. [14] studied the leader-following exponential consensus tracking problem with abrupt

Received 30 September 2021; accepted 25 May 2022.
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Recommended by Associate Editor: XU Jin-Ming.
Supported by National Natural Science Foundation of China (62133008, 61821004) and the Natural Science Foundation of Shandong Province, China (ZR2018MF021).

and incipient actuator faults under edge-fixed and edge-switching topologies. Cao and Song [15] developed a distributed adaptive control scheme to complete the consensus tracking problem for high-order multi-agent systems with consensus error transformation techniques.

However, the above literatures can only guarantee the realization of the desired consensus tracking asymptotically or in finite time. In practice, considering the safety and effectiveness of operation, a group of agents have to keep relative position during the whole process when performing specific repetitive tasks, such as a group of autonomous vehicles [16] and UAVs [17] cooperative to deliver huge goods and patrol in the air, respectively. Iterative learning control (ILC) [18] is an accurate technology by correcting the deviation between the output signal and the desired target to improve the performance of the system, which is suitable for solving the above problems. Recently, there are many researches on multi-agent systems consensus tracking control with ILC technology. Xiong et al. [19] presented quantized iterative learning controllers for digital networks to achieve the consensus tracking in a finite time interval with limited information communication. For a class of nonlinear multi-agent systems, Bu et al. [20] proposed a distributed model free adaptive ILC control protocol to solve the consensus tracking problem. To achieve the high precision consensus tracking, Zhang et al. [21] gave a unified ILC algorithm for heterogeneous multivehicle systems with switching topology and external disturbances.

In 2014, Khalil et al. [22] introduced the new concept of conformable derivative which is a natural extension of the usual derivative. The conformable derivative is well-behaved and obeys the chain rule and Leibniz rule. A rich number of relevant theoretical results are emerging [23–25]. It has attracted the attention of researchers due to its applications in various area, such as biology [26], physics [27], finance [28] and so on. Therefore, it is of great practical interest to study the distributed consensus tracking control for conformable multi-agent systems.

Motivated by the above discussion, the main purpose of this paper is to design appropriate protocols by using the ILC theory to achieve perfect tracking over finite time intervals. The main contributions of this paper can be summarized as follows: we considered a new simple well-behaved definition of derivative called conformable derivative in this paper. Different from the traditional difference method, conformable derivative can characterize a different step in real data sampling. The proposed distributed iterative learning-based scheme is a significant extension of the ILC approach to multi-agent systems and brings new alternatives to solve distributed consensus problems over finite time intervals.

The remainder of the paper is arranged as follows. The consensus tracking problem is formulated in Section 2. In Section 3, we present a distributed iterative learning scheme. Main results of this paper are given in Section 4, where the convergence conditions are analyzed. In Section 5, two simulation examples are given to illustrate the results. Finally, the conclusions are drawn in Section 6.

Notations: For a vector $\omega = (\omega_1, \cdots, \omega_n) \in \mathbb{R}^n$, we consider its vector norm $\|\omega\| = \sqrt{\sum\limits_{i=1}^{n} \omega_i^2}$. For a matrix $A \in \mathbb{R}^{m\times n}$, we consider its matrix norm $\|A\| = \sqrt{\lambda_{\max}(A^{\mathrm{T}}A)}$, where $\lambda_{\max}$ is the maximum eigenvalue of the matrix. The standard λ-norm and λ. α-norm for a function $g : [0,T] \to \mathbb{R}^n$ are defined as

$$\|g\|_\lambda = \sup_{t\in[0,T]} \{\mathrm{e}^{-\lambda t}\|g(t)\|\},$$

$$\|g\|_{\lambda.\alpha} = \sup_{t\in[0,T]} \{\mathrm{e}^{-\lambda \frac{t^\alpha}{\alpha}}\|g(t)\|\}$$

for some $\lambda > 0$ and $0 < \alpha < 1$, where $\|\cdot\|$ is any generic norm defined in the vector space $\mathbb{R}^n$. $\mathbb{N}^+$ stands for the set of positive integers. Given vectors or matrices A and B, $A \otimes B$ denotes the Kronecker product of A and B.

Preliminaries in graph theory: The communication topology of multi-agent systems composed of N agents can be described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, 2, \cdots, N\}$ is the set of vertices, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A}$ is the adjacency matrix. The set of neighboring nodes of the ith agent is denoted by $N_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$. If $j \in N_i$, the jth agent can receive the information from the ith agent. $\mathcal{A} = [a_{i,j}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix of $\mathcal{G}$ and $a_{i,i} > 0$. Set $a_{i,j} > 0$ for $(j, i) \in \mathcal{E}$ and $a_{i,j} = 0$ otherwise. Let $L = [l_{i,j}] \in \mathbb{R}^{N\times N}$ denote the Laplacian matrix of $\mathcal{G}$ where $l_{i,i} = \sum\limits_{j=1}^{N} a_{i,j}$ and $l_{i,j} = -a_{i,j}$ if $i \neq j$. Define a directed path as a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \cdots, (i_{n-1}, i_n)$. A directed graph is known as containing a spanning tree if the graph has at least one agent (as a root agent) with a directed path to any other agent.

2 Problem formulation

In this paper, we consider the iterative learning-based consensus tracking control for the following linear and nonlinear conformable multi-agent systems with repetitive properties as follows:

$$\begin{cases} \mathfrak{D}_\alpha^0 x_{k,i}(t) = Ax_{k,i}(t) + Bu_{k,i}(t), \\ y_{k,i}(t) = Cx_{k,i}(t) + Du_{k,i}(t), \\ t \in [0,T],\ T > 0,\ i \in \mathcal{V} \end{cases} \tag{1}$$

and

$$\begin{cases} \mathfrak{D}_\alpha^0 x_{k,i}(t) = f(x_{k,i}(t), t) + Bu_{k,i}(t), \\ y_{k,i}(t) = Cx_{k,i}(t),\ t \in [0,T],\ T > 0,\ i \in \mathcal{V}, \end{cases} \tag{2}$$

where $\mathfrak{D}_\alpha^0 (0<\alpha<1)$ denotes the conformable derivative with lower index zero (see Definition 1), $x_{k,i}(t) \in \mathbb{R}^n$, $u_{k,i}(t) \in \mathbb{R}^m$, $y_{k,i}(t) \in \mathbb{R}^m$, $f(x_{k,i}(t),t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times m}$ are real matrices.

Moreover, we rewrite the system in a compact form. For the kth iteration in the multi-agent systems, (1) and (2) can be rewritten as

$$\begin{cases} \mathfrak{D}_\alpha^0 x_k(t) = (I_N \otimes A)x_k(t) + (I_N \otimes B)u_k(t), \\ y_k(t) = (I_N \otimes C)x_k(t) + (I_N \otimes D)u_k(t), \\ t \in [0,T],\ T>0. \end{cases} \tag{3}$$

and

$$\begin{cases} \mathfrak{D}_\alpha^0 x_k(t) = \tilde{f}(x_k(t),t) + (I_N \otimes B)u_k(t), \\ y_k(t) = (I_N \otimes C)x_k(t), \\ t \in [0,T],\ T>0. \end{cases} \tag{4}$$

Definition 1[22, Definition 2.1] The conformable derivative with lower index a of a function $x: [a, \infty) \to \mathbb{R}$ is defined as

$$\mathfrak{D}_\beta^a x(t) = \lim_{\varepsilon\to 0} \frac{x(t+\varepsilon(t-a)^{1-\beta}) - x(t)}{\varepsilon}, \quad t>a,\ 0<\beta\leqslant 1,$$
$$\mathfrak{D}_\beta^a x(a) = \lim_{t\to a^+} \mathfrak{D}_\beta^a x(t).$$

Obviously, each state $x_k(t)$ of (3) and (4) with the initial state $x_k(0)$ and control function $u_k(t)$ have the form, respectively

$$x_k(t) = x_k(0)\mathrm{e}^{(I_N\otimes A)\frac{t^\alpha}{\alpha}} + \int_0^t \mathrm{e}^{(I_N\otimes A)(\frac{t^\alpha}{\alpha}-\frac{\tau^\alpha}{\alpha})}(I_N\otimes B)u_k(\tau)\tau^{\alpha-1}\mathrm{d}\tau, \tag{5}$$

and

$$x_k(t) = x_k(0) + \int_0^t (\tilde{f}(x_k(\tau),\tau) + (I_N\otimes B)u_k(\tau))\tau^{\alpha-1}\mathrm{d}\tau. \tag{6}$$

Let $y_\mathrm{d}(t)$ denote the desired trajectory for consensus tracking, which is regarded as a leader and index it by vertex 0 in the directed graph. Consequently, the united graph describing the information interaction between the leader and followers can be defined by $\mathcal{G}^+ = (\mathcal{V}\cup\{0\}, \mathcal{E}^+, \mathcal{A}^+)$, where $\mathcal{E}^+$ is the edge set and $\mathcal{A}^+$ is the adjacency matrix of graph $\mathcal{G}^+$. The communication topology of multi-agent systems is assumed to be described by graph $\mathcal{G}^+$, where each agent is corresponding to a node in $\mathcal{G}^+$. Meanwhile, we assume the virtual leader has at least one path to connect with any follower such that all the followers can receive the control objective from the leader. That is, the directed graph $\mathcal{G}^+$ contains a spanning tree with the virtual leader being the root. The main objective of this paper is to design appropriate distributed iterative learning schemes to guarantee all the agents implement the desired consensus tracking control over a finite time interval.

3 Distributed iterative learning scheme

ILC is used to realize the complete tracking task in a finite time interval by repeating the control attempt of the same trajectory and correcting the unsatisfactory control signal with the tracking error between the output signal and the desired trajectory. We defined the tracking error as the difference between the real-time relative outputs and the desired trajectory. In this section, we shall design distributed iterative learning schemes to drive the above tracking errors to converge to zero so that the multi-agent systems can implement the desired consensus tracking control objective.

We denote $\eta_{k,j}(t)$ as the available information at the $(k+1)$th iteration for the jth agent. Consider

$$\eta_{k,j}(t) = \sum_{w\in N_j} a_{j,w}(y_{k,w}(t) - y_{k,j}(t)) + s_j(y_\mathrm{d}(t) - y_{k,j}(t)), \tag{7}$$

where s_j equals 1 if the jth agent can access the desired trajectory and 0 otherwise. Let $e_{k,j}(t) = y_\mathrm{d}(t) - y_{k,j}(t)$ be the tracking error. Further, we can get

$$\eta_{k,j}(t) = \sum_{w\in N_j} a_{j,w}(e_{k,j}(t) - e_{k,w}(t)) + s_j e_{k,j}(t). \tag{8}$$

Remark 1 We shall design distribute protocols that only use the relative output instead of absolute measurements of output in the global framework. Each agent measures relative output errors through information interaction with neighbors in the local framework by limited communication.

For system (3) and (4), we consider the P-type and the PD$^\alpha$-type learning law with the initial state learning law, respectively

$$u_{k+1,j}(t) = u_{k,j}(t) + W_P \eta_{k,j}(t),$$
$$x_{k+1,j}(0) = x_{k,j}(0) + W_{P_0}\eta_{k,j}(0)$$

and

$$\begin{cases} u_{k+1,j}(t) = u_{k,j}(t) + W_{\mathrm{PD}_1}\eta_{k,j}(t) + W_{\mathrm{PD}_2}\mathfrak{D}_\alpha^0 \eta_{k,j}(t), \\ x_{k+1,j}(0) = x_{k,j}(0) + BW_{\mathrm{PD}_2}\eta_{k,j}(0), \end{cases}$$

where $W_P \in \mathbb{R}^{m\times m}$, $W_{P_0} \in \mathbb{R}^{n\times m}$, $W_{\mathrm{PD}_1} \in \mathbb{R}^{n\times m}$ and $W_{\mathrm{PD}_2} \in \mathbb{R}^{m\times m}$ are constant learning gain matrices.

Remark 2 Complete tracking can only be achieved under strict initial reset conditions, that is the initial state of the system is exactly equal to the expected initial state. However, it is difficult to meet the above conditions in actual situations. Therefore, we relax the initial value conditions and design the initial state learning laws.

Remark 3 Initial state learning laws can be recognized as discrete-time consensus protocols. That is, the iteration-axis can be treated as a discrete time-axis. under the proposed learning laws, the initial states of all agents can converge to the desired value over a finite interval.

For the kth iteration, we denote the column stack vectors: $\eta_k(t) = [\eta_{k,1}(t)^{\mathrm{T}} \cdots \eta_{k,N}(t)^{\mathrm{T}}]^{\mathrm{T}}$, $x_k(t) = [x_{k,1}(t)^{\mathrm{T}} \cdots x_{k,N}(t)^{\mathrm{T}}]^{\mathrm{T}}$, $y_k(t) = [y_{k,1}(t)^{\mathrm{T}} \cdots y_{k,N}(t)^{\mathrm{T}}]^{\mathrm{T}}$, $u_k(t) = [u_{k,1}(t)^{\mathrm{T}} \cdots u_{k,N}(t)^{\mathrm{T}}]^{\mathrm{T}}$, $e_k(t) = [e_{k,1}(t)^{\mathrm{T}} \cdots e_{k,N}(t)^{\mathrm{T}}]^{\mathrm{T}}$. Therefore, linking (8) and both P-type and PD$^\alpha$-type learning law by using Kronecker product, we obtain

$$\eta_k = ((L+S)\otimes I_m)e_k(t),$$

$$\begin{cases} u_{k+1}(t) = u_k(t) + ((L+S)\otimes W_P)e_k(t), \\ x_{k+1}(0) = x_k(0) + ((L+S)\otimes W_{P_0})e_k(0) \end{cases} \tag{9}$$

and

$$\begin{cases} u_{k+1}(t) = u_k(t) + ((L+S)\otimes W_{\mathrm{PD}_1})e_k(t) + \\ \qquad ((L+S)\otimes W_{\mathrm{PD}_2})\mathfrak{D}_\alpha^0 e_k(t), \\ x_{k+1}(0) = x_k(0) + ((L+S)\otimes BW_{\mathrm{PD}_2})e_k(0), \end{cases} \tag{10}$$

where I_m and L denote $m\times m$ identity matrix and graph Laplacian of $\mathcal{G}$, respectively, and $S = \mathrm{diag}\{s_1, \cdots, s_N\}$, $s_i \geqslant 0 (i = 1, 2, \cdots, N)$. Then, the convergence of this distributed iterative learning scheme will be analyzed in the next section.

Remark 4 In practical application, we can set to stop the iteration when the consensus tracking error is less than the actual required value. That is, we can stop the iteration if there exists a $k \in \mathbb{N}^+$ such that $|e_k(t)| < \epsilon$, where $\epsilon > 0$ is a preset parameter according to the actual demand.

4 Convergence analysis

In this section, we shall present two main results on the convergence of the proposed scheme.

4.1 Convergence analysis of P-type learning law for linear systems

First of all, we establish the following theorem by combining the P-type iterative learning law and multi-agent consensus tracking control of linear comformable systems (3).

Theorem 1 Consider the linear multi-agent systems (3) with P-type learning law (9). Suppose a directed graph $\mathcal{G}^+$ contains a spanning tree corresponding to the communication topology. If control gains satisfy

$$\|I_{mN} - (L+S)\otimes CW_{P_0} - (L+S)\otimes DW_P\| < 1 \tag{11}$$

and

$$\|I_{mN} - (L+S)\otimes DW_P\| < 1, \tag{12}$$

then the consensus tracking error $e_k(t) \to 0$ as iteration $k\to\infty$, i.e. $\lim\limits_{k\to\infty} y_{k,j}(t) = y_{\mathrm{d}}(t)$ for all $t\in[0,T]$.

Proof According to (9), the tracking error of the $(k+1)$th iteration can be written as

$$e_{k+1}(t) = y_{\mathrm{d}}(t) - y_{k+1}(t) = (I_{mN} - (L+S)\otimes DW_P)e_k(t) - (I_n\otimes C)(x_{k+1}(t) - x_k(t)), \tag{13}$$

which yields that

$$e_{k+1}(0) = (I_{mN} - (L+S)\otimes CW_{P_0} - (L+S)\otimes DW_P)e_k(0). \tag{14}$$

Then taking the matrix norm for the above equality, we have

$$\|e_{k+1}(0)\| \leqslant \|(I_{mN} - (L+S)\otimes CW_{P_0} - (L+S)\otimes DW_p)\| \times \|e_k(0)\|.$$

By condition (11), one can obtain

$$\lim_{k\to\infty}\|e_k(0)\| = 0. \tag{15}$$

Then using (9) to the states of all the agents (5), we get

$$x_{k+1}(t) = x_k(t) + ((L+S)\otimes W_{P_0})e_k(0)\mathrm{e}^{(I_N\otimes A)\frac{t^\alpha}{\alpha}} + \int_0^t \mathrm{e}^{(I_N\otimes A)(\frac{t^\alpha}{\alpha} - \frac{\tau^\alpha}{\alpha})}((L+S)\otimes BW_P)\cdot e_k(\tau)\tau^{\alpha-1}\mathrm{d}\tau.$$

Denoting $\delta x_k(t) = x_{k+1}(t) - x_k(t)$, we have

$$\|\delta x_k(t)\| \leqslant \|(L+S)\otimes W_{P_0}\| \times \|e_k(0)\mathrm{e}^{(I_N\otimes A)\frac{t^\alpha}{\alpha}}\| + \int_0^t \|\mathrm{e}^{(I_N\otimes A)(\frac{t^\alpha}{\alpha} - \frac{\tau^\alpha}{\alpha})}\| \times \|(L+S)\otimes BW_P\| \times \|e_k(\tau)\|\mathrm{d}\frac{\tau^\alpha}{\alpha}.$$

Next, multiplying both sides by $\mathrm{e}^{-\lambda\frac{t^\alpha}{\alpha}}$, it has

$$\begin{aligned} &\|\delta x_k(t)\|\mathrm{e}^{-\lambda\frac{t^\alpha}{\alpha}} \leqslant \\ &\|(L+S)\otimes W_{P_0}\| \times \|e_k(0)\mathrm{e}^{(I_N\otimes A)\frac{t^\alpha}{\alpha}}\|\mathrm{e}^{-\lambda\frac{t^\alpha}{\alpha}} + \\ &\int_0^t \|\mathrm{e}^{(I_N\otimes A)(\frac{t^\alpha}{\alpha} - \frac{\tau^\alpha}{\alpha})}\| \times \|(L+S)\otimes BW_P\| \times \\ &\|e_k(\tau)\|\mathrm{e}^{-\lambda\frac{t^\alpha}{\alpha}}\mathrm{d}\frac{\tau^\alpha}{\alpha} \leqslant \\ &\|(L+S)\otimes W_{P_0}\| \times \|e_k(0)\|\mathrm{e}^{-\lambda\frac{t^\alpha}{\alpha}}\mathrm{e}^{\|I_N\otimes A\|\frac{t^\alpha}{\alpha}} + \\ &\int_0^t \mathrm{e}^{(\|I_N\otimes A\| - \lambda)\frac{t^\alpha - \tau^\alpha}{\alpha}}\|(L+S)\otimes BW_P\| \times \\ &\|e_k\|_{\lambda.\alpha}\mathrm{d}\frac{\tau^\alpha}{\alpha}. \end{aligned} \tag{16}$$

Taking supremum, we get

$$\|\delta x_k\|_{\lambda.\alpha} \leqslant \|(L+S)\otimes W_{P_0}\| \times \|e_k(0)\|\mathrm{e}^{M\frac{T^\alpha}{\alpha}} + \frac{\|(L+S)\otimes BW_P\|\|e_k\|_{\lambda.\alpha}\Big(1 - \mathrm{e}^{-\lambda\frac{T^\alpha}{\alpha}}\Big)}{\lambda - M}, \tag{17}$$

where we denote $M = \|I_N\otimes A\|$.

For (13), taking the matrix norm, we can have

$$\|e_{k+1}(t)\| \leqslant \|I_{mN} - (L+S)\otimes DW_P\| \times \|e_k(t)\| + \|I_N\otimes C\| \times \|\delta x_k(t)\|. \tag{18}$$

Taking the $\lambda.\alpha$-norm and substituting (17) into (18) for the above inequality yield

$$\|e_{k+1}\|_{\lambda.\alpha} \leqslant (\|I_{mN}-(L+S)\otimes DW_P\| + \frac{\|I_N\otimes C\|\times\|(L+S)\otimes BW_P\|}{\lambda-M})\cdot (1-\mathrm{e}^{-\lambda\frac{T^\alpha}{\alpha}})\|e_k\|_{\lambda.\alpha} + \|I_N\otimes C\|\times\|(L+S)\otimes W_{P_0}\|\times\|e_k(0)\|\mathrm{e}^{M\frac{T^\alpha}{\alpha}}.$$

This implies that $\|e_{k+1}\|_{\lambda.\alpha}\to 0$ due to $\|e_k(0)\|\to 0$ and (12) when λ is sufficiently large, that is $\lim\limits_{k\to\infty}\|e_k\|_{\lambda.\alpha}=0$. □

4.2 Convergence analysis of PD$^\alpha$-type ILC for nonlinear systems

Next, we will give the following theorem for the combined studies of PD$^\alpha$-type ILC law and multi-agent consensus tracking control of nonlinear conformable systems (4). It is necessary to give the following assumption.

A1) Globally Lipschitz condition: The time-varying nonlinear function $f(x_z,t)$, satisfies

$$\|f(x_{z_1},t)-f(x_{z_2},t)\|\leqslant\gamma\|x_{z_1}-x_{z_2}(t)\|,\quad \forall x_{z_1},\ x_{z_2}\in\mathbb{R}^n, \tag{19}$$

where $\gamma\geqslant 0$ is constant.

Theorem 2 Consider the nonlinear multi-agent systems (4) with PD$^\alpha$-type learning law (10). Suppose assumption A1) holds and directed graph $\mathcal{G}^+$ contains a spanning tree corresponding to the communication topology. If control gains satisfy

$$\|I_{mN}-(L+S)\otimes CBW_{\mathrm{PD}_2}\|<1, \tag{20}$$

then the consensus tracking error $e_k(t)\to 0$ as iteration $k\to\infty$, i.e. $\lim\limits_{k\to\infty}y_{k,j}(t)=y_\mathrm{d}(t)$ for all $t\in[0,T]$.

Proof The tracking error of the $(k+1)$th iteration can be written as

$$e_{k+1}(t)=e_k(t)-(I_N\otimes C)\delta x_k(t). \tag{21}$$

Based on (10) and (21), one can obtain

$$e_{k+1}(0)=(I_{mN}-(L+S)\otimes CBW_{\mathrm{PD}_2})e_k(0).$$

Then, taking the matrix norm to both sides, it has

$$\|e_{k+1}(0)\| = \|I_{mN}-(L+S)\otimes CBW_{\mathrm{PD}_2}\|\times\|e_k(0)\|.$$

According to (20), we get

$$\lim_{k\to\infty}\|e_k(0)\|=0. \tag{22}$$

By the state equation (6) and PD$^\alpha$-type iterative learning law (10), we have

$$\begin{aligned}\delta x_k(t)=\,& x_{k+1}(0)-x_k(0)+\\ &\int_0^t(\tilde f(x_{k+1}(\tau),\tau)-\tilde f(x_k(\tau),\tau))\tau^{\alpha-1}\mathrm{d}\tau+\\ &\int_0^t(I_N\otimes B)(u_{k+1}(\tau)-u_k(\tau))\tau^{\alpha-1}\mathrm{d}\tau=\\ &((L+S)\otimes BW_{\mathrm{PD}_2})e_k(t)+\\ &\int_0^t(\tilde f(x_{k+1}(\tau),\tau)-\tilde f(x_k(\tau),\tau))\tau^{\alpha-1}\mathrm{d}\tau+\\ &(L+S)\otimes BW_{\mathrm{PD}_1}\int_0^t e_k(\tau)\tau^{\alpha-1}\mathrm{d}\tau.\end{aligned} \tag{23}$$

Then, taking norm for the above inequality and implementing into A1), we can get

$$\|\delta x_k(t)\|\leqslant (\|(L+S)\otimes BW_{\mathrm{PD}_2}\|+\gamma\int_0^t\|\delta x_k(\tau)\|\mathrm{d}\frac{\tau^\alpha}{\alpha}+ \|(L+S)\otimes BW_{\mathrm{PD}_1}\|t^\alpha)\mathrm{e}^{\lambda t}\|e_k\|_\lambda.$$

Note that $(\|(L+S)\otimes BW_{\mathrm{PD}_2}\|+\|(L+S)\otimes BW_{\mathrm{PD1}}\|t^\alpha)\mathrm{e}^{\lambda t}\|e_k\|_\lambda$ is a nondecreasing function on $[0,T]$. Applying the Gronwall inequality for $\|\delta x_k(t)\|$, we have

$$\begin{aligned}\|\delta x_k(t)\|\leqslant\,&\omega(t)\mathrm{e}^{\lambda t}\|e_k\|_\lambda+\gamma\int_0^t\|\delta x_k(\tau)\|\mathrm{d}\frac{\tau^\alpha}{\alpha}\leqslant\\ &\omega(t)\mathrm{e}^{\lambda t}\|e_k\|_\lambda\mathrm{e}^{\gamma\int_0^t\tau^\alpha\mathrm{d}\tau}\leqslant\\ &\omega(t)\mathrm{e}^{\lambda t}\|e_k\|_\lambda\mathrm{e}^{\gamma t^\alpha},\end{aligned}$$

where $\omega(t)=\|(L+S)\otimes BW_{\mathrm{PD}_2}\|+\|(L+S)\otimes BW_{\mathrm{PD}_1}\|t^\alpha$. Moreover, taking λ-norm, one can obtain

$$\|\delta x_k\|_\lambda\leqslant\sup_{t\in[0,T]}\|\omega(t)\mathrm{e}^{\gamma t^\alpha}\|\times\|e_k\|_\lambda. \tag{24}$$

By (21) and (23), it follows with

$$\begin{aligned}&e_{k+1}(t)=\\ &e_k(t)-(I_N\otimes C)\delta x_k(t)=\\ &(I_{mN}-(L+S)\otimes BW_{\mathrm{PD}_2})e_k(t)-\\ &(I_N\otimes C)\int_0^t(\tilde f(x_{k+1}(\tau),\tau)-\tilde f(x_k(\tau),\tau))\tau^{\alpha-1}\mathrm{d}\tau-\\ &(L+S)\otimes CBW_{\mathrm{PD}_1}\int_0^t e_k(\tau)\tau^{\alpha-1}\mathrm{d}\tau.\end{aligned} \tag{25}$$

Taking norm for (25), we have

$$\begin{aligned}&\|e_{k+1}(t)\|\leqslant\\ &\|I_{mN}-(L+S)\otimes BW_{\mathrm{PD}_2}\|\times\|e_k(t)\|+\\ &\|I_N\otimes C\|\gamma\int_0^t\mathrm{e}^{\lambda\tau}\mathrm{d}\frac{\tau^\alpha}{\tau}\|\delta x_k\|_\lambda\\ &\|(L+S)\otimes CBW_{\mathrm{PD}_1}\|\int_0^t\mathrm{e}^{\lambda\tau}\mathrm{d}\frac{\tau^\alpha}{\tau}\|e_k\|_\lambda.\end{aligned} \tag{26}$$

For any given $0<\alpha<1$, the existence of $p>1$ makes $\alpha>\frac{1}{p}$. Then we can see $\exists q>1$ makes $\frac{1}{p}+\frac{1}{q}=1$. By applying Hölder inequality,

$$\begin{aligned}\int_0^t\mathrm{e}^{\lambda\tau}\tau^{\alpha-1}\mathrm{d}\tau\leqslant\,&\frac{\mathrm{e}^{\lambda t}}{\sqrt[p]{p}\sqrt[p]{\lambda}}(\frac{t^{q\alpha-q+1}}{q\alpha-q+1})^{\frac{1}{q}}\leqslant\\ &\frac{1}{\lambda}\mathrm{e}^{\lambda t}\frac{T^{\alpha-\frac{1}{p}}}{\sqrt[q]{q\alpha-q+1}},(\lambda\geqslant 1).\end{aligned} \tag{27}$$

Substituting (27) into (26), we get

$$\|e_{k+1}(t)\|\leqslant\|I_{mN}-(L+S)\otimes BW_{\mathrm{PD}_2}\|\times$$

$$\|e_k(t)\|+\frac{\|I_N\otimes C\|\gamma e^{\lambda t}T^{\alpha-\frac{1}{p}}\|\delta x_k\|_\lambda}{\lambda\sqrt[q]{q\alpha-q+1}}+\frac{\|(L+S)\otimes CBW_{\mathrm{PD}_1}\|e^{\lambda t}T^{\alpha-\frac{1}{p}}\|e_k\|_\lambda}{\lambda\sqrt[q]{q\alpha-q+1}}. \tag{28}$$

Next, taking λ-norm and substituting (24) into (28)

$$\|e_{k+1}\|_\lambda\leqslant(\|I_{mN}-(L+S)\otimes BW_{\mathrm{PD}_2}\|+\frac{\|I_N\otimes C\|\gamma T^{\alpha-\frac{1}{p}}\sup\limits_{t\in[0,T]}\|\omega(t)e^{\gamma t^\alpha}\|}{\lambda\sqrt[q]{q\alpha-q+1}}+\frac{(\|L+S)\otimes CBW_{\mathrm{PD}_1}\|T^{\alpha-\frac{1}{p}}}{\lambda\sqrt[q]{q\alpha-q+1}})\|e_k\|_\lambda.$$

This implies that $\|e_{k+1}\|_\lambda\to 0$ due to (20) when λ is sufficiently large, i.e., $\lim\limits_{k\to\infty}\|e_k\|_\lambda=0$. □

5 Simulation examples

Two simulation examples are performed to illustrate the effectiveness of the proposed distributed iterative learning protocols.

The interaction graph among agents is described by an directed graph $\mathcal{G}^+=(\mathcal{V}\cup\{0\},\mathcal{E}^+,\mathcal{A}^+)$ in Fig. 1, where vertex 0 represents the virtual leader. We adopt $a_{i,j}=1$ if $(i,j)\in\mathcal{E}_\mathcal{G}$. It is easy to get the Laplacian matrix for followers

$$L=\begin{pmatrix}0&0&0&0\\0&2&-1&-1\\-1&0&1&0\\0&0&-1&1\end{pmatrix},$$

and $S=\mathrm{diag}\{1,1,0,0\}$,

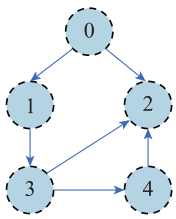


Fig. 1 Directed communication topology among agents in the network

In this section, we set $\alpha=0.7$. The norm of the tracking errors in each iteration is designated 2-norm in the following examples. The initial state at first iteration is chosen as $x_1=[1\ \ -3]^\mathrm{T}$, $x_2=[2\ \ -1]^\mathrm{T}$, $x_3=[0\ 4]^\mathrm{T}$, and $x_4=[-1\ 2]^\mathrm{T}$. The desired initial state is unique $x_d=0$. The initial control signal $u_{1,i}=0,\ i=1,2,3,4$ for all agents.

Example 1 Consider the multi-agent system (3) as follows:

$$\begin{cases}\mathfrak{D}_\alpha^0 x_{k,i}(t)=\begin{bmatrix}-1&1\\0&-2\end{bmatrix}x_{k,i}(t)+\begin{bmatrix}0.1&0\\0&-0.2\end{bmatrix}u_{k,i}(t),\\ y_{k,i}(t)=\begin{bmatrix}0.2&0\\0&0.1\end{bmatrix}x_{k,i}(t)+\begin{bmatrix}1&0\\0&0.2\end{bmatrix}u_{k,i}(t),\end{cases} \tag{29}$$

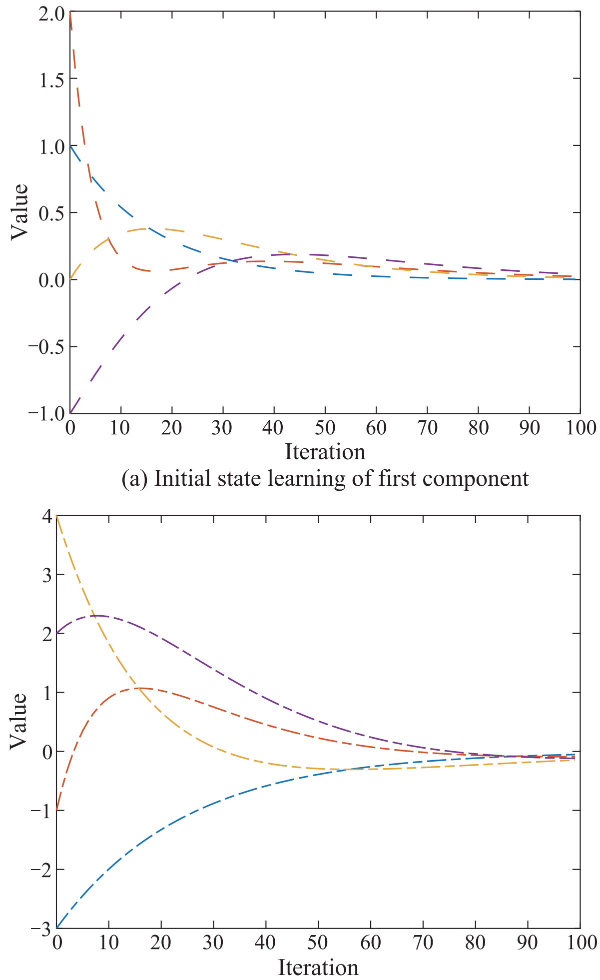
and the desired reference trajectory

$$y_\mathrm{d}=\begin{bmatrix}1-\cos(2\pi t)\\ \sin(2\pi t)\end{bmatrix},\ t\in[0,1].$$

To verify the contraction conditions in Theorem 1, we select the learning gain matrix

$$W_P=\begin{bmatrix}0.2&0\\0&0.4\end{bmatrix},\ W_{P_0}=\begin{bmatrix}0.3&0\\0&0.4\end{bmatrix}.$$

By explicit calculation, we can obtain that $\|I_{mN}-(L+S)\otimes DW_P\|=0.9895<1$ and $\|I_{mN}-(L+S)\otimes CW_{P_0}-(L+S)\otimes DW_P\|=0.9899<1$. The convergence condition in Theorem 1 is satisfied so that the consensus tracking can be achieved. Fig. 2 shows the initial state learning of agents. Fig. 3 shows the output of a leader and four agents at the 1st and 100th iteration. Fig. 4 depicts the tracking errors of each agent. It is easy to see all the initial states and outputs converge to the desired trajectory over a finite time interval, respectively.



(a) Initial state learning of first component

(b) Initial state learning of second component

Fig. 2 Initial state value at each iteration under P-type learning law

Example 2 Set $x_{k,i}(t):=[x_{k,i,1}(t)\ x_{k,i,2}(t)]^\mathrm{T}$ for each agent. Consider the multi-agent system (4) as follows:

$$\begin{cases} \mathfrak{D}_{\alpha}^{0} x_{k,i}(t) = \begin{bmatrix} \cos(x_{k,i,1}(t)) - 1.2x_{k,i,1}(t) \\ 0.8\sin(1.5x_{k,i,2}(t)) + 0.2x_{k,i,2}(t) \end{bmatrix} + \\ \qquad \begin{bmatrix} 1 & 1.4 \\ 0.5 & 1.2 \end{bmatrix} u_{k,i}(t), \\ y_{k,i}(t) = \begin{bmatrix} 1.8 & 1.2 \\ 0 & 2 \end{bmatrix} x_{k,i}(t), \end{cases} \tag{30}$$

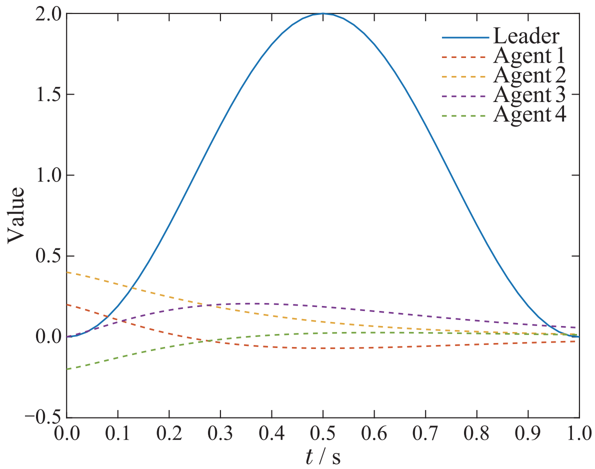
and the desired reference trajectory

$$y_{\rm d} = \begin{bmatrix} 2t + 2\cos(3t) \\ -2t - \sin(2t) \end{bmatrix},\ t \in \forall [0, 1].$$

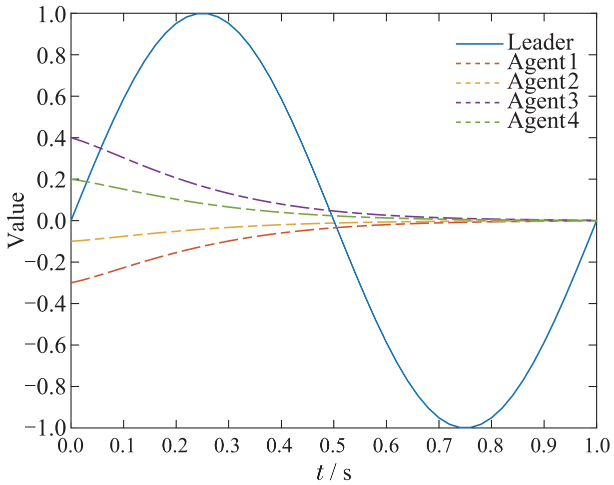
To verify the contraction conditions in Theorem 2, we select the learning gain matrix

$$W_{P1} = \begin{bmatrix} 0.125 & 0 \\ 0 & 0.125 \end{bmatrix},\quad W_{P_2} = \begin{bmatrix} 0.28 & -0.47 \\ -0.12 & 0.28 \end{bmatrix}.$$

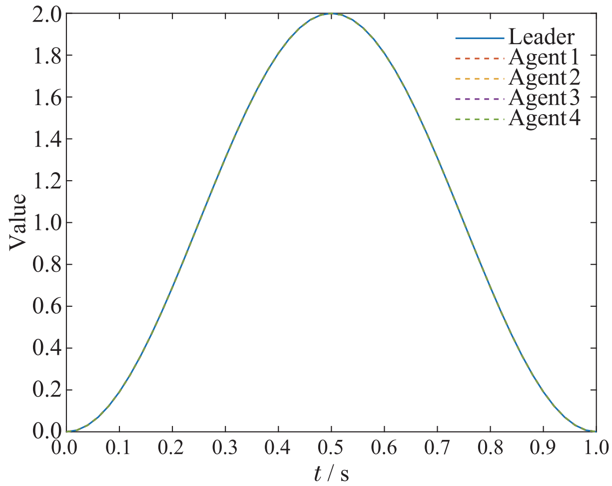
Through simple calculation, we can get $\|I_{mN} - (L+S) \otimes CBW_{P2}\| = 0.9318 < 1$. The convergence condition in Theorem 2 is satisfied so that the consensus tracking can be achieved. Fig. 5 shows the initial state learning of agents. Fig. 6 shows the output of a leader and four agents at the 1st and 50th iteration. Fig. 7 depicts the tracking errors of each agent. It is easy to see all the initial states and outputs converge to the desired trajectory in a finite time interval, respectively.



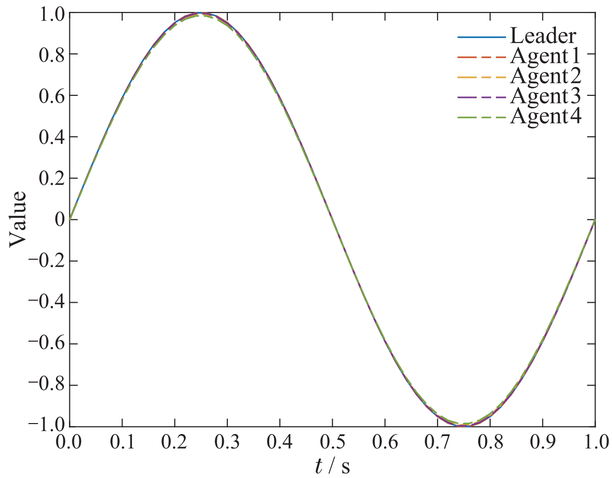
(a) Output of first component at 1st iteration



(b) Output of second component at 1st iteration



(c) Output of first component at 100th iteration



(d) Output of second component at 100th iteration

Fig. 3 Output trajectory at 1st and 100th iteration under P-type learning law

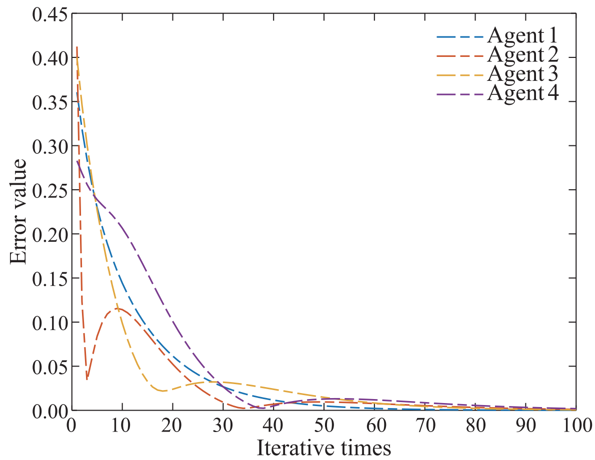
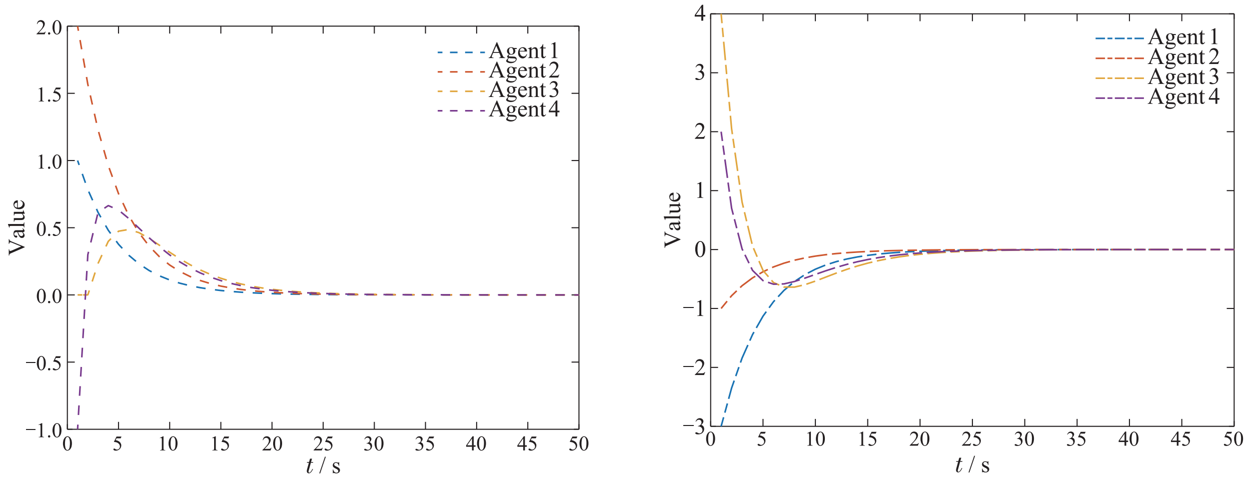


Fig. 4 The tracking error at each iteration under P-type learning law

6 Conclusions

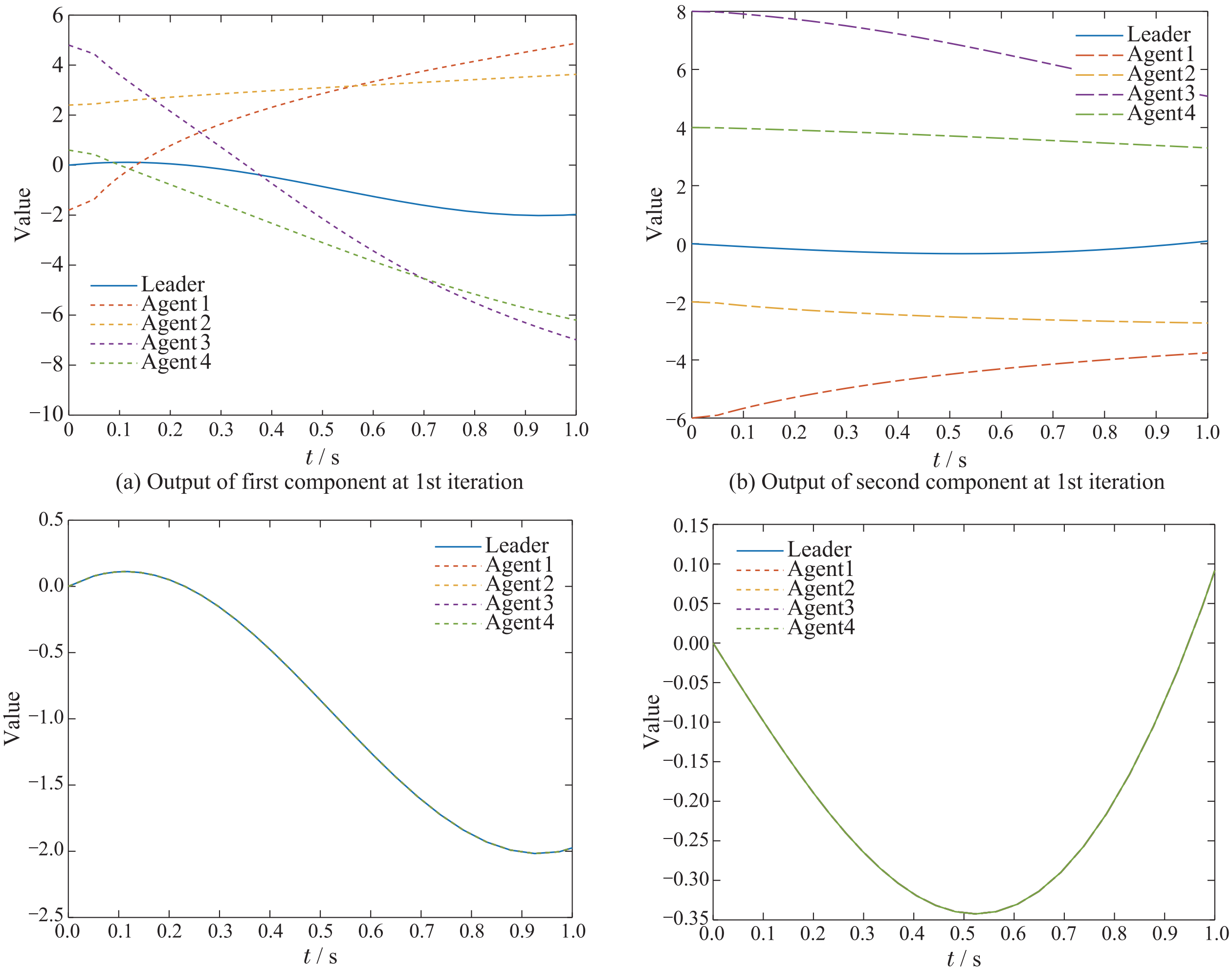
In this paper, the finite-time consensus tracking control problem for conformable multi-agent systems has been addressed. Under the proposed distributed iterative learning scheme, the desired consensus tracking can be achieved over a finite interval as the iteration increases. By using initial state learning laws, the performance of our protocol can be improved to reach the perfect tracking of the desired trajectory. Two simulations are given to verify the effectiveness of our results on iterative learning-based consensus tracking control. In our future work, the derived protocols will be further studied to provide them with explicit application validations by considering some practical applications such as biomedical science and intelligent unmanned systems.



(a) Initial state learning of first component

(b) Initial state learning of second component

Fig. 5 Initial state value at each iteration under PD$^{\alpha}$-type learning law



(a) Output of first component at 1st iteration

(b) Output of second component at 1st iteration

(c) Output of first component at 50th iteration

(d) Output of second component at 50th iteration

Fig. 6 Output trajectory at 1st, 10th, and 50th iteration under PD$^{\alpha}$-type learning law

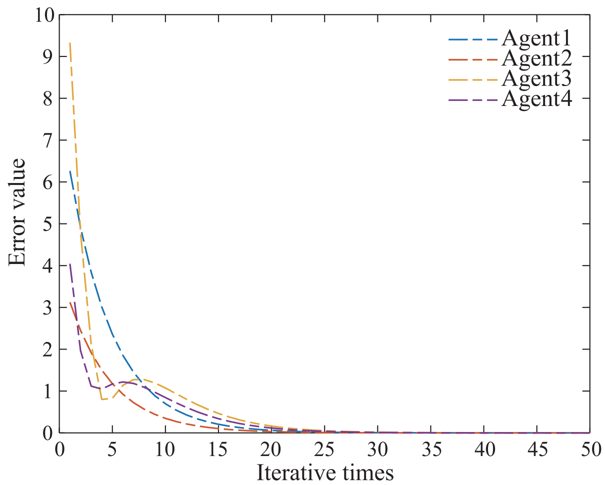


Fig. 7 The tracking error at each iteration under PD$^{\alpha}$-type learning law

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