第 34 卷第 2 期
2017 年 2 月

控 制 理 论 与 应 用
Control Theory & Applications

Vol. 34 No. 2
Feb. 2017

DOI: 10.7641/CTA.2017.16102

离散时间切换线性系统的最小状态超调设计

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摘要: 本文探讨离散时间切换线性系统的最小状态超调设计问题. 对有限时长情形, 给出基于穷尽搜索的构造性算法, 并采用降价法提高计算效率. 对无限时长情形, 给出基于分段总汇的次优设计方案.

关键词: 切换线性系统; 状态超调; 最优化

中图分类号: TP273 **文献标识码:** A

Minimum state-overshooting design for discrete-time switched linear systems

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Abstract: In this work, we address the problem of minimum state-overshooting for discrete-time switched linear systems. For finite time horizons, the problem is solved in a constructive manner by exhaustive search, and a reduced-order methodology is applied to improve the computational efficiency. For infinite time horizons, sub-optimal solutions are obtained based on proper aggregation of finite-time-horizon trajectories.

Key words: switched linear systems; state-overshooting; optimization

1 Introduction

A switched linear system is a hybrid system consisting of a set of linear subsystems and a rule that coordinates the switching among the subsystems. From modelling perspective, switched linear systems could represent/approximate a large class of real-world dynamical systems. From control perspective, the hybrid control strategy provides a powerful tool in achieving better adaptation/robustness/intelligence performances than the conventional non-switched control.

For switched linear systems where switching/control laws are design variables, how to properly design the laws to improve the system performances is a critical issue. In the literature, most works focused on asymptotic performances such as stability/stabilizability, and the reader is referred to [1–5]. It should be stressed that, the switching/control laws thus designed might produce bad transient behaviors such as large overshoot and/or high-frequency oscillation that damage the system. Therefore, it is important to design the switching/control laws to achieve acceptable transient/asymptotic behaviors simultaneously.

An important index of transient performance is overshoot that refers to an output exceeding its final, steady-state value. The minimum overshooting problem is a classical control problem, and many efforts have been taken mainly for linear systems[6–12] during the last two decades. Early in 1991, a bang-bang based damping switching controller was proposed to obtain non-overshooting control for second-order systems[13]. Zhu et al. used a similar approach to eliminate overshoot of the double integrator and the third-order integrator[14–15]. Santarelli & Dahleh showed that, for a class of linear plants, a particular switching architecture outperforms the (optimal) linear feedback controller in terms of overshoot and settling time, which indicates that hybrid control could provide transient performance benefits[16–19]. While the aforementioned progress was impressive, the problem of minimum overshooting is still largely open even for linear systems.

In this work, we are to investigate the problem of minimum state-overshooting for force-free discrete-time switched linear systems.

The problem is to find proper switching strategy that makes the switched system stable with least possible state overshoot. By introducing the state-

Received 14 July 2016; accepted 9 October 2016.
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Recommended by Associate Editor: HONG Yi-guang.
Supported by National Key Basic Research Program “973” Plan (2014CB845302) and National Natural Science Foundation of China (61273121).

overshoot measured by a norm, we divide the minimum overshooting problem into the finite-time horizon and infinite-time horizon cases, respectively. The former is solved via exhausted search, and the latter is converted into a sub-optimal concatenation of the finite-time horizon solutions.

Notations: Let $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times n}$ be the set of real numbers, the set of n-th dimensional real vectors, the set of $n\times n$ real matrices, respectively. Let $\boldsymbol{N}_+$ be the set of non-negative integers. Let $\|\cdot\|$ be an arbitrarily given but fixed vector (or the induced matrix) norm. Denote by $\boldsymbol{B}_{\mathrm{r}}$ and $\boldsymbol{H}_{\mathrm{r}}$ the ball and the sphere centered on the origin of state space with radius r, respectively. For any non-negative integer k, let $\overline{k}=\{0,1,\cdots,k\}$. For an indexed set of real numbers $\varDelta=\{\alpha_1,\alpha_2,\cdots,\alpha_j\}$, define $\arg\min\varDelta=\min\{j:\alpha_j\leqslant\alpha_i,\ \forall\, i\neq j\}$.

2 Preliminaries

A discrete-time force-free switched linear system is described by

$$x(t+1)=A_{\sigma(t)}x(t),\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $\sigma(t)\in M=\{1,2,\cdots,m\}$ is the switching law, and $A_i\in\mathbb{R}^{n\times n}$, $i=1,2,\cdots,m$ are known subsystem matrices.

Denote by $\phi(t;0,x,\sigma)$ the solution of system (1) at time t with initial condition $x(0)=x$ and switching law σ.

Definition 1 Switched system (1) is said to be

1) Switched stable if for any $\varepsilon>0$, there exist a $\delta>0$, and a switching law $\{\sigma^x: x\in\boldsymbol{B}_\delta\}$, such that

$$\|\phi(t;0,x,\sigma^x)\|\leqslant\varepsilon,\ \ \forall\, x\in\boldsymbol{B}_\delta,\ t\geqslant 0;\tag{2}$$

and

2) Switched convergent if for any $\epsilon>0$ and $\gamma>0$, there exist a switching law $\{\sigma^x: x\in\boldsymbol{B}_\gamma\}$, and a time $T>0$, such that

$$\|\phi(t;0,x,\sigma^x)\|\leqslant\epsilon,\ \forall x\in\boldsymbol{B}_\gamma,\ t\geqslant T.\tag{3}$$

It was proved that switched convergent implies exponential convergent that the system could be made exponentially stable by appropriately designing the switching laws [20].

For any natural number t_{f} and $x\neq 0$, define

$$\tau_x^{t_{\mathrm{f}}}=\min_{\sigma^x}\max_{t=0,1,\cdots,t_{\mathrm{f}}}\frac{\|\phi(t;0,x,\sigma^x)\|}{\|x\|},\tag{4}$$

and $\tau^{t_{\mathrm{f}}}=\sup\limits_{x\neq 0}\tau_x^{t_{\mathrm{f}}}$. Furthermore, define

$$\tau_x=\limsup_{t_{\mathrm{f}}\to+\infty}\tau_x^{t_{\mathrm{f}}},\ \ \tau=\sup_{x\neq 0}\{\tau_x\}.$$

It can be seen that $\tau=\limsup\limits_{t_{\mathrm{f}}\to+\infty}\tau^{t_{\mathrm{f}}}$, which we term as the system overshoot. It is clear that $\tau\in[1,+\infty]$, and $\tau<+\infty$ if and only if the system is switched stable. When $\tau>1$, the overshooting phenomenon occurs.

Proposition 1 Denote

$$\varGamma=\{x\in\mathbb{R}^n:\ \min_{i\in M}\|A_ix\|>\|x\|\}.\tag{5}$$

Then we have

1) $\tau=1$ if and only if $\varGamma$ is the empty set;

2) The switched system is switched stable if and only if $\tau_x<\infty$;

3) If the system is switched convergent, then $\tau=\max\limits_{x\in\varGamma}\tau_x$.

Proof It follows from the definition that $\tau_x\geqslant 1$ for all $x\neq 0$. Note that $\tau=1$ means that the system does not admit overshoot, which in turn implies that $\varGamma$ is empty. On the contrary, if there is an x in $\varGamma$, then $\tau_x>1$ and hence $\tau>1$.

The second statement is straightforward.

For the third statement, note that switched convergence is equivalent to exponential stability, which further implies the existence of a positive real number $\bar\tau$ such that $\tau_x\leqslant\bar\tau$. On the other hand, it follows from the radial-invariance property that we could focus on the unit sphere, $\boldsymbol{H}_1$. By the compactness of the unit sphere and the continuity of τ_x on $\boldsymbol{H}_1$, the maximum of τ_x on $\boldsymbol{H}_1$ could be reached.

Remark 1 When the switched system is switched marginally stable (switched stable but not switched convergent), we do not know yet whether $\tau=\max\limits_{x\in\varGamma}\tau_x$ holds true or not. This seems quite involved and we leave it open for further investigation.

In this work, we are to investigate the following problems of minimum overshooting.

Finite time minimum state-overshooting problem (FTMSOP): Fix a terminal time t_{f}. For any initial state $0\neq x\in\mathbb{R}^n$, find a switching law σ^x over $\overline{t_{\mathrm{f}}-1}$ such that the maximum state norm over $\overline{t_{\mathrm{f}}}$ is minimized.

Infinite time minimum state-overshooting problem (ITMSOP): Suppose that system (1) is switched convergent. For any initial state $0\neq x\in\mathbb{R}^n$, find a switching law σ^x such that the state converges to the origin with least possible state overshoot.

Note that in the latter problem, we need to find (common) switching laws that achieve exponential stability with least possible system overshoot.

3 FTMSOP

As the time horizon is finite, for any given initial state, the minimum overshooting problem could be solved by exhausted searching. However, when the initial state varies, exhausted searching is not feasible. For this, we are to find a computational procedure to calculate the minimum overshoot over all state trajectories within the time horizon.

Let $\varTheta$ be the set of switching paths defined over $\overline{t_{\rm f}-1}$. For a switched system with m subsystems, the cardinality of $\varTheta$ is $m^{t_{\rm f}}$. For any non-origin x and switching path $\vartheta \in \varTheta$, denote

$$\nu_{\rm x}^{\vartheta} = \max_{i=0}^{t_{\rm f}} \|\phi(i;0,x,\vartheta)\|.$$

It is clear that

$$\tau_{\rm x}^{t_{\rm f}} = \min_{\vartheta\in\varTheta}\{\nu_{\rm x}^{\vartheta}\}/\|x\|,\ \tau^{t_{\rm f}} = \max_{x\neq 0}\{\tau_{\rm x}^{t_{\rm f}}\}.$$

A switching path ϑ is said to be minimum-overshooting w.r.t. x if $\nu_{\rm x}^{\vartheta} = \tau_{\rm x}^{t_{\rm f}}\|x\|$. Let $\varTheta_{\rm x}^{t_{\rm f}}$ be the set of such minimum-overshooting switched laws. Furthermore, define $\varOmega_{\vartheta} = \{x \neq 0 : \nu_{\rm x}^{\vartheta} = \tau_{\rm x}^{t_{\rm f}}\|x\|\}$. It follows that $\bigcup\limits_{\vartheta\in\varTheta} \varOmega_{\vartheta} = \mathbb{R}^n - \{0\}$.

As analyzed in [21], the region $\varOmega_{\vartheta}$ is norm-dependent. For ℓ_1-norm and ℓ_∞-norm, the region is a union of polyhedrons that are easily representative.

Note that, while feasible, the determination of $\varOmega_{\vartheta}$ by exhausted computation is quite time-consuming. In many cases, this is not necessary as some pruning algorithms could be applied to reduce the computational burden. For instance, if follows from Proposition 1 that, when $\min\limits_{i\in M}\|A_i x\| \leqslant \|x\|$ holds true for any $x \in \mathbb{R}^n$, then there is no overshoot, and hence the computing of $\varOmega_{\vartheta}$ needs not to proceed. For the purpose of computing τ, we have the following proposition that could reduce the computational load.

Proposition 2 Suppose that $\varGamma$ defined as in (5) is non-empty. Then we have $\tau^{t_{\rm f}} = \max\{\tau_{\rm x}^{t_{\rm f}} : x \in \varGamma\}$.

Proof We proceed by contradiction. It follows from Proposition 1 that $\tau^{t_{\rm f}} > 1$. Define $\bar{\tau} = \sup\{\tau_{\rm x}^{t_{\rm f}} : x \in \varGamma\}$. Suppose that there is an $x \notin \varGamma$, such that $\tau_{\rm x}^{t_{\rm f}} > \bar{\tau}$. For $x_0 = x$, define a switching law and the corresponding state trajectory recursively by

$$\sigma_{x_j} = \begin{cases} \arg\min\limits_{i\in M}\{\|A_i x_j\|\}, & \text{if } x_j \notin \varGamma, \\ \varpi_j, & \text{otherwise}, \end{cases}$$
$$x_{j+1} = A_{\sigma_{x_j}} x_j,\ j^* = \min\{j : x_j \in \varGamma\},$$
$$\varpi_j \in \varTheta_{x_{j^*}}^{t_{\rm f}-j^*},\ j = 0,1,2,\cdots.$$

It can be seen that $\|x_j\| \leqslant \|x_{j^*}\|$ and $\tau_{x_{j^*}}^{t_{\rm f}-j^*} \leqslant \tau_{x_{j^*}}^{t_{\rm f}}$, which means that $\tau_{\rm x}^{t_{\rm f}} \leqslant \tau_{x_{j^*}}^{t_{\rm f}} \leqslant \bar{\tau}$. This leads to contradiction.

Finally, the above analysis clearly shows that

$$\sup\{\tau_{\rm x}^{t_{\rm f}} : x \in \varGamma\} = \max_{x\neq 0}\{\tau_{\rm x}^{t_{\rm f}}\},$$

which means that

$$\sup\{\tau_{\rm x}^{t_{\rm f}} : x \in \varGamma\} = \max\{\tau_{\rm x}^{t_{\rm f}} : x \in \varGamma\}.$$

4 ITMSOP

When the time horizon is infinite, we are looking for a switching law that makes the switched system stable while achieving least possible overshoot. For this, we assume a prior that the switched system is switched convergent throughout this section.

Lemma 1 For any switched convergent system and any positive real number μ, there exist a positive integer L, a set of switching paths $\{\theta_1,\cdots,\theta_k\}$, and set of nonnegative integers $\{t_1,\cdots,t_k\}$ with $t_i \leqslant L$, $i = 1,\cdots,k$, such that

$$\min_{i=1}^{k}\|\phi(t_i;0,x,\theta_i)\| \leqslant \mu\|x\|,\ \ \forall\, x \neq 0. \tag{6}$$

Fix a $\mu < 1$, and let $t_{\rm f} = L$. The switching paths $\{\theta_1,\cdots,\theta_k\}$ and the corresponding set $\{t_1,\cdots,t_k\}$ could be computed by the computational procedure presented in [21].

For any $x \neq 0$, define $\upsilon_{\rm x} = \min\limits_{\substack{\|\phi(t_i;0,x,\theta_i)\|\leqslant\mu\|x\| \\ i=1,\cdots,k}}\{\nu_{\rm x}^{\theta_i}\}$.

With the above preparations, we have the main result as follows.

Theorem 1 The system overshoot satisfies the estimations

$$\tau \geqslant \max\{\tau_{\rm x}^{t_{\rm f}} : x \in \varGamma\}, \tag{7}$$

and

$$\tau \leqslant \max\{\max_{x\in\varGamma}\{\upsilon_{\rm x}\}, \mu\max_{x\notin\varGamma}\{\upsilon_{\rm x}\}\}. \tag{8}$$

Proof The lower-bound estimate follows directly from Proposition 2 and the fact that $\tau^{t_{\rm f}} \leqslant \tau$. We now proceed to establish the upper-bound estimate.

Borrowing the idea of path-wise state feedback switching law presented in [21], for any initial state x, we define a modified path-wise switching law σ^x recursively by

$$\begin{cases} \kappa_j = \arg\min\limits_{\substack{\|\phi(t_i;0,x_j,\theta_i)\|\leqslant\mu\|x_j\| \\ i=1,\cdots,k}}\{\nu_{x_j}^{\theta_i}\}, \\ l_{j+1} = l_j + t_{\kappa_j}, \\ \sigma^x(t) = \theta_{\kappa_j}(t-l_j),\ t = l_j,\cdots,l_{j+1}-1, \\ x_{j+1} = \phi(t_{\kappa_j};0,x_j,\theta_{\kappa_j}),\ j = 0,1,\cdots, \end{cases} \tag{9}$$

where $x_0 = x$ and $l_0 = 0$. It can be seen that the switching law is well defined over the infinite time horizon for any non-origin initial state. Furthermore, for any non-origin initial state, we have

$$\max_{i=l_0}^{l_1-1}\|\phi(i;0,x,\sigma^x)\| = \upsilon_{\rm x},\ \|\phi(l_1;0,x,\sigma^x)\| \leqslant \mu\|x\|,$$

which could be further extended to

$$\max_{i=l_0}^{l_j-1}\|\phi(i;0,x,\sigma^x)\| \leqslant \max\{\upsilon_{\rm x},\cdots,\mu^{j-1}\upsilon_{x_{j-1}}\},$$

and $\|\phi(l_j;0,x,\sigma^x)\| \leqslant \mu^j\|x\|$. Based on the above reasonings, we conclude that the state trajectory is exponentially convergent, and the overshoot is less than or equal to $\max\{\upsilon_{\rm x},\mu\upsilon_{x_1},\cdots\}$ that is bounded from above by $\max\{\max\limits_{x\in\varGamma}\{\upsilon_{\rm x}\},\mu\max\limits_{x\notin\varGamma}\{\upsilon_{\rm x}\}\}$ when $x \in \boldsymbol{H}_1$. This completes the proof.

Remark 2 Note that the theorem does not provide an accurate computing of the system overshoot. It is conjectured that the lower bound is the true value of system overshoot, but we are currently unable to affirm the conjecture.

5 Computational procedure

Combining the above analysis together, we are able to provide a computational procedure to estimate the system overshoot.

Schematic Procedure for computing the System Overshoot

Step 1 Compute region Γ.

Step 2 Compute switching paths $\{\theta_1, \cdots, \theta_k\}$ and the corresponding set $\{t_1, \cdots, t_k\}$ that satisfy Lemma 1.

Step 3 Compute $\tau_* = \max\{\tau_{\mathrm{x}}^{t_{\mathrm{f}}} : x \in \Gamma\}$.

Step 4 Compute

$$\tau^* = \max\{\max_{x\in\Gamma}\{\upsilon_{\mathrm{x}}\}, \mu \max_{x\notin\Gamma}\{\upsilon_{\mathrm{x}}\}\}.$$

Step 5 Output the estimate $\tau \in [\tau_*, \tau^*]$.

When we fix the ℓ_1-norm, by virtual of the homogeneity τ_{x} on x, and borrow the ideas from Ref. [21], a computational procedure could be developed. Here we present a numerical example to illustrate the procedure.

Example 1 Consider the system (1) with three planar subsystems given by

$$A_1 = \begin{bmatrix} -0.7113 & 0.5333 \\ 1.8498 & 0.0968 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.0378 & 0.4588 \\ 2.4130 & 0.4437 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} -0.7714 & 0.2266 \\ -0.8239 & -1.4026 \end{bmatrix}.$$

By applying the computational procedure presented in [21], we confirm that the system is switched convergent. When further applying the above schematic procedure, we obtain the upper and lower bounds for any initial state, which is shown in Fig.1.

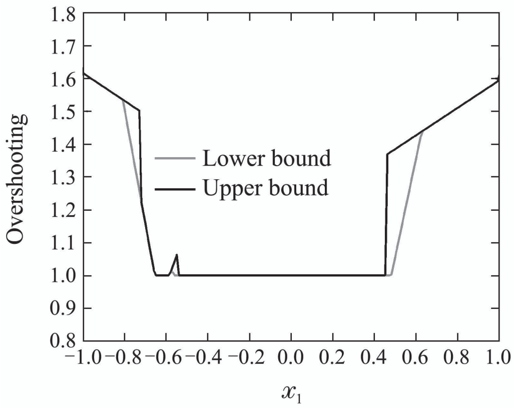

Fig. 1 The overshooting bounds

Note that the figure depicts initial states on the upper semi-part of the unit circle, that is, $x_0 = [x_1; 1 - |x_1|]$ with $x_1 \in [-1, 1)$, and any other initial state could be expressed by a scalar multiplication of these points. It can be seen that the upper bounds coincide with the lower bounds for a large part of the initial states, indicating that the overshooting estimates are with nice accuracy.

Fig.2 shows the state trajectories under switching law (9) and the original path-wise state-feedback switching law of [21]. It is clear that both switching laws produce convergent state trajectories with possible unequal overshoots.

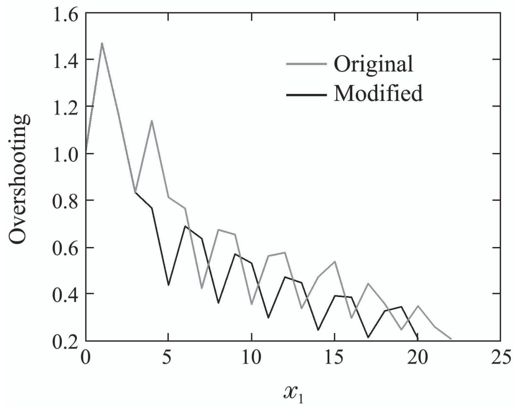

Fig. 2 State norms under different switching laws

6 Conclusions

In this work, the problem of minimum-overshooting problem has been investigated for force-free general discrete-time switched linear systems. Some properties of minimum overshooting were revealed both for the finite time and infinite time horizons. By properly modifying the path-wise state-feedback switching law developed in our earlier work[21], we obtained new switching law that achieves exponential stability with sub-minimum system overshoot. A computational procedure was also presented to approximate the overshoot, and a numerical example was given to illustrate the effectiveness of the proposed scheme.

As a final remark, note that the results could not be directly extended to the continuous-time case. First, without sampling, the computation of the least overshooting is intractable. Second, when the system is sampled, then the overshooting among the sampling intervals is difficult to exactly computed, and the accumulation might make effective estimates impossible. Nevertheless, some ideas in the work could possibly be extended to the continuous-time counterpart, and we will investigate this in a further work.

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