计及随机传感器时滞的不确定半Markov跳变系统鲁棒滑模控制

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摘要: 在实际系统中，系统参数与结构随机变化、未知外界干扰、传感器时滞等现象时有发生并严重影响了系统的稳定运行。为了解决这一问题，本文提出计及随机传感器时滞的一类不确定半Markov 跳变系统鲁棒滑模控制方法，其中系统的传感器时滞通过使用Bernoulli随机分布进行描述。考虑系统状态信息不可测量条件下，设计模态依赖Luenberger观测器去估计半Markov跳变系统的运行状态。然后，构造一个积分滑模面并借助随机Lyapunov理论，提出两种半Markov跳变系统的随机稳定性分析方法。进而，提出基于观测器的滑模控制方法使得系统状态能够在有限时间内到达滑模面上以及滑模动态在H_∞性能指标γ下是随机稳定的。最后，通过一种基于他励直流电动机模型的数值仿真例子验证所设计的滑模控制方法的有效性与正确性。

关键词: 半Markov跳变系统; 滑模控制; 鲁棒控制; 模态依赖Luenberger观测器; 随机传感器时滞

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Robust Sliding Mode Control for Uncertain Semi-Markov Jump Systems with Random Sensor Time Delay

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Abstract: In practical systems, random changes of system parameters and structures, unknown external disturbance, sensor time delay and other phenomena occur from time to time, which seriously affect the stable operation of the system. In order to solve this problem, this paper proposes a robust sliding mode control method for a class of uncertain semi-Markov jump systems with stochastic sensor time delay, in which the sensor time delay is described by Bernoulli stochastic distribution. Considering that the system state information cannot be measured, the mode-dependent Luenberger observer is designed to estimate the operating state of the semi-Markov jump system. Then, an integral sliding mode surface is constructed and two stochastic stability analysis methods for semi-Markov jump systems are proposed based on stochastic Lyapunov theory. Furthermore, the observer-based sliding mode control method is proposed to make the system states reach the sliding mode surface in finite time and the sliding mode dynamic is stochastically stable with H_∞ performance index γ. Finally, the effectiveness and correctness of the proposed sliding mode control method are verified by a numerical simulation example based on the separately excited DC motor model.

Key words: Semi-Markov jump systems; sliding mode control; robust control; mode-dependent Luenberger observer; random sensor time delay

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1 引言

近年来，作为混合系统的一个重要分支，半Markov跳变系统因其能够准确刻画不同工况下系统的状态以及各个工况之间的关联而得到了广泛的关注与研究，并且成功地应用到诸多领域中，如电力系统 [1]，认知无线电网络 [2]，医疗系统 [3]。在连续型半Markov跳变系统中，模态转移速率的驻留时间分布函数克服了传统Markov跳变系统转移速率驻留时间只服从指数分布的局限性 [4–5]，更加适用于实际系统。随后，文献 [6] 提出了一个浴盆曲线来描述随驻留时间变化的转移速率（连续两次跳跃之间的持续时间），并给出了确保半Markov跳变系统鲁棒稳定性的充分条件。进一步，针对系统转移概率部分未知、系统与控制器/滤波器模态异步、系统非线性特征等情况，半Markov跳变系统分析、控制和滤波设计等方面已取得了丰硕的研究成果 [7–12]。

另一方面，滑模控制方法，作为有效的鲁棒控制方法之一，能够对模型参数的不确定性与外界干扰起到很好地调节作用。综合半Markov跳变系统与滑模控制方法的优势，文献 [13]提出了非线性奇异摄动半Markov跳变系统的滑模控制方法。文献 [14] 利用Takagi-Sugeno模糊方法构造了非线性半Markov跳变系统模型，并基于平均驻留时间方法提出了非线性半Markov 跳变系统的滑模控制策略。文献 [15]借助补充变量技术和系统转化方法提出了半Markov跳变系统的状态估计与滑模控制框架。文献 [16]提出了一种连续型不确定半Markov 跳变系统的输出反馈滑模控制框架。上述研究成果均是在系统状态完全可测量的情况下进行的，而在实际系统中，由于系统的复杂性、测量技术的限制，实际系统的状态往往是不易于测量的。随后，基于观测器的半Markov跳变系统滑模控制方法相继提出 [17–19]。

此外，系统的输出信号通常基于传感器进行传输，考虑当前技术的限制，系统的测量输出可能会产生随机传感器时滞现象。为了解决这一问题，大量考虑传感器时滞现象的成果应运而生 [20–21]。同时，外部干扰与参数不确定性也是对控制器性能产生影响的重要因素。基于上述内容与现有滑模控制研究成果，考虑系统的参数不确定性、外部干扰、随机传感器时滞、系统状态不可测等情况，如何为连续型不确定半Markov跳变系统设计一个有效的滑模面与滑模控制律目前仍没有彻底解决，这推动了本文的研究。

本文研究了计及随机传感器时滞与状态不可测条件下连续型不确定半Markov跳变系统的鲁棒滑模控制问题。与现有研究成果相比，本文主要创新如下：

1. 针对连续型不确定半Markov跳变系统，提出基于模态依赖Luenberger 观测器的滑模控制方案避免随机传感器时滞、不确定参数以及状态不可测等情况对系统的影响。与文献 [20] 相比，本文考虑的系统模型更一般且更可用于实际系统。

2. 通过构造积分滑模面与Luenberger观测器，提出两种系统滑模动态随机稳定的充分条件，确保系统的状态能够在有限时间内被吸引到预先设定的滑模面上。

2 系统描述与基本知识

在实际系统运行过程中，经常伴随着多种工况随机切换、传感器时滞、外部干扰等现象。因此，本文考虑一类定义在概率空间$(\Theta，\Lambda，\mathrm{Pr}(\bullet))$上的不确定半Markov 跳变系统模型去模拟上述现象，具体形式表示如下：

$$\begin{cases} \dot{z}(t) = (\mathcal{A}_{\kappa(t)} + \Gamma\mathcal{A}_{\kappa(t)})z(t) + \mathcal{B}(u(t) + d(t)), \\ y_1(t) = \mathcal{E}_{\kappa(t)}z(t) + \hat{\mathcal{D}}_{\kappa(t)}d(t), \\ y_2(t) = \alpha(t)(\mathcal{C}_{\kappa(t)}z(t) - \mathcal{D}_{\kappa(t)}z(t-\eta(t))) \\ \qquad +\mathcal{D}_{\kappa(t)}z(t-\eta(t)), \end{cases} \tag{1}$$

式中：$z(t)$表示系统的状态向量且属于$\mathbb{R}^{n_z}$, $u(t)$ 表示系统的控制输入且属于$\mathbb{R}^{n_u}$，$d(t) \in \mathbb{R}^{n_u}$表示外部的干扰输入且属于$\ell_2[0,\infty)$，$y_1(t)$表示系统的控制输出且属于$\mathbb{R}^{n_{y_1}}$，$y_2(t)$表示系统的测量输出且属于$\mathbb{R}^{n_{y_2}}$。$\Theta$，$\Lambda$和$\mathrm{Pr}(\bullet)$分别表示样本空间，样本空间子集的$\sigma$- 代数和事件概率。$\kappa(t)(t \geq 0)$ 表示有限状态的半Markov过程且取值在集合$\mathbb{N} = \{1,2,3,\ldots,\mathcal{N}\}$内。对于$\kappa(t) = m$，$\mathcal{A}_m$，$\mathcal{B}$，$\mathcal{E}_m$，$\hat{\mathcal{D}}_m$，$\mathcal{C}_m$ 以及$\mathcal{D}_m$表示具有适当维数的系统参数矩阵且依赖于随机过程$\kappa(t)$。$\Gamma\mathcal{A}_{\kappa(t)}$是系统的不确定参数矩阵且满足$\Gamma\mathcal{A}_{\kappa(t)} \triangleq \mathcal{E}_{\mathcal{A}\kappa(t)}\Gamma(t)\mathcal{F}_{\mathcal{A}\kappa(t)}$。$\mathcal{E}_{\mathcal{A}\kappa(t)}$和$\mathcal{F}_{\mathcal{A}\kappa(t)}$表示具有适当维数的已知常数矩阵。未知时变矩阵$\Gamma(t)$满足$\Gamma^T(t)\Gamma(t) \leq \mathcal{I}$。基于上述描述，系统模态的转移速率能够表示为$\Pi = (\vartheta_{mn}(\tau))_{\mathcal{N}\times\mathcal{N}}(m,n = 1,2,3,\ldots,\mathcal{N})$。同时，模态转移速率与转移概率之间的关系能够表示为下列形式。

$$\begin{aligned} &\mathrm{Pr}\left\{\kappa_{t+\mu} = n \mid \kappa_t = m\right\} \\ &= \begin{cases} \vartheta_{mn}(\tau)\mu + o(\mu),\ m \neq n, \\ 1 + \vartheta_{mm}(\tau)\mu + o(\mu),\ m = n, \end{cases} \end{aligned}$$

式中：$\vartheta_{mn}(\tau)(m \neq n)$表示系统从模态$m$到模态$n$的转移速率且满足$\vartheta_{mm}(\tau) = \sum_{m=1,m\neq n}^{\mathcal{N}}\vartheta_{mn}(\tau)$，$\tau$表示系统两次相邻跳跃的驻留时间，$o(\mu)(\mu > 0)$表示无穷小转移区间且被定义为$\lim_{\mu\to 0}\frac{o(\mu)}{\mu} = 0$。针对系统模型的测量输出$y_2(t)$，本文引入伯努利随机变量$\alpha(t)$刻画系

统传感器时滞的发生，即当$\alpha(t)=0$时，系统存在传感器时滞。当$\alpha(t)=1$时，系统不存在传感器时滞。然后，能够推出相关事件的概率为$\Pr\{\alpha(t)=1\}=\mathrm{E}\{\alpha(t)\}=\hat{\alpha}$，$\Pr\{\alpha(t)=0\}=1-\mathrm{E}\{\alpha(t)\}=1-\hat{\alpha}$，$\hat{\alpha}\in[0,1]$。此外，$\eta(t)$表示时变时滞且满足条件$0<\eta(t)\leq\bar{\eta}$和$|\dot{\eta}(t)|\leq\eta_1\leq 1$，$\bar{\eta}$和$\eta_1$ 表示已知的标量。

3　基于模态依赖Luenberger观测器的滑模控制

考虑系统状态是不可测量的情况，本节需要设计状态观测器去克服这一困难。在此基础上，还需要设计一个滑模面使系统状态能够驱动到所设定的滑模面上以实现本文主要目标。因此，对于半Markov跳变系统模型(1)，设计Luenberger观测器如下：

$$\begin{cases}\dot{z}_l(t)=\mathcal{A}_m z_l(t)+\mathcal{B}u(t)+\mathcal{H}_m(y_2(t)-\hat{\alpha}y_{z_l}(t)),\\ y_{z_l}(t)=\mathcal{C}_m z_l(t),\end{cases}\tag{2}$$

式中：$z_l(t)\in\mathbb{R}^{n_z}$表示系统状态$z(t)$的估计，矩阵$\mathcal{H}_m\in\mathbb{R}^{n_z\times n_{y_2}}$表示观测器的待确定增益。

结合半Markov跳变系统模型(1)和Luenberger观测器(2)，得到误差系统如下：

$$\begin{aligned}\dot{r}(t)=&\mathcal{A}_m r(t)+\Gamma\mathcal{A}_m z(t)+\mathcal{B}d(t)\\&-\alpha(t)\mathcal{H}_m\mathcal{C}_m z(t)+\mathcal{H}_m\hat{\alpha}\mathcal{C}_m z_l(t)\\&-(1-\alpha(t))\mathcal{H}_m\mathcal{D}_m z(t-\eta(t)),\end{aligned}\tag{3}$$

式中：$r(t)=z(t)-z_l(t)$，$r(t-\eta(t))=z(t-\eta(t))-z_l(t-\eta(t))$。然后，构造积分滑模面如下：

$$\phi(t)=\mathcal{F}z_l(t)-\int_0^t\mathcal{F}(\mathcal{A}_m+\mathcal{B}\mathcal{K}_m)z_l(s)ds,\tag{4}$$

式中：$\mathcal{F}$表示具有适当维数的常数矩阵且属于$\mathbb{R}^{n_u\times n_z}$。$\mathcal{K}_m\in\mathbb{R}^{n_u\times n_z}$表示预先设定的参数矩阵且使得$\mathcal{A}_m+\mathcal{B}\mathcal{K}_m$是Hurwitz矩阵。$\mathcal{F}\mathcal{B}$ 是正定对称矩阵。

进一步，能够推导出积分滑模面的导数为：

$$\begin{aligned}\dot{\phi}(t)=&\mathcal{F}\dot{z}_l(t)-\mathcal{F}(\mathcal{A}_m+\mathcal{B}\mathcal{K}_m)z_l(t)\\=&\mathcal{F}\mathcal{A}_m z_l(t)+\mathcal{F}\mathcal{B}u(t)+\mathcal{F}\mathcal{H}_m[\alpha(t)\mathcal{C}_m r(t)\\&+(1-\alpha(t))\mathcal{D}_m z(t-\eta(t))\\&+(\alpha(t)-\hat{\alpha})\mathcal{C}_m z_l(t)]-\mathcal{F}(\mathcal{A}_m+\mathcal{B}\mathcal{K}_m)z_l(t).\end{aligned}$$

定义$\dot{\phi}(t)=0$，能够计算出等效控制律：

$$\begin{aligned}u_{eq}(t)=&-\alpha(t)(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\mathcal{C}_m r(t)\\&-(1-\alpha(t))(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\mathcal{D}_m z(t-\eta(t))\\&-(\alpha(t)-\hat{\alpha})(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\mathcal{C}_m z_l(t)+\mathcal{K}_m z_l(t).\end{aligned}\tag{5}$$

根据Luenberger观测器(2)和等效控制律(5)，能够得出滑动模态动力学方程：

$$\begin{aligned}\dot{z}_l(t)=&\mathcal{A}_m z_l(t)+\mathcal{B}u_{eq}(t)+\mathcal{H}_m(y_2(t)-\hat{\alpha}y_{z_l}(t))\\=&(\mathcal{A}_m+\mathcal{B}\mathcal{K}_m)z_l(t)\\&+\alpha(t)(\mathcal{I}-\mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{C}_m r(t)\\&+(1-\alpha(t))(\mathcal{I}-\mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{D}_m z(t-\eta(t))\\&+(\alpha(t)-\hat{\alpha})(\mathcal{I}-\mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{C}_m z_l(t).\end{aligned}\tag{6}$$

为了实现本文的主要目标，一些重要的引理与定义给出如下：

引理 1　[20] 如果存在向量p和q以及矩阵$\mathcal{Q}>0$,则能够找到下列不等式成立：

$$sym(p^Tq)\leq p^T\mathcal{Q}p+q^T\mathcal{Q}^{-1}q.$$

引理 2　[8] 如果存在具有适当维数的实矩阵$\mathcal{W}$，$\mathcal{G}$和$\mathcal{H}$，时变矩阵$\Gamma(t)$,当且仅当存在标量$\delta>0$，使得

$$\mathcal{M}+sym\{\mathcal{G}\Gamma(t)\mathcal{H}\}<0,$$

则有

$$\mathcal{M}+\delta\mathcal{G}^T\mathcal{G}+\delta^{-1}\mathcal{H}^T\mathcal{H}<0,$$

成立，式中：$\Gamma(t)$满足$\Gamma^T(t)\Gamma(t)\leq\mathcal{I}$，$\mathcal{M}=\mathcal{M}^T$。

引理 3　[7] 如果同时满足下列条件，可称误差系统（3）和滑动模态系统（6）组成的扩维系统是随机稳定的且满足H_∞性能指标γ。

（1）针对系统状态与模态的任意初始条件且干扰信号$d(t)$为零时，如果

$$||\zeta(t)||_2^2=\int_0^\infty \boldsymbol{E}\{||\zeta(t)||^2|(\zeta(0),\kappa_0)\}\mathrm{d}t<\infty,$$

则误差系统（3）和滑动模态系统（6）组成的扩维系统是随机稳定的。

（2）对于标量$\gamma>0$，所有非零的干扰信号$d(t)\in\ell_2[0,\infty)$，以及零初始条件下，如下条件成立：

$$\int_0^\infty y_1^T(t)y_1(t)\mathrm{d}t\leq\gamma^2\int_0^\infty d^T(t)d(t)\mathrm{d}t,$$

则系统满足H_∞性能指标γ。

4　主要结果

4.1　稳定性分析

本节给出误差系统(3)和滑动模态系统(6)所组成扩维系统的随机稳定性的充分条件。

定理 1　针对已知的时变时滞上界$\bar{\eta}$，标量η_1和$\hat{\alpha}$，如果存在正定矩阵$\mathcal{P}_m$，$\mathcal{M}_{1m}$，$\mathcal{M}_{2m}$，$\mathcal{R}_1$，$\mathcal{R}_2$，常数$\gamma>0$和$\delta>0$，以及矩阵$\mathcal{Y}_m$，使得下列线性矩阵

不等式成立：

$$\begin{bmatrix} \hat{\Psi}_m^{11} & \hat{\Psi}_m^{12} & 0 & 0 & \hat{\Psi}_m^{15} & \hat{\Psi}_m^{16} & \hat{\Psi}_m^{17} \\ \star & \hat{\Psi}_m^{22} & \hat{\Psi}_m^{23} & \hat{\Psi}_m^{24} & \hat{\Psi}_m^{25} & \hat{\Psi}_m^{26} & \hat{\Psi}_m^{27} \\ \star & \star & \hat{\Psi}_m^{33} & 0 & 0 & \hat{\Psi}_m^{36} & 0 \\ \star & \star & \star & \hat{\Psi}_m^{44} & 0 & \hat{\Psi}_m^{46} & 0 \\ \star & \star & \star & \star & \hat{\Psi}_m^{55} & 0 & 0 \\ \star & \star & \star & \star & \star & \hat{\Psi}_m^{66} & 0 \\ \star & \star & \star & \star & \star & \star & \hat{\Psi}_m^{77} \end{bmatrix} < 0, \tag{7}$$

$$\sum_{n=1}^{\mathcal{N}} \hat{\vartheta}_{mn}\mathcal{M}_{1n} - \mathcal{R}_1 \leq 0, \tag{8}$$

$$\sum_{n=1}^{\mathcal{N}} \hat{\vartheta}_{mn}\mathcal{M}_{2n} - \mathcal{R}_2 \leq 0, \tag{9}$$

式中：

$$\hat{\Psi}_m^{11} = sym\{\mathcal{P}_m(\mathcal{A}_m + \mathcal{B}\mathcal{K}_m)\} + \sum_{n=1}^{\mathcal{N}} \hat{\vartheta}_{mn}\mathcal{P}_n + \mathcal{M}_{1m},$$
$$+\bar{\eta}\mathcal{R}_1 + \mathcal{E}_m^T\mathcal{E}_m,\ \hat{\Psi}_m^{12} = \mathcal{E}_m^T\mathcal{E}_m,\ \hat{\alpha}_1 = \frac{1}{1-\hat{\alpha}},$$
$$\hat{\Psi}_m^{22} = \sum_{n=1}^{\mathcal{N}} \hat{\vartheta}_{mn}\mathcal{P}_n + \mathcal{M}_{2m} + sym\{\mathcal{P}_m\mathcal{A}_m - \hat{\alpha}\mathcal{Y}_m\mathcal{C}_m\}$$
$$+\bar{\eta}\mathcal{R}_2 + \mathcal{E}_m^T\mathcal{E}_m,\ \hat{\Psi}_m^{23} = \hat{\Psi}_m^{24} = (\hat{\alpha}-1)\mathcal{Y}_m\mathcal{D}_m,$$
$$\Psi_m^{33} = -(1-\eta_1)\mathcal{M}_{1m},\ \Psi_m^{44} = -(1-\eta_1)\mathcal{M}_{2m},$$
$$\Psi_m^{15} = \mathcal{E}_m^T\hat{\mathcal{D}}_m,\ \Psi_m^{25} = \mathcal{E}_m^T\hat{\mathcal{D}}_m + \mathcal{P}_m\mathcal{B},$$
$$\Psi_m^{16} = \left[\mathcal{P}_m\Delta\mathcal{F}\,0\,0\,0\right],\ \Psi_m^{26} = \left[0\,\mathcal{C}_m^T\mathcal{Y}_m^T\,0\,0\right],$$
$$\Psi_m^{36} = \left[0\,0\,\mathcal{D}_m^T\mathcal{Y}_m^T\,0\right],\ \Psi_m^{46} = \left[0\,0\,0\,\mathcal{D}_m^T\mathcal{Y}_m^T\right],$$
$$\Psi_m^{55} = \hat{\mathcal{D}}_m^T\hat{\mathcal{D}}_m - \gamma^2\mathcal{I},\ \Psi_m^{77} = \mathrm{diag}\{-\delta\mathcal{I}, -\delta\mathcal{I}\},$$
$$\Psi_m^{66} = \mathrm{diag}\{-\frac{1}{2-\hat{\alpha}}\mathcal{P}_m, -\frac{1}{\hat{\alpha}}\mathcal{P}_m, -\hat{\alpha}_1\mathcal{P}_m, -\hat{\alpha}_1\mathcal{P}_m\},$$
$$\Psi_m^{17} = \left[0\,\delta\mathcal{F}_{\mathcal{A}m}^T\right],\ \Psi_m^{27} = \left[\mathcal{P}_m\mathcal{E}_{\mathcal{A}m}\,\delta\mathcal{F}_{\mathcal{A}m}^T\right],$$

则误差系统(3)和滑动模态系统(6)所组成的扩维系统是随机稳定的且满足H_∞性能指标γ。然后，状态观测器增益能够确定为：

$$\mathcal{H}_m = \mathcal{P}_m^{-1}\mathcal{Y}_m. \tag{10}$$

证明：选取依赖于半Markov随机过程的Lyapunov-Krasovskii泛函：

$$V(z_l(t), r(t), \kappa(t), t) = \sum_{i=1}^{5} V_i(z_l(t), r(t), \kappa(t), t),$$

式中：

$$V_i(t) = V_i(z_l(t), r(t), \kappa(t), t),$$
$$V_1(t) = z_l^T(t)\mathcal{P}_{\kappa(t)}z_l(t) + r^T(t)\mathcal{P}_{\kappa(t)}r(t),$$
$$V_2(t) = \int_{t-\eta(t)}^{t} z_l^T(t)\mathcal{M}_{1\kappa(t)}z_l(t)\mathrm{d}t,$$
$$V_3(t) = \int_{t-\eta(t)}^{t} r^T(t)\mathcal{M}_{2\kappa(t)}r(t)\mathrm{d}t,$$
$$V_4(t) = \int_{t-\eta(t)}^{t}\int_{\theta}^{t} z_l^T(s)\mathcal{R}_1 z_l(s)\mathrm{d}s\mathrm{d}_\theta,$$
$$V_5(t) = \int_{t-\eta(t)}^{t}\int_{\theta}^{t} r^T(s)\mathcal{R}_2 r(s)\mathrm{d}s\mathrm{d}_\theta.$$

根据文献[7,22]，能够得出$V_i(z_l(t), r(t), \kappa(t), t)$的无穷小算子为：

$$\mathrm{E}\{\mathcal{L}V_1(z_l(t), r(t), \kappa(t), t)\} = sym\{z_l^T(t)\mathcal{P}_m\dot{z}_l(t)\}$$
$$+sym\{r^T(t)\mathcal{P}_m\dot{r}(t)\} + z_l^T(t)\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{P}_n z_l(t)$$
$$+r^T(t)\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{P}_n r(t),$$

$$\mathrm{E}\{V_2(z_l(t), r(t), \kappa(t), t)\} = z_l^T(t)\mathcal{M}_{1m}z_l(t)$$
$$+\int_{t-\eta(t)}^{t} z_l^T(t)\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{M}_{1n}z_l(t)\mathrm{d}t$$
$$-(1-\dot{\eta}(t))z_l^T(t-\eta(t))\mathcal{M}_{1m}z_l(t-\eta(t)),$$

$$\mathrm{E}\{\mathcal{L}V_3(z_l(t), r(t), \kappa(t), t)\} = r^T(t)\mathcal{M}_{2m}r(t)$$
$$+\int_{t-\eta(t)}^{t} r^T(t)\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{M}_{2n}r(t)\mathrm{d}t$$
$$-(1-\dot{\eta}(t))r^T(t-\eta(t))\mathcal{M}_{2m}r(t-\eta(t)),$$

$$\mathrm{E}\{\mathcal{L}V_4(z_l(t), r(t), \kappa(t), t)\} = \eta(t)z_l^T(t)\mathcal{R}_1 z_l(t)$$
$$-\int_{t-\eta(t)}^{t} z_l^T(\theta)\mathcal{R}_1 z_l(\theta)\mathrm{d}_\theta,$$

$$\mathrm{E}\{\mathcal{L}V_5(z_l(t), r(t), \kappa(t), t)\} = \eta(t)r^T(t)\mathcal{R}_2 r(t)$$
$$-\int_{t-\eta(t)}^{t} r^T(\theta)\mathcal{R}_2 r(\theta)\mathrm{d}_\theta,$$

式中：$\hat{\vartheta}_{mn}$表示系统模态转移速率$\vartheta_{mn}(\tau)$的期望且满足$\hat{\vartheta}_{mn} = \mathrm{E}\{\vartheta_{mn}(\tau)\} = \int_0^\infty \vartheta_{mn}(\tau)f_m(\tau)\mathrm{d}_\tau$，$f_m(\tau)$表示系统在模态$m$上的概率密度函数。

然后，对误差系统(3)和滑动模态系统(6)进行等价变换，得到：

$$\begin{aligned}\dot{r}(t) &= \mathcal{A}_m r(t) + \Gamma\mathcal{A}_m z(t) + \mathcal{B}d(t) \\ &-\hat{\alpha}\mathcal{H}_m\mathcal{C}_m r(t) - (\alpha(t)-\hat{\alpha})\mathcal{H}_m\mathcal{C}_m r(t) \\ &-(1-\hat{\alpha})\mathcal{H}_m\mathcal{D}_m z(t-\eta(t)) \\ &-(\hat{\alpha}-\alpha(t))\mathcal{H}_m\mathcal{D}_m z(t-\eta(t)) \\ &+(\hat{\alpha}-\alpha(t))\mathcal{H}_m\mathcal{C}_m z_l(t),\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{z}_l(t) &= (\mathcal{A}_m + \mathcal{B}\mathcal{K}_m)z_l(t) \\ &+\hat{\alpha}(\mathcal{I} - \mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{C}_m r(t) \\ &+(\alpha(t)-\hat{\alpha})(\mathcal{I} - \mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{C}_m r(t) \\ &+(1-\hat{\alpha})(\mathcal{I} - \mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{D}_m z(t-\eta(t)) \\ &-(\alpha(t)-\hat{\alpha})(\mathcal{I} - \mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{D}_m z(t-\eta(t)) \\ &+(\alpha(t)-\hat{\alpha})(\mathcal{I} - \mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F})\mathcal{H}_m\mathcal{C}_m z_l(t),\end{aligned} \tag{12}$$

式中：$\mathrm{E}\{\alpha(t)-\hat{\alpha}\} = 0$和$\mathrm{E}\{(\alpha(t)-\hat{\alpha})^2\} = \hat{\alpha}(1-\hat{\alpha})$。根据引理1，对

于$\Delta\mathcal{F}=\mathcal{I}-\mathcal{B}(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}$，能够得出下列不等式：

$$\begin{aligned}&sym(z_l^T(t)\mathcal{P}_m\Delta\mathcal{F}\mathcal{H}_m\mathcal{C}_m r(t))\\&\le z_l^T(t)\mathcal{P}_m\Delta\mathcal{F}\mathcal{P}_m^{-1}\Delta\mathcal{F}^T\mathcal{P}_m z_l(t)\\&+r^T(t)\mathcal{C}_m^T\mathcal{H}_m^T\mathcal{P}_m\mathcal{H}_m\mathcal{C}_m r(t),\end{aligned}\tag{13}$$

$$\begin{aligned}&sym(z_l^T(t)\mathcal{P}_m\Delta\mathcal{F}\mathcal{H}_m\mathcal{D}_m r(t-\eta(t)))\\&\le z_l^T(t)\mathcal{P}_m\Delta\mathcal{F}\mathcal{P}_m^{-1}\Delta\mathcal{F}^T\mathcal{P}_m z_l(t)\\&+r^T(t-\eta(t))\mathcal{D}_m^T\mathcal{H}_m^T\mathcal{P}_m\mathcal{H}_m\mathcal{D}_m r(t-\eta(t)).\end{aligned}\tag{14}$$

定义

$$\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{M}_{1n}-\mathcal{R}_1\le 0,\tag{15}$$

$$\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{M}_{2n}-\mathcal{R}_2\le 0.\tag{16}$$

当$d(t)=0$时，结合无穷小算子$\mathcal{L}V_i(z_l(t),r(t),\kappa(t),t)$与条件(11) − (16)，能够得到下列不等式：

$$\mathrm{E}\{\mathcal{L}V(z_l(t),r(t),\kappa(t),t)\}\le\mathrm{E}\{\zeta^T(t)\Psi_m\zeta(t)\},$$

式中：

$$\zeta^T(t)=\left[z_l^T(t)\ r^T(t)\ z_l^T(t-\eta(t))\ r^T(t-\eta(t))\right],$$

$$\Psi_m=\begin{bmatrix}\Psi_m^{11}&\Psi_m^{12}&0&0\\\star&\Psi_m^{22}&\Psi_m^{23}&\Psi_m^{24}\\\star&\star&\Psi_m^{33}&0\\\star&\star&\star&\Psi_m^{44}\end{bmatrix},$$

$$\begin{aligned}\Psi_m^{11}=&sym\{\mathcal{P}_m(\mathcal{A}_m+\mathcal{B}\mathcal{K}_m)\}+(2-\hat{\alpha})\mathcal{P}_m\Delta\mathcal{F}\\&\times\mathcal{P}_m^{-1}\Delta\mathcal{F}^T\mathcal{P}_m+\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{P}_n+\mathcal{M}_{1m}\\&+\bar{\eta}\mathcal{R}_1,\Psi_m^{12}=\Gamma\mathcal{A}_m^T\mathcal{P}_m,\end{aligned}$$

$$\begin{aligned}\Psi_m^{22}=&\hat{\alpha}\mathcal{C}_m^T\mathcal{H}_m^T\mathcal{P}_m\mathcal{H}_m\mathcal{C}_m+\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{P}_n+\mathcal{M}_{2m}\\&+sym\{\mathcal{P}_m\mathcal{A}_m-\hat{a}\mathcal{P}_m\mathcal{H}_m\mathcal{C}_m+\Gamma\mathcal{A}_m^T\mathcal{P}_m\}\\&+\bar{\eta}\mathcal{R}_2,\end{aligned}$$

$$\Psi_m^{23}=\Psi_m^{24}=(\hat{\alpha}-1)\mathcal{P}_m\mathcal{H}_m\mathcal{D}_m,$$

$$\Psi_m^{33}=(1-\hat{\alpha})\mathcal{D}_m^T\mathcal{H}_m^T\mathcal{P}_m\mathcal{H}_m\mathcal{D}_m-(1-\eta_1)\mathcal{M}_{1m},$$

$$\Psi_m^{44}=(1-\hat{\alpha})\mathcal{D}_m^T\mathcal{H}_m^T\mathcal{P}_m\mathcal{H}_m\mathcal{D}_m-(1-\eta_1)\mathcal{M}_{2m}.$$

结合条件(7)-(10)，引理2以及Schur补引理，可得: $\mathrm{E}\{\zeta^T(t)\Psi_m\zeta(t)\}<0$. 然后，对于常数$a>0$，能够满足条件：$\mathrm{E}\{\zeta^T(t)\Psi_m\zeta(t)\}<-a\parallel\zeta(t)\parallel^2$. 使用Dynkin公式，对于常数$\mathrm{T}_\star>0$，能够得到：$\mathrm{E}\{\zeta^T(\mathrm{T}_\star)\Psi_m\zeta(\mathrm{T}_\star)\}-\mathrm{E}\{\zeta^T(0)\Psi_m\zeta(0)\}<\int_0^{\mathrm{T}_\star}-a\parallel\zeta(t)\parallel^2\mathrm{d}t$。也就是说，$\int_0^{\mathrm{T}_\star}\parallel\zeta(t)\parallel^2\mathrm{d}t<\mathrm{E}\{\frac{1}{a}\zeta^T(0)\Psi_m\zeta(0)\}<\infty$.

然后，我们可以说由误差系统(3)和滑动模态系统(6)组成的扩维系统在$d(t)=0$时是随机稳定的。

当$d(t)\neq 0$时，随机Lyapunov泛函的无穷小算子能被重写为：$\mathrm{E}\{\mathcal{L}V(z_l(t),r(t),\kappa(t),t)\}\le\mathrm{E}\{\zeta^T(t)\Psi_m\zeta(t)\}+sym\{r^T(t)\mathcal{P}_m\mathcal{B}d(t)\}$.

根据引理3，系统在零初始条件下，其H_∞性能指标能够转换为：

$$\begin{aligned}&\mathrm{E}\left\{\int_0^\infty y_1^T(t)y_1(t)-\gamma^2d^T(t)d(t)\mathrm{d}t\right\}\\\le&\mathrm{E}\left\{\int_0^\infty y_1^T(t)y_1(t)-\gamma^2d^T(t)d(t)\right.\\&\left.+\zeta^T(t)\Psi_m\zeta(t)\mathrm{d}t\right\}\\=&\mathrm{E}\left\{\int_0^\infty\zeta_1^T(t)\Psi_{1m}\zeta_1(t)\mathrm{d}t\right\},\end{aligned}$$

式中：

$$\zeta_1^T(t)=\left[\zeta^T(t)\ d^T(t)\right],\ \Psi_{1m}=\begin{bmatrix}\Psi_{1m}^{11}&\Psi_{1m}^{12}\\\star&\Psi_{1m}^{22}\end{bmatrix},$$

$$\Psi_{1m}^{11}=\begin{bmatrix}\Psi_m^{111}&\Psi_m^{112}&0&0\\\star&\Psi_m^{122}&\Psi_m^{23}&\Psi_m^{24}\\\star&\star&\Psi_m^{33}&0\\\star&\star&\star&\Psi_m^{44}\end{bmatrix},$$

$$\begin{aligned}\Psi_m^{111}=&sym\{\mathcal{P}_m(\mathcal{A}_m+\mathcal{B}\mathcal{K}_m)\}+(2-\hat{\alpha})\mathcal{P}_m\Delta\mathcal{F}\\&\times\mathcal{P}_m^{-1}\Delta\mathcal{F}^T\mathcal{P}_m+\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{P}_n+\mathcal{M}_{1m}\\&+\bar{\eta}\mathcal{R}_1+\mathcal{E}_m^T\mathcal{E}_m,\end{aligned}$$

$$\Psi_m^{112}=\Gamma\mathcal{A}_m^T\mathcal{P}_m+\mathcal{E}_m^T\mathcal{E}_m,$$

$$\begin{aligned}\Psi_m^{122}=&\hat{\alpha}\mathcal{C}_m^T\mathcal{H}_m^T\mathcal{P}_m\mathcal{H}_m\mathcal{C}_m+\sum_{n=1}^{\mathcal{N}}\hat{\vartheta}_{mn}\mathcal{P}_n+\mathcal{M}_{2m}\\&+sym\{\mathcal{P}_m\mathcal{A}_m-\hat{a}\mathcal{P}_m\mathcal{H}_m\mathcal{C}_m+\Gamma\mathcal{A}_m^T\mathcal{P}_m\}\\&+\bar{\eta}\mathcal{R}_2+\mathcal{E}_m^T\mathcal{E}_m,\end{aligned}$$

$$\Psi_{1m}^{12}=\left[(\mathcal{E}_m^T\hat{\mathcal{D}}_m)^T\ (\mathcal{P}_m\mathcal{B})^T+(\mathcal{E}_m^T\hat{\mathcal{D}}_m)^T\ 0\ 0\right]^T,$$

$$\Psi_{1m}^{22}=\hat{\mathcal{D}}_m^T\hat{\mathcal{D}}_m-\gamma^2\mathcal{I}.$$

根据条件(7)-(10)，引理(2)-(3)以及Schur补，可得$\zeta_1^T(t)\Psi_{1m}\zeta_1(t)<0$并可称由误差系统(3)和滑动模态系统(6) 组成的扩维系统在$d(t)\neq 0$时是随机稳定的且能够维持H_∞性能指标γ。证毕。

在实际系统中，基于上述定理进行系统稳定性分析时，需要先模拟出系统在不同模态下驻留时间所遵循的概率分布，计算出期望值以处理时变模态转移速率$\vartheta_{mn}(\tau)$。显然，这种方法需要花费大量的测量成本。为了降低测量成本，我们可以根据实际系统所处的环境信息，预先设置转移速率的上下界，使得系统转移速率在该设置范围内能够保持稳定。于是，在了解系统所处环境信息以及确定系统模态与转移率上下边界后，我们能够给出如下定理。

定理 2　针对已知的时变时滞上界$\bar{\eta}$，标

量η_1和$\hat{\alpha}$，转移速率的上下边界$\bar{\vartheta}_{mn}$和$\underline{\vartheta}_{mn}$，如果存在正定矩阵$\mathcal{P}_m$，$\mathcal{M}_{1m}$，$\mathcal{M}_{2m}$，$\mathcal{R}_1$，$\mathcal{R}_2$，常数$\gamma > 0$和$\delta > 0$，以及矩阵$\mathcal{Y}_m$，使得下列线性矩阵不等式成立：

$$\underline{\Theta}_m < 0, \tag{17}$$

$$\bar{\Theta}_m < 0, \tag{18}$$

$$\sum_{n=1}^{\mathcal{N}} \underline{\vartheta}_{mn}\mathcal{M}_{1n} - \mathcal{R}_1 \le 0, \tag{19}$$

$$\sum_{n=1}^{\mathcal{N}} \bar{\vartheta}_{mn}\mathcal{M}_{1n} - \mathcal{R}_1 \le 0, \tag{20}$$

$$\sum_{n=1}^{\mathcal{N}} \underline{\vartheta}_{mn}\mathcal{M}_{2n} - \mathcal{R}_2 \le 0, \tag{21}$$

$$\sum_{n=1}^{\mathcal{N}} \bar{\vartheta}_{mn}\mathcal{M}_{2n} - \mathcal{R}_2 \le 0, \tag{22}$$

式中：

$$\underline{\Theta}_m = \begin{bmatrix} \underline{\Theta}_m^{11} & \hat{\Psi}_m^{12} & 0 & 0 & \hat{\Psi}_m^{15} & \hat{\Psi}_m^{16} & \hat{\Psi}_m^{17} \\ \star & \underline{\Theta}_m^{22} & \hat{\Psi}_m^{23} & \hat{\Psi}_m^{24} & \hat{\Psi}_m^{25} & \hat{\Psi}_m^{26} & \hat{\Psi}_m^{27} \\ \star & \star & \hat{\Psi}_m^{33} & 0 & 0 & \hat{\Psi}_m^{36} & 0 \\ \star & \star & \star & \hat{\Psi}_m^{44} & 0 & \hat{\Psi}_m^{46} & 0 \\ \star & \star & \star & \star & \hat{\Psi}_m^{55} & 0 & 0 \\ \star & \star & \star & \star & \star & \hat{\Psi}_m^{66} & 0 \\ \star & \star & \star & \star & \star & \star & \hat{\Psi}_m^{77} \end{bmatrix},$$

$$\bar{\Theta}_m = \begin{bmatrix} \bar{\Theta}_m^{11} & \hat{\Psi}_m^{12} & 0 & 0 & \hat{\Psi}_m^{15} & \hat{\Psi}_m^{16} & \hat{\Psi}_m^{17} \\ \star & \bar{\Theta}_m^{22} & \hat{\Psi}_m^{23} & \hat{\Psi}_m^{24} & \hat{\Psi}_m^{25} & \hat{\Psi}_m^{26} & \hat{\Psi}_m^{27} \\ \star & \star & \hat{\Psi}_m^{33} & 0 & 0 & \hat{\Psi}_m^{36} & 0 \\ \star & \star & \star & \hat{\Psi}_m^{44} & 0 & \hat{\Psi}_m^{46} & 0 \\ \star & \star & \star & \star & \hat{\Psi}_m^{55} & 0 & 0 \\ \star & \star & \star & \star & \star & \hat{\Psi}_m^{66} & 0 \\ \star & \star & \star & \star & \star & \star & \hat{\Psi}_m^{77} \end{bmatrix},$$

$$\underline{\Theta}_m^{11} = sym\{\mathcal{P}_m(\mathcal{A}_m + \mathcal{B}\mathcal{K}_m)\} + \sum_{n=1}^{\mathcal{N}} \underline{\vartheta}_{mn}\mathcal{P}_n + \mathcal{M}_{1m} + \bar{\eta}\mathcal{R}_1 + \mathcal{E}_m^T\mathcal{E}_m,$$

$$\underline{\Theta}_m^{22} = \sum_{n=1}^{\mathcal{N}} \underline{\vartheta}_{mn}\mathcal{P}_n + \mathcal{M}_{2m} + sym\{\mathcal{P}_m\mathcal{A}_m - \hat{\alpha}\mathcal{Y}_m\mathcal{C}_m\} + \bar{\eta}\mathcal{R}_2 + \mathcal{E}_m^T\mathcal{E}_m,$$

$$\bar{\Theta}_m^{11} = sym\{\mathcal{P}_m(\mathcal{A}_m + \mathcal{B}\mathcal{K}_m)\} + \sum_{n=1}^{\mathcal{N}} \bar{\vartheta}_{mn}\mathcal{P}_n + \mathcal{M}_{1m} + \bar{\eta}\mathcal{R}_1 + \mathcal{E}_m^T\mathcal{E}_m,$$

$$\bar{\Theta}_m^{22} = \sum_{n=1}^{\mathcal{N}} \bar{\vartheta}_{mn}\mathcal{P}_n + \mathcal{M}_{2m} + sym\{\mathcal{P}_m\mathcal{A}_m - \hat{\alpha}\mathcal{Y}_m\mathcal{C}_m\} + \bar{\eta}\mathcal{R}_2 + \mathcal{E}_m^T\mathcal{E}_m,$$

则误差系统(3)和滑动模态系统(6)所组成的扩维系统是随机稳定的且满足H_∞性能指标γ。然后，状态观测器增益能够确定为：$\mathcal{H}_m = \mathcal{P}_m^{-1}\mathcal{Y}_m$.

证明：根据定理1，可得：

$$\Theta_m < 0, \tag{23}$$

$$\sum_{n=1}^{\mathcal{N}} \vartheta_{mn}(\tau)\mathcal{M}_{1n} - \mathcal{R}_1 \le 0, \tag{24}$$

$$\sum_{n=1}^{\mathcal{N}} \vartheta_{mn}(\tau)\mathcal{M}_{2n} - \mathcal{R}_2 \le 0, \tag{25}$$

式中：

$$\Theta_m = \begin{bmatrix} \Theta_m^{11} & \hat{\Psi}_m^{12} & 0 & 0 & \hat{\Psi}_m^{15} & \hat{\Psi}_m^{16} & \hat{\Psi}_m^{17} \\ \star & \Theta_m^{22} & \hat{\Psi}_m^{23} & \hat{\Psi}_m^{24} & \hat{\Psi}_m^{25} & \hat{\Psi}_m^{26} & \hat{\Psi}_m^{27} \\ \star & \star & \hat{\Psi}_m^{33} & 0 & 0 & \hat{\Psi}_m^{36} & 0 \\ \star & \star & \star & \hat{\Psi}_m^{44} & 0 & \hat{\Psi}_m^{46} & 0 \\ \star & \star & \star & \star & \hat{\Psi}_m^{55} & 0 & 0 \\ \star & \star & \star & \star & \star & \hat{\Psi}_m^{66} & 0 \\ \star & \star & \star & \star & \star & \star & \hat{\Psi}_m^{77} \end{bmatrix},$$

$$\Theta_m^{11} = sym\{\mathcal{P}_m(\mathcal{A}_m + \mathcal{B}\mathcal{K}_m)\} + \sum_{n=1}^{\mathcal{N}} \vartheta_{mn}(\tau)\mathcal{P}_n + \mathcal{M}_{1m} + \bar{\eta}\mathcal{R}_1 + \mathcal{E}_m^T\mathcal{E}_m,$$

$$\Theta_m^{22} = \sum_{n=1}^{\mathcal{N}} \vartheta_{mn}(\tau)\mathcal{P}_n + sym\{\mathcal{P}_m\mathcal{A}_m - \hat{\alpha}\mathcal{Y}_m\mathcal{C}_m\} + \mathcal{M}_{2m} + \bar{\eta}\mathcal{R}_2 + \mathcal{E}_m^T\mathcal{E}_m.$$

假设系统的模态转移速率$\vartheta_{mn}(\tau)$满足$\vartheta_{mn}(\tau) \in [\underline{\vartheta}_{mn}, \bar{\vartheta}_{mn}](\underline{\vartheta}_{mn} - \bar{\vartheta}_{mn} < 0)$，$\underline{\vartheta}_{mn}$和$\bar{\vartheta}_{mn}$分别表示转移速率的下边界和上边界。然后，时变转移速率$\vartheta_{mn}(\tau)$能够通过下列线性组合表示：$\vartheta_{mn}(\tau) = \beta_1\underline{\vartheta}_{mn} + \beta_2\bar{\vartheta}_{mn}$，式中：$\beta_1 > 0$，$\beta_2 > 0$，$\beta_1 + \beta_2 = 1$。通过调节$\beta_1$和$\beta_2$ 的值，所有可能的$\vartheta_{mn}(\tau) \in [\underline{\vartheta}_{mn}, \bar{\vartheta}_{mn}]$能够被实现。对不等式(17)的两边分别乘以$\beta_1$和(18)的两边分别乘以$\beta_2$，并对其结果进行加和。显然，不等式(24) 成立。同理，不等式(19)-(22)能够与条件(25)-(26)实现等价变换。因此，此方法也能够保障误差系统(3)和滑动模态系统(6)所组成的扩维系统是随机稳定的且能够维持H_∞ 性能指标γ。证毕。

5 滑模控制器设计

本节旨在设计一个能够确保系统状态在有限时间内到达预先设定滑模面上的滑模控制率。

定理 3 对于半Markov跳变系统(1)，观测器(2)与滑模面(4)，通过构造滑模控制律

$$u(t) = -\hat{c}_m\phi(t) + \mathcal{K}_m z_l(t) - c_2\mathrm{sgn}(\phi(t)), \tag{26}$$

式中：$\hat{c}_m > 0, c_2 > 0$，然后状态能够在有限时间内被吸引到滑模面上。

证明：首先，我们选取Lyapunov函数：$S(t) = \phi^T(t)(2\mathcal{F}\mathcal{B})^{-1}\phi(t)$.

然后，能够得出$S(t)$的无穷小算子如下：

$$\begin{aligned}&\mathrm{E}\{\mathcal{L}\{\mathcal{S}(t)\}\}\\=&\mathrm{E}\{\phi^T(t)(\mathcal{F}\mathcal{B})^{-1}[\mathcal{F}\mathcal{B}u(t)+\mathcal{F}\mathcal{H}_m[\alpha(t)\mathcal{C}_m r(t)\\&+(1-\alpha(t))\mathcal{D}_m z(t-\eta(t))\\&+(\alpha(t)-\hat{\alpha})\mathcal{C}_m z_l(t)]-\mathcal{F}\mathcal{B}\mathcal{K}_m z_l(t)]\}.\end{aligned}$$

结合$\phi^T(t)\mathrm{sgn}((\phi(t)))\le\|\phi(t)\|_1$，标量$c_3>0$与控制律(27)，可得：

$$\begin{aligned}&\mathrm{E}\{\mathcal{L}\{\mathcal{S}(t)\}\}\\=&\phi^T(t)(\mathcal{F}\mathcal{B})^{-1}[\mathcal{F}\mathcal{H}_m[\hat{\alpha}\mathcal{C}_m r(t)\\&+(1-\hat{\alpha})\mathcal{D}_m z(t-\eta(t))]\\&-\hat{c}_m\phi^T(t)\phi(t)-c_2\phi^T(t)\mathrm{sgn}(\phi(t))\\\le&-\hat{c}_m\|\phi(t)\|^2-\|\phi(t)\|(c_2-\|(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\hat{\alpha}\mathcal{C}_m\|\\&\times\|r(t)\|-c_3-\|(1-\hat{\alpha})(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\mathcal{D}_m\|\times\\&(\|z_l(t-\eta(t))\|+\|r(t-\eta(t))\|))-c_3\|\phi(t)\|.\end{aligned}$$

定义$\Pi_m=\{c_2-\|(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\hat{\alpha}\mathcal{C}_m\|\|r(t)\|-c_3-\|(1-\hat{\alpha})(\mathcal{F}\mathcal{B})^{-1}\mathcal{F}\mathcal{H}_m\mathcal{D}_m\|(\|z_l(t-\eta(t))\|+\|r(t-\eta(t))\|)\ge 0\}$，于是有$\mathrm{E}\{\mathcal{L}\{\mathcal{S}(t)\}\}\le-\hat{c}_m\|\phi(t)\|^2-c_3\|\phi(t)\|\le-c_3\|\phi(t)\|\le\sqrt{2c_3^2\mathcal{S}(t)/\lambda_{\max}[(\mathcal{F}\mathcal{B})^{-1}]}$(此时状态轨迹达到区域$\Pi_m$)，式中：$\hat{c}_m$表示正常数，$\sqrt{2c_3^2/\lambda_{\max}((\mathcal{F}\mathcal{B})^{-1})}>0$。由此可知，当系统在时刻$T^\star=2\mathcal{S}^{1/2}(0)/\sqrt{2c_3^2/\lambda_{\max}((\mathcal{F}\mathcal{B})^{-1})}$时，$\mathcal{S}(t)=0$和$\phi(t)=0$。由此可知，系统的状态轨迹能够在有限时间内到达所设计的滑模面上。证毕。

6　仿真验证与分析

在本节中，旨在给出数值仿真例子验证所提滑模控制方法的有效性与正确性。根据文献 [23]，考虑一种他励直流电动机模型，其动态制动的动态等效电路如图1 所示。然后，该电路模型能够被描述为：

$$\begin{cases}R_{am}i_a+L_a\frac{\mathrm{d}i_a}{\mathrm{d}t}+C_e w=0,\\J\frac{\mathrm{d}w}{\mathrm{d}t}+fw-C_M i_a=0,\end{cases}\tag{27}$$

式中：R_{am}表示制动电阻，i_a表示制动电流，L_a表示电枢电感，C_e表示反电动势系数，w表示轴角速度，J表示电机惯量，f表示粘滞摩擦系数，C_M表示电磁转矩系数，m的值为1 或2。假设$m=\kappa(t)$是遵循半Markov过程随机变化的。因此，能够选择系统的转移速率矩阵为：$[\vartheta_{mn}(\tau)]=[-2\tau\ 2\tau;3\tau^2\ -3\tau^2]$.

根据文献 [11]，考虑模态驻留时间遵循Weibull分布并通过选取适当的形状参数与尺度参数，能够得出上述转移速率矩阵的期望为：$\mathrm{E}[\vartheta_{mn}(\tau)]=[-1.7725\ 1.7725;2.7082\ -2.7082]$.

定义下列变量：$z_1(t)=w$，$z_2(t)=i_a$，$y_1(t)=w$，$z(t)=\left[z_1^T(t)\ z_2^T(t)\right]^T$，可得：

$$\begin{cases}\dot{z}(t)=\mathcal{A}_m z(t),\\y_2(t)=C_m z(t),\end{cases}$$

式中：

$$\mathcal{A}_m=\begin{bmatrix}-\frac{f}{J}&\frac{C_M}{J}\\-\frac{C_e}{L_a}&-\frac{R_{am}}{L_a}\end{bmatrix},\ C_m=\begin{bmatrix}1\\0\end{bmatrix}^T,$$

$$f=0.015\ \mathrm{N\cdot m\cdot s/rad},\ J=0.003\ \mathrm{kg\cdot m^2},$$
$$C_M=300\ \mathrm{N\cdot m/A},\ C_e=6\ \mathrm{V/K\cdot RPM},$$
$$R_{a1}=2\,\Omega,L_a=5\,\mathrm{H},R_{a2}=1\,\Omega.$$

考虑半Markov跳变系统模型(1)以及上述他励直流电动机模型，系统中其他参数选择如下：

$$\mathcal{D}_1=\left[-0.1\ 0.3\right],\ \mathcal{E}_1=\left[0.5\ -0.1\right],$$
$$\mathcal{E}_{\mathcal{A}1}=\left[-0.01\ 0.01\right]^T,\ \mathcal{F}_{\mathcal{A}1}=\left[-0.01\ 0.01\right],$$
$$\hat{\mathcal{D}}_1=-0.05,\ \hat{\mathcal{D}}_2=-0.02,\ \mathcal{B}=\left[1\ 2\right]^T,$$
$$\mathcal{D}_2=\left[0.2\ 0.01\right],\ \mathcal{E}_2=\left[0.2\ -0.5\right],$$
$$\mathcal{E}_{\mathcal{A}2}=\begin{bmatrix}-0.2\\0.1\end{bmatrix},\ \mathcal{F}_{\mathcal{A}2}=\begin{bmatrix}-0.2\\0.1\end{bmatrix}^T,\ \bar{\eta}_1=0.5,$$
$$\mathcal{F}=\left[-0.3\ -0.1\right],\ \hat{\alpha}=0.6,\ \eta_1=0.8.$$

考虑$\mathcal{A}_m+\mathcal{B}\mathcal{K}_m$是Hurwitz矩阵，依据上述参数值可求得：

$$\mathcal{K}_1=\left[1.9514\ -0.4876\right],\ \mathcal{K}_2=\left[1.9228\ -0.5583\right].$$

系统干扰信号，时变时滞，以及控制律中的相关参数选择如下：$d(t)=e^{-\sqrt{(x_1^2(t)+x_2^2(t))t^2}}$, $\eta(t)=0.25+0.25\sin t$, $\hat{c}_1=0.005$, $\hat{c}_2=0.003$, $c_2=0.001$ 和$\mathrm{sgn}(\phi(t))=\frac{\phi(t)}{0.001+\|\phi(t)\|}$.

求解定理1中的不等式可得: $\gamma_{min}=1.1971$, $\mathcal{H}_1=[0.0151\ 0.0575]$ 和$\mathcal{H}_2=[0.1533\ 0.5923]$.

考虑系统与观测器的初始状态$z_0=z_{l0}=[1\ 1]^T$，得出仿真结果如图2-7所示。图2表示半Markov跳变系统的模态变化情况。图4表示闭环半Markov跳变系统的状态变化。图3和图5分别表示系统状态的估计误差以及观测器的状态变化。图6表示系统的测量输出。图7 表示系统的控制输入信号。由图4可知，在所设计的滑模控制律下，闭环半Markov跳变系统的运行状态能够保持稳定。换句话说，本文所提的滑模控制方法是有效的。

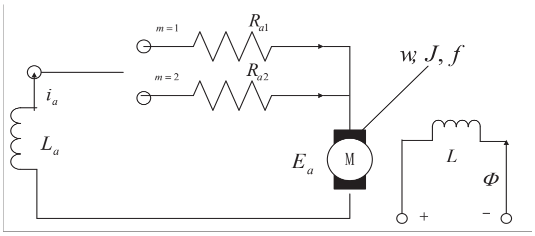

图 1 他励直流电动机动态等效电路

Fig. 1 Dynamic equivalent circuit of separately excited DC motor

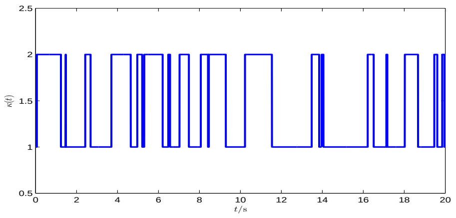

图 2 半Markov跳变系统的模态变化

Fig. 2 Mode changes of semi-Markov jump systems

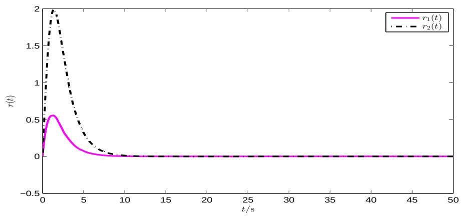

图 3 系统状态的估计误差

Fig. 3 Estimation error of system states

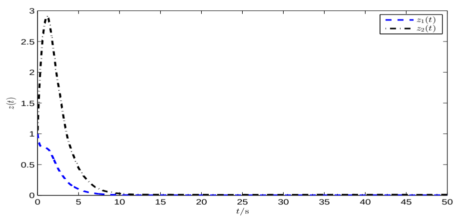

图 4 闭环半Markov跳变系统的状态轨迹

Fig. 4 State trajectories of closed-loop semi-Markov jump systems

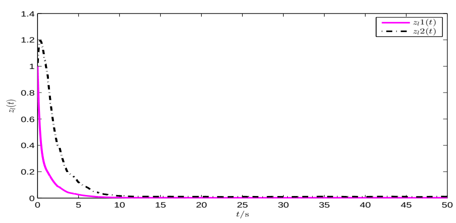

图 5 观测器的状态变化

Fig. 5 State changes of observer

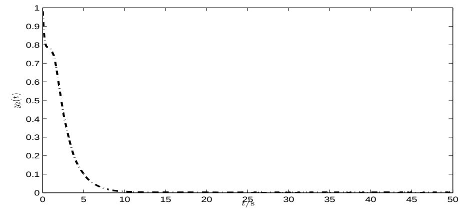

图 6 半Markov跳变系统的测量输出

Fig. 6 Measurement output of semi-Markov jump systems

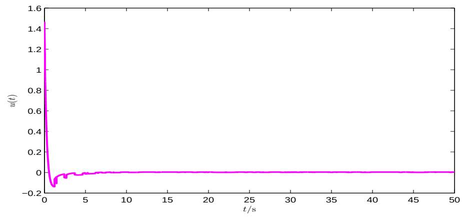

图 7 系统的控制输入信号

Fig. 7 Control input signal of systems

7 结论

考虑随机传感器时滞、参数不确定性以及状态不可测等条件，本文提出了一类不确定半Markov 跳变系统的鲁棒滑模控制方法。首先，设计了模态依赖Luenberger观测器去估计半Markov跳变系统的运行状态。然后，构造了一个积分滑模面并借助随机Lyapunov理论，给出了两种半Markov跳变系统随机稳定的充分条件。同时，设计了相应滑模控制律，确保所提滑模控制方案能够使得系统的状态在有限时间内吸引到预先设定的积分滑模面上。最后，考虑了一种他励直流电动机模型并通过数值仿真例子验证了所提滑模控制方法的有效性与正确性。在将来的研究工作中，希望将本文提出的控制方案扩展到更多的系统，例如，电力系统 [24]，微电网 [25–26]，奇异系统 [27–29]，多智能体系统 [30–31]等。

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