基于三阶超螺旋滑模观测器算法的机器人力矩估计方法

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摘要: 本文研究了六自由度(6-DOF)机器人关节力矩估计问题. 为了提高机器人关节力矩估计精度, 设计一种三阶超螺旋滑模关节力矩观测器. 在对观测器系统进行稳定性分析时, 本文设计了一种新的Lyapunov 函数证明了观测器闭环系统的稳定性. 与已有的机器人力矩观测器结果不同, 该力矩观测器仅以机器人关节角度作为观测器输入信号, 减少了对机器人系统信息的依赖. 最后, 以6-DOF工业机器人为对象, 开展实验验证. 实验结果表明, 与传统线性力矩观测器相比, 本文提出的滑模力矩观测器估计误差小, 验证理论的正确性.

关键词: 六自由度机器人; 三阶超螺旋滑模观测器; 有限时间收敛; 力矩估计.

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Torque estimation algorithm of robot based on third-order super-twisting sliding mode observer

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Abstract: In this paper, the joint torque estimation of the six-degree-of-freedom (6-DOF) robots is studied. In order to improve the accuracy of the robot joint torque estimation, a third-order super-twisting sliding mode joint torque observer is designed. In the stability analysis of the observer system, a new Lyapunov function is designed to prove the stability of the closed-loop system of the observer. Unlike the results of existing robot joint torque observers, the torque observer proposed in this paper uses only the robot joint angle as the observer input signal, which reduces the reliance on the robot state information. Finally, the 6-DOF industrial robot is used as the object for experimental verification. The experimental results show that the estimation error of the sliding mode torque observer is smaller than that of the linear torque observer, which verifies the correctness of the theory.

Key words: 6-DOF robots; Third-order super-twisting sliding mode observer; Finite-time convergence; Torque estimation

1 引言

机械电子技术的进步促进了工业自动化技术的发展, 使得工业机器人广泛应用于各种工业现场. 然而一些特殊应用场合, 如修边打磨 [1]、碰撞检测 [2] 等, 需要与周围环境及操作物相接触, 此时需要考虑机器人与外部环境之间的相互作用力. 这种情况下传统的位置控制方法是不能适用的, 往往需要采用力控制方式. 对于力控制来说, 机器人各关节力矩值是至关重要的信息. 实际应用中, 获取机器人关节力矩最直接最准确的方法就是在机器人末端或底座安装多轴力传感器获得力信息, 或者直接在机器人各关节内安装力矩传感器. 然而由于力传感器的昂贵价格, 基于力传感器的力控机器人未能得到广泛应用. 因此, 开发设计一种结构简单、性能出色的力矩观测器具有重要意义.

从控制理论角度考虑,可将求解机器人关节力矩问题转化为求解系统状态问题. 针对系统变量估计, 许多学者采用状态观测器的方式进行估计, 如自适应观测器 [3–7], 滑模观测器 [8–10], 扩张状态观测器 [11–12] 等, 并在一些实际系统中加以应用. 如文献[7] 采用全

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阶自适应观测器估计电机系统转速. 目前相关研究成果大多为渐近收敛的, 相比之下，具有有限时间收敛性的观测器具有更快的收敛速度和抗干扰能力. 相关学者基于有限时间控制理论设计了有限时间观测器[13–17]. 如文献[17] 采用基于加幂积分方法设计有限时间观测器用以估计DC-AC 变换器系统的状态信息. 然而, 此类型观测器结构较为复杂, 不利于在实际工业机器人系统中应用. 滑模观测器由于鲁棒性好, 结构简洁, 实用性强, 受到研究者的欢迎. 依据滑模观测器理论[18], 不少学者基于Levant的成果设计了有限时间滑模观测器[19–20, 28]. 文献[28] 采用高阶有限时间观测器估计工业机器人关节力矩, 取得较好的效果. 但是此观测器的输入信号不仅包含了机器人关节角度信号, 还含有角速度信号. 由于机器人关节角速度是通过关节位置差分滤波得到, 这增加了工业机器人控制系统的计算量. 考虑此情况, 本文考虑仅已知关节角度情况下的关节力矩估计问题, 相比文献[28] 更具有挑战性.

本文主要研究了基于三阶超螺旋滑模观测器的六自由度工业机器人力矩估计问题. 针对六自由度机器人系统, 首先, 依据机器人动力学模型, 将含有机器人关节力矩的项扩张为第三个系统变量, 设计三阶超螺旋滑模观测器. 其次, 与其他研究者对滑模观测器稳定性分析过程不同, 本文设计一种新的Lyapunov 函数, 并采用基于有限时间Lyapunov 稳定性判据的方法分析了滑模观测器有限时间收敛性. 最后, 以六自由度工业机器人为对象,开展力矩估计实验.

本文的主要创新点如下:

(1)本文采用观测器的方式估计机器人关节力矩和角速度值, 避免了采用差分方法带来的噪声.

(2)本文在观测器设计时仅将机器人关节位置作为输入信号, 减少了对实际系统的信息需求.

(3)采用一种新的Lyapunov函数, 依据有限时间的Lyapunov稳定性判据证明了三阶超螺旋观测器闭环系统的有限时间稳定性. 此外, 本文提供的证明方法还能为观测器增益值选取提供参考范围.

2 问题描述与准备知识

本文主要研究了如何在不使用外部传感器的情况下估计机器人关节力矩值. 在本节中, 首先介绍了六自由度工业机器人动力学模型的基本结构; 然后给出了研究过程中用到的引理.

2.1 工业机器人动力学模型

一般来说，六自由度工业机器人的动力学方程如下 [21]:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) + f(\dot{\theta}) = \tau, \quad (1)$$

其中,$\theta \in R^{6\times1}$为机器人关节位置矢量, $\tau \in R^{6\times1}$ 为机器人关节力矩,$M(\theta) \in R^{6\times6}$ 为机器人惯性矩阵, $C(\theta,\dot{\theta}) \in R^{6\times6}$为机器人向心力和科氏力矢项,$G(\theta) \in R^{6\times1}$ 为机器人重力项矢量, $f(\dot{\theta}) \in R^{6\times1}$为机器人关节摩擦力矢量. 在实际机器人系统中, $M(\theta)$, $C(\theta,\dot{\theta})$ 以及$G(\theta)$ 均为光滑的 [21].

机器人关节的摩擦力模型通常可以表示如下[27]:

$$f(\dot{\theta}) = f_c \cdot \mathrm{sgn}(\dot{\theta}) + f_v \cdot \dot{\theta} + f_b, \quad (2)$$

其中, f_c为库伦摩擦系数, f_v为粘性摩擦系数, f_b 为摩擦力偏置值, $\mathrm{sgn}(\cdot)$ 为符号函数.

假设 1 假设机器人关节角度、角速度是光滑变化且有界的, 即存在正实数$N > 0$, 满足$|\theta_{1,i}| < N$, $|\theta_{2,i}| < N, i = 1, 2, \cdots, 6$.

假设 2 假设机器人关节力矩是光滑变化且有界的, 综合假设1 可知θ_3是光滑有界的, 且可得$\dot{\theta}_3 < L$, 其中L 为正常数.

2.2 相关定义与引理

定义 1 定义符号$\mathrm{sig}^\alpha(\sigma) = |\sigma|^\alpha \mathrm{sgn}(\sigma)$, 其中$\alpha > 0$.当$\sigma = [\sigma_1, \cdots, \sigma_n]^T$, $\mathrm{sig}^\alpha(\sigma) = [\mathrm{sig}^\alpha(\sigma_1), \cdots, \mathrm{sig}^\alpha(\sigma_n)]^T$.

定义 2 定义符号$\mathrm{sat}_N(\sigma) = N \cdot \mathrm{sat}(\frac{\sigma}{N})$, 当$\sigma = [\sigma_1, \cdots, \sigma_n]^T$ 时, $\mathrm{sat}_N(\sigma) = [N \cdot \mathrm{sat}(\frac{\sigma_1}{N}), \cdots, N \cdot \mathrm{sat}(\frac{\sigma_n}{N})]^T$, 其中$N$为一正常数.

引理 1 [21] 对于机器人系统(1), 其惯性矩阵$M(\theta)$是有界的正定矩阵,且满足关系: $\lambda_m I_{6\times6} \leqslant M(\theta) \leqslant \lambda_M I_{6\times6}$, 其中$\lambda_m, \lambda_M$为常数, $I_{6\times6}$ 为6 阶单位矩阵.

引理 2 [23] 若$x, y \in \mathbb{R}$, 且$\epsilon, \bar{p}, \bar{q}$ 是正实数, 则存在如下关系:

$$|x|^{\bar{p}}|y|^{\bar{q}} \le \frac{\bar{p}}{\bar{p}+\bar{q}}\epsilon|x|^{\bar{p}+\bar{q}} + \frac{\bar{q}}{\bar{p}+\bar{q}}\epsilon^{-\bar{p}/\bar{q}}|y|^{\bar{p}+\bar{q}}.$$

引理 3 [22] 对$\forall\sigma_i \in R$,$i = 1, 2, \cdots, n$, 以及实数p, 则有:

(1) 当$0 < p < 1$时, $(\sum_{i=1}^n |\sigma_i|)^p \le \sum_{i=1}^n |\sigma_i|^p$,

(2) 当$p > 1$时, $n^{1-p}\sum_{i=1}^n |\sigma_i|^p \le \sum_{i=1}^n |\sigma_i|^p \le (\sum_{i=1}^n |\sigma_i|)^p$.

引理 4 [23] 对于实数x, y, p, 存在如下性质:

(1) 当$0 < p < 1$时, $|x^p - y^p| \le 2^{1-p}|x - y|^p$,

(2) 当$p > 1$时, $|x^p - y^p| \ge 2^{1-p}|x - y|^p$,

(3) 当$p > 1$时, $|x^p - y^p| \le h|x - y|^p + h|x - y||y|^{p-1}$, 其中$h = p(2^{p-2} + 2)$.

引理 5 [25] 若存在实数s_1,s_2,$N > 0$, 假设$|s_1| < N$, 则$|s_1 - \mathrm{sat}_N(s_2)| \le |s_1 - s_2|$.

3 三阶超螺旋滑模力矩观测器设计与分析

记$\theta_1 = \theta, \theta_2 = \dot{\theta}$,取$\theta_3 = M^{-1}(\theta_1)\tau$, 机器人系统

方程(1) 可重写为

$$\begin{cases} \dot{\theta}_1 = \theta_2, \\ \dot{\theta}_2 = \theta_3 + g(\theta_1, \theta_2), \end{cases} \tag{3}$$

其 中,$g(\theta_1, \theta_2) = -M^{-1}(\theta_1)(C(\theta_1, \theta_2)\theta_2 + G(\theta_1) + f(\theta_2))$.

引理 6　对于$g(\theta_1, \theta_2)$, 在满足假设1的条件下, 根据引理5可得如下结论:

$$\begin{aligned} &|g(\theta_1, \theta_2) - g(\mathrm{sat}_N(\tilde{\theta}_1, \mathrm{sat}_N(\tilde{\theta}_2))| \\ \le& a_1|\theta_1 - \tilde{\theta}_1|^{\frac{1}{3}} + a_2|\theta_2 - \tilde{\theta}_2|^{\frac{1}{2}} + a_3, \end{aligned} \tag{4}$$

其中, a_1, a_2, a_3 为合适的正常数. 证明过程详见附录.

定理 1　对系统(3), 设计如下三阶超螺旋滑模观测器

$$\begin{cases} \dot{\tilde{\theta}}_1 = \tilde{\theta}_2 + k_1 \cdot \mathrm{sig}^{\frac{2}{3}}(\theta_1 - \tilde{\theta}_1), \\ \dot{\tilde{\theta}}_2 = \tilde{\theta}_3 + g(\mathrm{sat}_N(\tilde{\theta}_1), \mathrm{sat}_N(\tilde{\theta}_2)) \\ \qquad + k_2 \cdot \mathrm{sig}^{\frac{1}{3}}(\theta_1 - \tilde{\theta}_1), \\ \dot{\tilde{\theta}}_3 = k_3 \cdot \mathrm{sgn}(\theta_1 - \tilde{\theta}_1), \end{cases} \tag{5}$$

其 中, $\tilde{\theta}_i$为θ_i的 估 计 值, $i = 1, 2, 3$. 当$k_1, k_2, k_3 > 1$满足一定条件时,估计误差可以有限时间收敛到如下区域内

$$\Omega_1 = \{(z_1, z_2, z_3) | V(z) \le \gamma_3(\frac{\gamma_2}{\gamma_1})^{\frac{6}{5}}\}, \tag{6}$$

其中, $V(z)$定义如(10)所示,$z_1 = \theta_1 - \tilde{\theta}_1$,$z_2 = \frac{\theta_1 - \tilde{\theta}_1}{k_1}$, $z_3 = \frac{\theta_3 - \tilde{\theta}_3}{k_2}$, $\gamma_1, \gamma_2, \gamma_3$为正的常数.

证明: 定 义 观 测 误 差$e_i = \theta_i - \tilde{\theta}_i, i = 1, 2, 3$.简 记$\bar{g} = g(\theta_1, \theta_2) - g(\mathrm{sat}_N(\tilde{\theta}_1), \mathrm{sat}_N(\tilde{\theta}_2))$. 综 合 观 测 器(5) 和机器人系统(3) , 可得闭环系统

$$\begin{cases} \dot{e}_1 = e_2 - k_1 \mathrm{sig}^{\frac{2}{3}}(e_1), \\ \dot{e}_2 = e_3 + \bar{g} - k_2 \mathrm{sig}^{\frac{1}{3}}(e_1), \\ \dot{e}_3 = \dot{\theta}_3 - k_3 \mathrm{sgn}(e_1). \end{cases} \tag{7}$$

令$z_1 = e_1$, $z_2 = \frac{e_2}{k_1}$, $z_3 = \frac{e_3}{k_2}$. 则(7) 可写为

$$\begin{cases} \dot{z}_1 = c_1(z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)), \\ \dot{z}_2 = c_2(z_3 - \mathrm{sig}^{\frac{1}{3}}(z_1)) + \dfrac{\bar{g}}{k_1}, \\ \dot{z}_3 = c_3(d_1 - \mathrm{sgn(z_1)}), \end{cases} \tag{8}$$

其中, $c_1 = k_1$, $c_2 = \frac{k_2}{k_1}$, $c_3 = \frac{k_3}{k_2}$, $d_1 = \frac{\dot{\theta}_3}{k_3}$.

取$k_3 > L > \dot{\theta}_3$, 故可得$d_1 < \rho < 1$.

基于引理6 以及变量e_i与z_i之间的变换关系, 可得

$$\frac{\bar{g}}{k_1} \le \sigma_1|z_1|^{\frac{1}{3}} + \sigma_2|z_2|^{\frac{1}{2}} + \sigma_3, \tag{9}$$

其中, $\sigma_1 = \frac{a_1}{k_1}$, $\sigma_2 = \frac{a_2}{\sqrt{k_1}}$, $\sigma_3 = \frac{a_3}{\sqrt{k_1}}$.

取如下Lyapunov函数:

$$V = V_1 + V_2 + V_3, \tag{10}$$

其中

$$\begin{cases} V_1 = \displaystyle\int_{\mathrm{sig}^{\frac{3}{2}}(z_2)}^{z_1} (s - \mathrm{sig}^{\frac{3}{2}}(z_2))ds, \\ V_2 = \displaystyle\int_{\mathrm{sig}^2(z_3)}^{z_2} (\mathrm{sig}^2(s) - \mathrm{sig}^4(z_3))ds, \\ V_3 = \dfrac{1}{6}|z_3|^6. \end{cases} \tag{11}$$

步骤 1　首先, 对V_1进行讨论.

对V_1求导可得

$$\begin{aligned} \dot{V}_1 =& (z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2))\dot{z}_1 - \frac{3}{2}|z_2|^{\frac{1}{2}}\dot{z}_2(z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2)) \\ \le& \frac{3}{2}|z_2|^{\frac{1}{2}} \cdot |z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2)| \cdot \big(c_2|z_3 - \mathrm{sig}^{\frac{1}{3}}(z_1)| + \\ & \sigma_1|z_1|^{\frac{1}{3}} + \sigma_2|z_2|^{\frac{1}{2}} + \sigma_3\big) + c_1(z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2)) \times \\ & (z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)) \\ \le& \frac{3}{2}|z_2|^{\frac{1}{2}}|z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2)|\big((\sigma_1 + \sigma_2)|z_3| + \sigma_3 \\ & + (c_2 + \sigma_1 + \sigma_2)|z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)| \\ & + 2^{\frac{1}{2}}(c_2 + \sigma_1)|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|\big) \\ & + c_1(z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2))(z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)). \end{aligned} \tag{12}$$

根据引理4可得

$$\begin{aligned} & c_1(z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2))(z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)) \\ =& -c_1|z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2)| \cdot |z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)| \\ \le& -2^{-\frac{1}{2}}c_1|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}. \end{aligned} \tag{13}$$

考虑$|z_2|^{\frac{1}{2}}|z_3 - \mathrm{sig}^{\frac{1}{3}}(z_1)||z_1 - \mathrm{sig}^{\frac{3}{2}}(z_2)|$ 各部分:

$$|z_2|^{\frac{1}{2}} \le |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)| + |z_3|. \tag{14}$$

根据引理4可得

$$\begin{aligned} & |\mathrm{sig}^{\frac{3}{2}}(z_2) - z_1| \\ \le& l_1(|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{3}{2}} + |z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)||z_2|^{\frac{1}{2}}), \end{aligned} \tag{15}$$

其中, $l_1 = \frac{3}{2} \cdot (2^{-\frac{1}{2}} + 2)$.

同理, 由引理4可得

$$\begin{aligned} & |z_3 - \mathrm{sig}^{\frac{1}{3}}(z_1)| \\ =& |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2) + \mathrm{sig}^{\frac{1}{2}}(z_2) - \mathrm{sig}^{\frac{1}{3}}(z_1)| \\ \le& |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)| + |\mathrm{sig}^{\frac{1}{2}}(z_2) - \mathrm{sig}^{\frac{1}{3}}(z_1)| \\ \le& |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)| + 2^{\frac{1}{2}}|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{1}{2}}. \end{aligned} \tag{16}$$

由(14)-(16)和引理2可得

$$\begin{aligned}&\frac{3}{2}|z_2|^{\frac{1}{2}}|z_1-\mathrm{sig}^{\frac{3}{2}}(z_2)|\big((\sigma_1+\sigma_2)|z_3|+\sigma_3\\&\quad+(c_2+\sigma_1+\sigma_2)|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|\\&\quad+2^{\frac{1}{2}}(c_2+\sigma_1)|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|\big)\\\leq&\varphi_1|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5+\varphi_3(c_2)|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}\\&\quad+\varphi_2|z_3|^5+\varphi_4.\end{aligned}\tag{17}$$

综合(13)和(17)可得

$$\begin{aligned}\dot{V}_1\leq&(-2^{\frac{1}{2}}c_1+\varphi_3(c_2))|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}\\&+\varphi_1|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5+\varphi_2|z_3|^5+\varphi_4,\end{aligned}\tag{18}$$

其中, $\varphi_1,\varphi_2,\varphi_4$为常数; φ_3是与c_2有关的常数.

步骤 2 其次, 对V_2进行讨论.

对V_2求导可得

$$\begin{aligned}\dot{V}_2=&(\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3))\dot{z}_2-4|z_3|^3(z_2-\mathrm{sig}^2(z_3))\dot{z}_3\\\leq&(\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3))\cdot\big(c_2(z_3-\mathrm{sig}^{\frac{1}{3}}(z_1))+\sigma_3\\&+\sigma_1|z_1|^{\frac{1}{3}}+\sigma_2|z_2|^{\frac{1}{2}}\big)\\&+4c_3|z_3|^3\cdot|1+\rho|\cdot|z_2-\mathrm{sig}^2(z_3)|\\\leq&c_2(\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3))(z_3-\mathrm{sig}^{\frac{1}{2}}(z_2))\\&+|\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3)|\cdot\big(c_2|\mathrm{sig}^{\frac{1}{2}}(z_2)-\mathrm{sig}^{\frac{1}{3}}(z_1)|\\&+\sigma_1|z_3-\mathrm{sig}^{\frac{1}{3}}(z_1)|+\sigma_2|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|+(\sigma_1\\&+\sigma_2)|z_3|+\sigma_3\big)+4c_3(1+\rho)|z_3|^3|z_2-\mathrm{sig}^2(z_3)|.\end{aligned}\tag{19}$$

对于$c_2(\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3))(z_3-\mathrm{sig}^{\frac{1}{2}}(z_2))$部分, 根据引理4 可得

$$\begin{aligned}&c_2(\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3))(z_3-\mathrm{sig}^{\frac{1}{2}}(z_2))\\=&-c_2|(z_3)^4-\mathrm{sig}^2(z_2)||z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|\\\leq&-c_2\cdot2^{1-4}|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5.\end{aligned}\tag{20}$$

根据引理4, 可知

$$\begin{aligned}&|\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3)|\\&\leq l_2(|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^4+|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)||z_3|^3,\end{aligned}\tag{21}$$

其中, $l_2=4\cdot(2^{4-2}+2)=24$. 进而结合引理2可得

$$\begin{aligned}&\sigma_3|\mathrm{sig}^2(z_2)-\mathrm{sig}^4(z_3)|\\\leq&l_2\sigma_3(\frac{4}{5}|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5+\frac{1}{5}|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5\\&+\frac{3}{5}|z_3|^5+\frac{2}{5}),\end{aligned}\tag{22}$$

同理,由引理4, 可知

$$|\mathrm{sig}^{\frac{1}{2}}(z_2)-\mathrm{sig}^{\frac{1}{3}}(z_1)|\leq2^{\frac{1}{2}}(|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{1}{2}},\tag{23}$$

以及

$$\begin{aligned}|z_2-\mathrm{sig}^2(z_3)|\leq&l_3(|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^2+\\&|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)||z_3|),\end{aligned}\tag{24}$$

其中, $l_3=2\cdot(2^{2-2}+1)=4$.

综合(16)、(22-24), 结合引理2和引理3, 可得

$$\begin{aligned}&|\mathrm{sig}^4(z_2)-\mathrm{sig}^4(z_3)|(c_2|\mathrm{sig}^{\frac{1}{2}}(z_2)-\mathrm{sig}^{\frac{1}{2}}(z_3)|\\&+\sigma_1|z_3-\mathrm{sig}^{\frac{1}{3}}(z_1)|+\sigma_2|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|+(\sigma_1\\&+\sigma_2)|z_3|+\sigma_3)+4(1+\rho)c_3|z_3|^3|z_2-\mathrm{sig}^2(z_3)|\\\leq&\varphi_5|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5+\varphi_6|z_3|^5+\varphi_7(c_2)|z_2\\&-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}+\varphi_8(c_3)|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5+\varphi_9,\end{aligned}\tag{25}$$

其中, φ_5,φ_6 和φ_9是符合要求的常数, $\varphi_7(c_2)$ 是与c_2相关的常数, $\varphi_8(c_3)$ 是与c_3 相关的常数.

联合(20)和(25)可得

$$\begin{aligned}\dot{V}_2\leq&(-\frac{c_2}{8}+\varphi_4+\varphi_7(c_3))|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5\\&+\varphi_5|z_3|^5+\varphi_6(c_2)|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}+\varphi_9.\end{aligned}\tag{26}$$

步骤 3 然后对V_3进行讨论.

对V_3求导可得

$$\begin{aligned}\dot{V}_3=&\mathrm{sig}^5(z_3)\dot{z}_3=\mathrm{sig}^5(z_3)c_3(d_2-\mathrm{sgn}(z_1))\\=&c_3\mathrm{sig}^5(z_3)(\mathrm{sgn}(z_3)-\mathrm{sgn}(z_3)-\mathrm{sgn}(z_1)+d_2)\\\leq&-c_3|z_3|^5(1-\rho)+c_3|z_3|^5|\mathrm{sgn}(z_3)-\mathrm{sgn}(z_1)|.\end{aligned}\tag{27}$$

设$h(z_1,z_3)=c_3|z_3|^5|\mathrm{sgn}(z_3)-\mathrm{sgn}(z_1)|$, 接下来对$h(z_1,z_3)$ 进行分类讨论:

(i): 当$z_1z_3>0$时, 可得$\mathrm{sgn}(z_3)-\mathrm{sgn}(z_1)=0$, 即$h(z_1,z_3)=0$.

(ii): 当$z_1z_3\leq0$时, 可得$|\mathrm{sgn}(z_3)-\mathrm{sgn}(z_1)|\leq2$, 且存在如下关系:

$$|z_3|\leq|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|+|\mathrm{sig}^{\frac{1}{2}}(z_2)-\mathrm{sig}^{\frac{1}{3}}(z_1)|.\tag{28}$$

根据引理3和4, 以及上述讨论内容, 对于$h(z_1,z_3)$均可得如下结论:

$$\begin{aligned}h\leq&2c_3(|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|+|\mathrm{sig}^{\frac{1}{2}}(z_2)-\mathrm{sig}^{\frac{1}{3}}(z_1)|)^5\\\leq&\varphi_{10}(c_3)|z_3-\mathrm{sig}^{\frac{1}{2}}(z_2)|^5+\varphi_{11}(c_3)|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}.\end{aligned}\tag{29}$$

综合(27)和(29), 可得

$$\begin{aligned}\dot{V}_3\leq&-c_3|z_3|^5(1-\rho)+\varphi_{10}(c_3)|z_3-\mathrm{sig}^{\frac{1}{2}}(z_1)|^5+\\&\varphi_{11}(c_3)|z_2-\mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}}.\end{aligned}\tag{30}$$

其中, $\varphi_{10}(c_3), \varphi_{11}(c_3)$ 是与c_3 相关的常数.

步骤 4　最后,综合讨论.

综合(18)、(25)和(30), 可得

$$
\begin{aligned}
\dot V =&\dot V_1 + \dot V_2 + \dot V_3\\
\le& -(c_3(1-\rho) - \varphi_2 - \varphi_5)|z_3|^5 - (2^{-\frac{1}{2}}c_1 - \varphi_3(c_2)\\
&- \varphi_6(c_2) - \varphi_{11}(c_3))|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}} - (\frac{c_2}{8} - \varphi_1\\
&- \varphi_4 - \varphi_7(c_3) - \varphi_{10}(c_3))|z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)|^5\\
&+ \varphi_4 + \varphi_9.
\end{aligned}
\tag{31}
$$

根据预先设定的参数$\rho(0<\rho<1)$, 设c_1, c_2, c_3按如下要求取值:

$$
\begin{cases}
c_3 > \dfrac{\varphi_2+\varphi_5}{1-\rho},\\
c_2 > 8(\varphi_1 + \varphi_4 + \varphi_7(c_3) + \varphi_{10}(c_3)),\\
c_1 > \max\{2^{\frac{1}{2}}(\varphi_3(c_2) + \varphi_6(c_2) + \varphi_{11}(c_3)), \dfrac{l}{\rho c_2 c_3}\}.
\end{cases}
\tag{32}
$$

当c_1,c_2,c_3取值按(32)时, 则存在正常数β_1, β_2, 使得(31) 变为

$$
\begin{aligned}
\dot V \le& -\beta_1(|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^{\frac{5}{2}} + |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)|^5 + |z_3|^5)\\
&+ \beta_2.
\end{aligned}
\tag{33}
$$

考虑式(11), 根据积分中值定理可知, 存在常数$\beta_3>0, \beta_4>0$ 使得以下成立:

$$V_1 = \beta_3|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^3, \tag{34}$$

$$V_2 = \beta_4|\mathrm{sig}^2(z_2) - \mathrm{sig}^4(z_3)||z_2 - \mathrm{sig}^2(z_3)|. \tag{35}$$

综合式(11),(34)和(35), 易知存在正常数β_5, 使得

$$V \le \beta_5(|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^3 + |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)|^6 + |z_3|^6). \tag{36}$$

根据引理3, 式(33)可进一步写为

$$
\begin{aligned}
\dot V \le& -\beta_1(|z_2 - \mathrm{sig}^{\frac{2}{3}}(z_1)|^3 + |z_3 - \mathrm{sig}^{\frac{1}{2}}(z_2)|^6 + |z_3|^6)^{\frac{5}{6}}\\
&+ \beta_2.
\end{aligned}
\tag{37}
$$

联立式(36)和(37) 可得

$$\dot V \le -\beta_1(\frac{V}{\beta_5})^{\frac{5}{6}} + \beta_2. \tag{38}$$

根据式(38)可知, 估计误差可以在有限时间内收敛到区域Ω_2中, 其中

$$\Omega_2 = \{z_i | V(z) \le \beta_5(\frac{\beta_2}{\beta_1})^{\frac{6}{5}}, i = 1,2,3\}. \tag{39}$$

当c_1, c_2, c_3取值较大时, 则$\beta_2 \ll \beta_1$, 此时z_i可收敛至较小区域.

定理1证明完毕.■

注 1　由观测器的结构可以看出, 本文提出的滑模观测器依赖机器人模型. 因此, 观测器估计效果与机器人模型的准确性相关. 此外,机器人是一个强耦合、非线性系统, 即使通过参数辨识的方法估计动力学参数 [28–29], 依旧很难得到精确到模型.

注 2　根据式(39), 可知观测器闭环系统状态可以在有限时间内收敛到预定域内. 当观测器增益选择较大时, 该预定域将缩小, 使得观测器闭环系统误差更小, 估计效果更佳. 此外, 当机器人关节角速度已知时, 观测器闭环系统可实现有限时间收敛到原点.

注 3　当滑模观测器(5) 中sig(·) 函数的指数均为1 时, 可得到机器人系统的线性观测器, 即

$$
\begin{cases}
\dot{\tilde\theta}_1 = \tilde\theta_2 + k_1(\theta_1 - \tilde\theta_1),\\
\dot{\tilde\theta}_2 = \tilde\theta_3 + g(\mathrm{sat}_N(\tilde\theta_1), \mathrm{sat}_N(\tilde\theta_2)) + k_2(\theta_1 - \tilde\theta_1),\\
\dot{\tilde\theta}_3 = k_3(\theta_1 - \tilde\theta_1),
\end{cases}
\tag{40}
$$

当k_1,k_2,k_3满足一定条件时, 线性观测器(40)可以观测出系统(3) 的状态,即当$t\to\infty$ 时, $\tilde\theta_1 \to \theta, \tilde\theta_2 \to \dot\theta, \tilde\theta_3 \to M^{-1}\tau$.

4　实验与分析

4.1　实验仪器

本文以六自由度工业机器人为研究对象, 设备采用埃夫特ER3 系列工业机器人, 其DH 参数如表1 所示. 实验所用机器人系统由机器人本体、驱动与伺服控制单元以及相应的上位机组成, 如图1 所示.

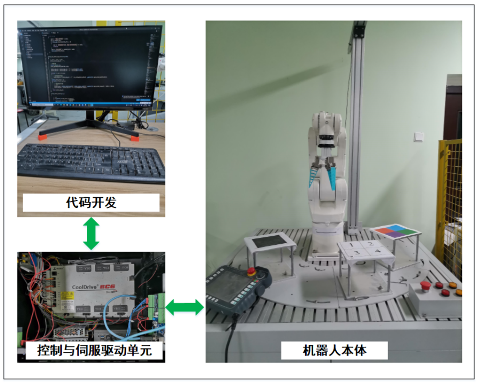

图 1　实验平台

Fig. 1　Experimental platform

表 1　机器人DH参数

Table 1　Robot DH parameters

连杆i	$a_{i-1}(m)$	$\alpha_{i-1}(rad)$	$d_i(m)$	$\theta_i(rad)$
1	0	π/2	0.3675	0
2	0.295	0	0	π/2
3	0.037	π/2	0	0
4	0	π/2	0.2955	0
5	0	-π/2	0	0
6	0	0	0.0785	0

4.2 三阶超螺旋滑模观测器力矩估计实验

进行力矩估计实验时, 采用两种估计方法进行机器人关节力矩估计, 分别是线性观测器(40)和滑模观测器(5). 线性观测器的控制参数设置为$k_1 = 400$, $k_2 = 120000$, $k_3 = 8000000$, 滑模观测器的控制参数设置为$k_1 = 50$, $k_2 = 700$, $k_3 = 1800$. 此外,观测器参数N 均设置为10. 机器人关节角度和力矩数据采集频率为1000Hz, 滑模力矩观测器基于simulink 实现.

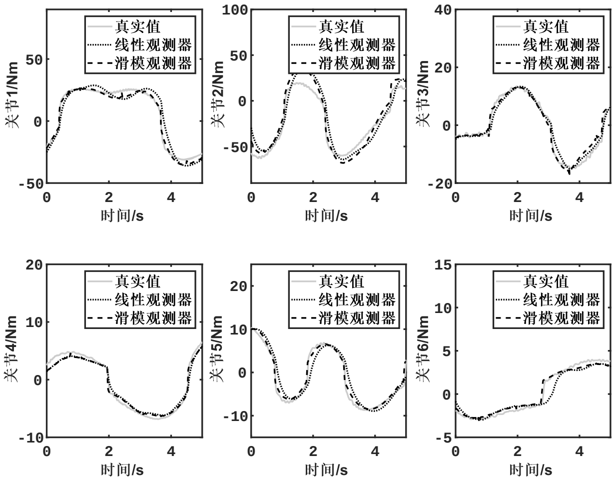

图 2 关节力矩估计结果

Fig. 2 Estimation results of joint torque

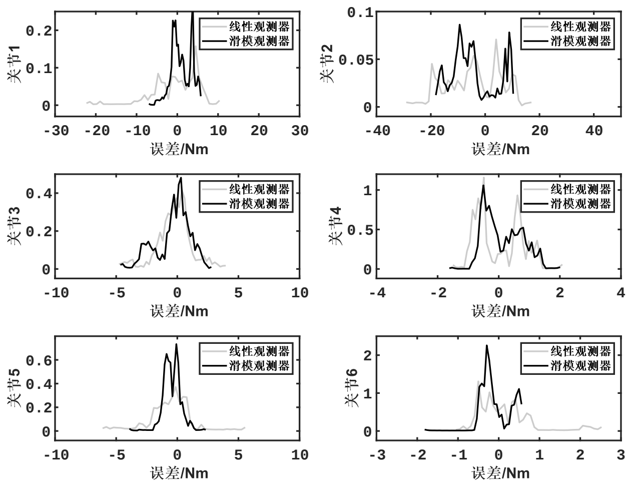

图 3 机器人力估计误差概率分布

Fig. 3 Probability distribution of estimation errors

机器人关节力矩估计结果如图2所示, 力矩估计误差的概率分布图如图3所示. 从整体上看, 两种方法都可以估计关节力矩, 但是通过估计误差的概率分布图可以看出, 线性观测器在误差分布上具有更广泛的区域, 也就是具有较大的极差. 相比于线性观测器,滑模观测器更集中在小误差区域, 整体上看, 滑模观测器的估计结果更令人满意.

表2 给出了两种力矩估计方法的估计误差数据分析, 可以明显看出, 与线性观测器相比, 滑模观测器具有更好的估计结果最佳. 综合以上, 基于三阶超螺旋滑模观测器的机器人力矩估计方法具有优异的力矩估计效果.

根据式(39)的结果可知, 观测器增益满足使得观测器系统有限时间收敛的前提下, 增大控制增益, 可降低估计误差. 此外, 本文设计的滑模观测器不仅可以估计关节力矩, 还可以估计关节角速度, 避免了对关节角度进行差分、滤波等数值处理. 以关节1和3为例, 角速度估计结果如图4 所示. 从图中可以看出, 采用观测器的方法估计出的角速度更为光滑, 这在实际应用中具有重要意义.

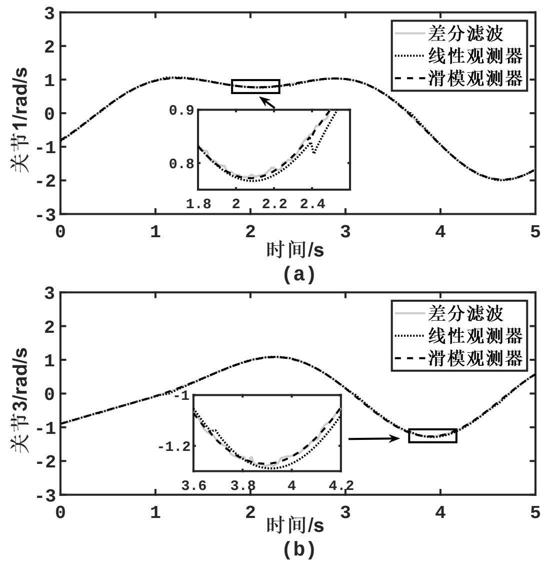

图 4 机器人角速度估计值

Fig. 4 Estimated results of robot angular velocity

5 结论

本文主要研究了六自由度工业机器人关节力矩估计问题, 设计一种基于三阶超螺旋滑模算法的机器人关节力矩观测器, 并构建新的Lyapunov 函数分析了观测器闭环系统的稳定性. 通过实验方法验证了可行性. 研究表明, 本文的方法可以在不需要安装力矩传感器的情况下估计机器人关节力矩值. 相比于线性观测器方法, 采用滑模观测器方法的力矩估计结果具有更好的估计效果. 此外, 采用观测器方法还能得到平滑的角速度估计值.

附录

引理6的证明: 首先将$g(\theta_1, \theta_2)$分为两个部分, 即

$$g(\theta_1, \theta_2) = g_1(\theta_1, \theta_2) + g_2(\theta_1, \theta_2), \qquad (A.1)$$

表 2 力矩估计误差分析(N · m)

Table 2 Analysis of torque estimation errors

类型	力矩估计方法	关节1	关节2	关节3	关节4	关节5	关节6
最大估计误差	线性观测器	22.2890	29.0495	4.7695	2.1740	6.1091	2.6147
	滑模观测器	6.9645	18.1476	4.6568	2.1022	3.9190	1.8180
	改进	68.75%	37.53%	2.36%	3.30%	35.85%	30.47%
估计误差均方根	线性观测器	5.9239	10.8568	1.4951	0.7629	1.9525	0.7924
	滑模观测器	2.7364	9.1417	1.4407	0.6511	0.8224	0.3857
	改进	57.70%	15.80%	3.64%	14.66%	6.66%	51.33%

其中

$$\begin{cases} g_1(\theta_1,\theta_2) = -M^{-1}(\theta_1)(C(\theta_1,\theta_2)+G(\theta_1)), \\ g_2(\theta_1,\theta_2) = -M^{-1}(\theta_1)f(\theta_2). \end{cases} \tag{A.2}$$

因此, 式(A.1)可重写为

$$\begin{aligned} & g(\theta_1,\theta_2) - g(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2)) \\ =& g_1(\theta_1,\theta_2) - g_1(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2)) + g_2(\theta_1,\theta_2) \\ & - g_2(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2)). \end{aligned} \tag{A.3}$$

由于$g_1(\theta_1,\theta_2)$ 满足Lipschitz条件, 故可得

$$\begin{aligned} & |g_1(\theta_1,\theta_2) - g_1(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2))| \\ \le & b_1|\theta_1 - \mathrm{sat}_N(\tilde{\theta}_1)| + b_2|\theta_2 - \mathrm{sat}_N(\tilde{\theta}_2)|, \end{aligned} \tag{A.4}$$

其中, $b_1 > 0$, $b_2 > 0$ 均为常数.

对于$g_2(\theta_1,\theta_2)$, 由于机器人关节摩擦力不连续特性, 故不满足Lipschitz条件. 因此, 首先做以下处理

$$\begin{aligned} & |g_2(\theta_1,\theta_2) - g_2(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2))| \\ \le & |g_2(\theta_1,\theta_2) - g_2(\theta_1,\mathrm{sat}_N(\tilde{\theta}_2))| + |g_2(\theta_1,\mathrm{sat}_N(\tilde{\theta}_2)) \\ & - g_2(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2))| \end{aligned} \tag{A.5}$$

基于式(2) 和引理1, 可得

$$\begin{aligned} & |g_2(\theta_1,\theta_2) - g_2(\theta_1,\mathrm{sat}_N(\tilde{\theta}_2))| \\ \le & |M^{-1}(\theta_1)(f_v\cdot\mathrm{sgn}(\theta_2) + f_c\cdot\theta_2 + f_b) - M^{-1}(\theta_1)\times \\ & (f_v\cdot\mathrm{sgn}(\mathrm{sat}_N(\tilde{\theta}_2)) + f_c\cdot\mathrm{sat}_N(\tilde{\theta}_2) + f_b)| \\ \le & \frac{1}{\lambda_m}\cdot|f_v|\cdot|\mathrm{sgn}(\theta_2) - \mathrm{sgn}(\mathrm{sat}_N(\tilde{\theta}_2))| \\ & + \frac{1}{\lambda_m}\cdot|f_c|\cdot|\theta_2 - \mathrm{sat}_N(\tilde{\theta}_2)| \\ \le & b_3 + b_4|\theta_2 - \mathrm{sat}_N(\tilde{\theta}_2)| \end{aligned} \tag{A.6}$$

其中, $b_3 > 0$和$b_4 > 0$均是常数. 同理可得,

$$\begin{aligned} & |g_2(\theta_1,\mathrm{sat}_N(\tilde{\theta}_2)) - g_2(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2))| \\ \le & |(f_c\cdot\mathrm{sgn}(\mathrm{sat}_N(\tilde{\theta}_2)) + f_v\cdot\mathrm{sat}_N(\tilde{\theta}_2) + f_b)(M^{-1}(\theta_1) \\ & - M^{-1}(\mathrm{sat}_N(\tilde{\theta}_2)))| \\ \le & b_5|\theta_1 - \mathrm{sat}_N(\tilde{\theta}_2)| \end{aligned} \tag{A.7}$$

其中, $b_5 > 0$为常数. 结合式(A.4),(A.6)和(A.7), 可得

$$\begin{aligned} & |g(\theta_1,\theta_2) - g(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2))| \\ \le & (b_1+b_5)|\theta_1 - \mathrm{sat}_N(\tilde{\theta}_1)| + (b_2+b_4)|\theta_2 \\ & - \mathrm{sat}_N(\tilde{\theta}_2)| + b_3. \end{aligned} \tag{A.8}$$

基于引理5, 分类讨论如下:

(1) 当$|\theta_i - \tilde{\theta}_i| \ge 1$, 则$|\theta_i - \mathrm{sat}_N(\tilde{\theta}_i)| \le 2N|\theta_i - \tilde{\theta}_i|^{r_1}$,

(2) 当$|\theta_i - \tilde{\theta}_i| < 1$, 则$|\theta_i - \mathrm{sat}_N(\tilde{\theta}_i)| \le |\theta_i - \tilde{\theta}_i|^{r_1}$,

其中, $0 < r_1 < 1$, $i = 1, 2$.

综上所述可得, $|\theta_i - \tilde{\theta}_i| \le \Psi\cdot|\theta_i - \tilde{\theta}_i|^{r_1}$, 其中 $\Psi = \max\{2N, 1\}$. 分 别 取$r_1 = 1/3$和$r_2 = 1/2$, 可得

$$\begin{cases} |\theta_1 - \mathrm{sat}_N(\tilde{\theta}_1)| \le \Psi\cdot|\theta_1 - \tilde{\theta}_1|^{\frac{1}{3}}, \\ |\theta_2 - \mathrm{sat}_N(\tilde{\theta}_2)| \le \Psi\cdot|\theta_2 - \tilde{\theta}_2|^{\frac{1}{2}}. \end{cases} \tag{A.9}$$

结合式(A.8) 和(A.9)可得

$$\begin{aligned} & |g(\theta_1,\theta_2) - g(\mathrm{sat}_N(\tilde{\theta}_1),\mathrm{sat}_N(\tilde{\theta}_2))| \\ \le & b_6|e_1|^{\frac{1}{3}} + b_7|e_2|^{\frac{1}{2}} + b_3, \end{aligned} \tag{A.10}$$

其中, $b_6 = (b_1+b_5)\cdot\Psi$, $b_7 = (b_2+b_4)\cdot\Psi$.

引理6证明完毕. ■

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