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多输出非线性系统在数据截断协议下的时滞多采样率输出反馈镇定

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摘要: 本文研究一类多输出非线性系统基于一类数据截断协议的时滞多采样率输出反馈控制. 所考虑的非线性系统的输出以异步方式进行采集, 通过网络传输到输出反馈控制端后立即用于构造输出反馈控制器. 当控制输入信号可用时, 立即用来更新被控制系统. 这样, 就存在两种传输时滞, 一种是非线性系统输出端到输出反馈控制器端的传输时滞, 另一种是输出反馈控制器端到非线性系统输入端的传输时滞. 从而被控系统的更新和输出反馈控制器的更新不在同一个时间区间. 利用区间分解, 可以在同一个区间考虑被控系统和输出反馈控制器并分析闭环系统的稳定性. 最后, 一个算例用来验证所提方法的有效性.

关键词: 输出反馈镇定器; 采样数据; 数据截断; 多采样率; 时滞

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Multi-rate sampled and delayed output feedback stabilization for multi-output nonlinear systems under data truncation protocol

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Abstract: Multi-rate sampled and delayed output feedback control law is presented for a class of multi-output nonlinear systems under a kind of data truncation protocol. The system outputs are sampled in a fully asynchronous way, transmitted through communication network, and used to construct the output feedback stabilizer immediately when they are available. The nonlinear system is updated by the system inputs once they are available. There exist two kinds of transmission delays, i.e., transmission delays from the nonlinear system to the output feedback stabilizer, and transmission delays from the output feedback stabilizer to the nonlinear system. Therefore, the output feedback stabilizer and the nonlinear system are updated at different time instants. Based on interval decomposition, we put the output feedback stabilizer and the nonlinear system on the same interval to analyze stability of the closed-loop system. An example is provided to illustrate the efficiency of the proposed methods.

Key words: output feedback stabilizer; sampled-data; data truncation; multi-rate; time delay

1 Introduction

The problem of global stabilization of nonlinear systems has made great progress in the past years[1–9]. For instance, the author in [1] investigated global stabilization for a class of single-input nonlinear systems by state feedback. By constructing a linear state feedback control law, the closed-loop systems could be globally exponentially stabilized. In [2], state feedback controller was constructed for a class of nonlinear continuous systems based on a simple methodology. However, it is not easy to extend the result from state feedback to output feedback for nonlinear systems with uncertain nonlinearities. By presenting a “Tikhonov-like” theorem, an observer-based controller was designed to stabilize a fully linearizable nonlinear system in which there exist uncertain nonlinearities[3]. This idea was also applied for a class of single-input-single-output (SISO) systems[4], which was then extended to multi-input-multi-output (MIMO) systems[5].

However, the aforementioned studies about output

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feedback stabilization are on the basis of continuous time analysis. More recently, sampled-data observer and sampled-data output feedback stabilization for nonlinear systems have received more and more attention. For linear systems, it is usual to design sampled-data observers and sampled-data output feedback control laws on the basis of the exact discretized model of the continuous systems. For nonlinear system, its exact discretized model is usually difficult to obtain. In order to overcome the difficulties, three main approaches are introduced as follows: i) The first approach is discrete-time design in view of an approximate exact discretized model[10–11]. ii) The second approach is continuous-time design and discrete-time implementation[12–14]. iii) The third approach is continuous-discrete design based on continuous-time model without discretization[15–22]. For instance, an explicit formula has been proposed in [15] to guarantee stability of nonlinear sampled-data systems with an emulated controller. The closed-loop system is required to be transformed into the form of a hybrid system introduced in [23]. In [24], sampled-data and delayed observer is designed for nonlinear systems under two different kinds of truncation protocols, static truncation and uniform truncation. A sampled-data output feedback stabilizer has been proposed for a class of single-output nonlinear systems via a linear sampled-data control without considering time delay[19]. The results in [20] are the extension of those in [19] by taking time delay into account. However, the upper bound of time delays is assumed to less than the sampling period. In [21], a relaxed condition of time delay is derived for the single-output nonlinear system, that is, the time delay may be larger than the sampling period. But, even if the sampled and delayed outputs are available, they aren't applied to update the stabilizer immediately. In [25], an observer design was proposed for multi-output nonlinear systems, in which there exist multi-rate sampled and delayed outputs. The sampled and delayed outputs are used to design the observer immediately once they are available.

For a sampled-data system, its outputs are usually sampled by transducers and transmitted via communication channels. The values of sampled data from transducer are usually not the true values of the original data with measurement errors. However, in [18, 20–21, 25–27], the values of sampled data are directly utilized to design observers or output feedback control laws, neglect of the effect of measurement errors. Therefore, it is of great practical significance to research output feedback stabilization for multi-output nonlinear systems with multi-rate sampled and delayed measurements by taking the effect of measurement accuracy into account. In this paper, the considered system outputs are sampled in a fully asynchronous way and transmitted under a kind of data truncation protocol as in [24]. They are used to update the output feedback stabilizer whenever they are available. The nonlinear system is also updated by the system inputs once they are available. There exist two kinds of transmission delays, i.e., transmission delays from the nonlinear system to the output feedback stabilizer, and transmission delays from the output feedback stabilizer to the nonlinear system. Therefore, the output feedback stabilizer and the nonlinear system are updated at different time instants. In order to analyze stability, we propose a method of interval decomposition and take the output feedback stabilizer and the nonlinear system on the same time interval into consideration. Based on a Lypunonv-Krasovskii function, sufficient condition is also derived to guarantee that the proposed output feedback stabilizer can globally uniformly stabilize the multi-output nonlinear systems.

This paper is organized as follows. In Section 2, we present our main aim of this paper, and some definitions and some useful results. An output feedback stabilizer is proposed for a class of multi-output nonlinear systems with multi-rate sampled and delayed measurements under the static data truncation in Section 3. In Section 4, an example is provided to illustrate the validity of the proposed design methods. This paper concludes in Section 5.

2 Preliminaries

In this paper, our purpose is to design an output feedback controller to stabilize a class of multi-output nonlinear systems in the form of

$$\begin{cases} \dot{x}(t) = Ax(t) + B(x(t), t) + E(u(t)), \\ \tilde{y}(t) = Cx(t) = [C_1 x^1(t) \ \ \cdots \ \ C_m x^m(t)]^{\mathrm{T}}, \end{cases} \tag{1}$$

where

$$A = \mathrm{diag}\{A_1, A_2, \cdots, A_m\},\ A_i = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}_{\lambda_i \times \lambda_i},$$

$$B(x(t), t) = [b^1(x(t), t)^{\mathrm{T}} \ \ \cdots \ \ b^m(x(t), t)^{\mathrm{T}}]^{\mathrm{T}},$$
$$E(u(t)) = [(E_1 u_1(t))^{\mathrm{T}} \ \ \cdots \ \ (E_m u_m(t))^{\mathrm{T}}]^{\mathrm{T}},$$
$$E_i = [0 \ \ 0 \ \ \cdots \ \ 1]^{\mathrm{T}}_{1\times\lambda_i},\ C = \mathrm{diag}\{C_1, C_2, \cdots, C_m\},$$
$$C_i = [1 \ \ 0 \ \ \cdots \ \ 0]_{1\times\lambda_i},$$

$x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the system input, $\tilde{y}(t) \in \mathbb{R}^m$ is the system output, $x(t) = [x^1(t)^{\mathrm{T}} \ \ \cdots \ \ x^m(t)^{\mathrm{T}}]^{\mathrm{T}}$, $x^i(t) \in \mathbb{R}^{\lambda_i} (1 \leqslant i \leqslant m)$ is the ith partition of the state $x(t)$, $u(t) = [u_1(t) \ \ \cdots \ \ u_m(t)]^{\mathrm{T}}$, $\tilde{y}(t) = [\tilde{y}_1(t) \ \ \cdots \ \ \tilde{y}_m(t)]^{\mathrm{T}}$, $\tilde{y}_i(t) = C_i x^i(t)$.

The ith block of system (1) can be rewritten by

$$\begin{cases} \dot{x}^i_j(t) = x^i_{j+1}(t) + b^i_j(x(t)^{[1,i-1]}; x^i_{[1,j]}(t), t), \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{x}^i_{\lambda_i}(t) = u_i(t) + b^i_{\lambda_i}(x(t)^{[1,i-1]}; x(t)^i_{[1,\lambda_i]}, t), \end{cases} \tag{2}$$

where $x^i_j(t)$ is the jth element of the ith block $x^i(t)$, and $x(t)^{[1,k]} := [x^1(t)^{\mathrm{T}} \ \cdots \ x^k(t)^{\mathrm{T}}]^{\mathrm{T}}$ and $x(t)^i_{[1,j]} := [x^i_1(t) \ \ \cdots \ \ x^i_j(t)]^{\mathrm{T}}$. The unknown nonlinear terms $b^i_j(\cdot)$ are assumed to satisfy the following condition with a constant $l_1 > 0$,

$$|b^i_j(\cdot)| = |b^i_j(x^1, \cdots, x^{i-1}; x^i_1, \cdots, x^i_j, t)| \leqslant l_1(|x^1_1| + |x^1_2| + \cdots + |x^i_j|). \tag{3}$$

We also assumed that the system output $\tilde{y}_i(t)$ is measured at the sampling instants t^i_k and only $y_i(t^i_k) = h_i(\tilde{y}_i(t^i_k))$ can be available. The sequences $\{t^i_k\}$ $(i = 1, 2, \cdots, m)$ are strictly increasing and satisfy $\lim\limits_{k\to+\infty} t^i_k = +\infty$ for $i = 1, 2, \cdots, m$, and $T^i = t^i_{k+1} - t^i_k$. In addition, each function $h_i(\cdot)$ denotes a data truncation of the sampled data, whose definition is given as follows.

Definition 1[25] The functions $h_i(\cdot)(i = 1, \cdots, m)$ are said to be data truncation at the sampling instants t^i_k or static data truncation, if the following inequalities

$$|h_i(x^i_1(t^i_k)) - x^i_1(t^i_k)| \leqslant c^i_1, \ i = 1, \cdots, m \tag{4}$$

hold, where $c^i_1 > 0$ denote absolute error limit.

The sampled-data output feedback control system with network-induced delay is shown in Fig. 1. τ^i_k $(k \geqslant 0)$ is the network-induced delay from sampler to the stabilizer of block i and d^i_k is the network-induced delay from the stabilizer to the zero-order hold (ZOH). We assume that $\tau^i_k \leqslant \bar{\tau}_i$ and $d^i_k \leqslant \bar{d}_i$. Note that the output updating signal at the instant t^i_k has experienced signal transmission delay τ^i_k from sampler to the stabilizer and then the control updating signal experienced signal transmission delay d^i_k from stabilizer to the ZOH (see Fig. 1).

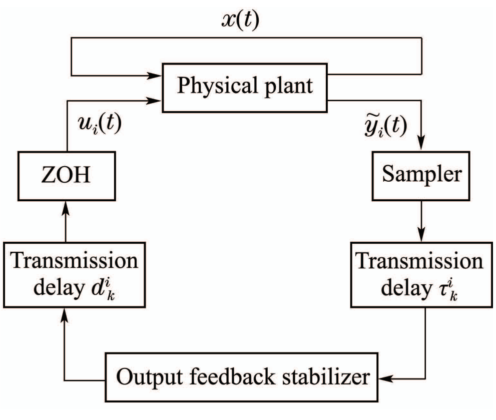

Fig. 1 Output feedback system with network-induced delays

The following inequality is also useful for our main results.

Lemma 1 [28] For any positive definite matrix $U \in \mathbb{R}^{n\times n}$, scalar $\gamma > 0$, vector function $w : [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds

$$[\int_0^\gamma w(s)\mathrm{d}s]^{\mathrm{T}} U [\int_0^\gamma w(s)\mathrm{d}s] \leqslant \gamma [\int_0^\gamma w(s)^{\mathrm{T}} U w(s)\mathrm{d}s].$$

3 Multi-rate sampled and delayed output feedback stabilization under the static data truncation

In this section, the output feedback control law in the form of

$$\begin{cases} \dot{\hat{x}}^i_j(t) = \hat{x}^i_{j+1}(t) + L^j_i a^i_j (y_i(t^i_k) - \hat{x}^i_1(t^i_k)), \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{\hat{x}}^i_{\lambda_i}(t) = u_i(t) + L^{\lambda_i}_i a^i_{\lambda_i} (y_i(t^i_k) - \hat{x}^i_1(t^i_k)), \\ \qquad t \in [t^i_k + \tau^i_k, t^i_{k+1} + \tau^i_{k+1}), \\ \hat{x}^i_j(t^i_{k+1} + \tau^i_{k+1}) = \lim\limits_{t\to t^i_{k+1}+\tau^i_{k+1}{}^-} \hat{x}^i_j(t), \\ i = 1, 2, \cdots, m, \ j = 1, \cdots, \lambda_i, \ k \geqslant 0, \end{cases} \tag{5}$$

and

$$u_i(t) = -[L^{\lambda_i}_i k^i_1 \hat{x}^i_1(t^i_k) + L^{\lambda_i - 1}_i k^i_2 \hat{x}^i_2(t^i_k) + \cdots + L_i k^i_{\lambda_i} \hat{x}^i_{\lambda_i}(t^i_k)], \ t \in [t^i_k + \tau^i_k, t^i_{k+1} + \tau^i_{k+1}), \ k \geqslant 0 \tag{6}$$

is proposed to stabilize the system (2) with the data truncation (4), where $t_0 = t^i_0$ and $L_i \geqslant 1$, a^i_j, k^i_j $(1 \leqslant i \leqslant m, 1 \leqslant j \leqslant \lambda_i)$ are some parameters. Since the outputs $y_i(t)$ are sampled at different rate, (5)–(6) is called multi-rate sampled and delayed output feedback stabilizer.

From (2)(5)–(6) and the control scheme shown in Fig. 1, we have

$$\begin{cases} \dot{x}^i_j(t) = x^i_{j+1}(t) + b^i_j(x(t)^{[1,i-1]}; x^i_{[1,j]}(t), t), \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{x}^i_{\lambda_i}(t) = -[L^{\lambda_i}_i k^i_1 \hat{x}^i_1(t^i_k) + \cdots + L_i k^i_{\lambda_i} \hat{x}^i_{\lambda_i}(t^i_k)] + \\ \qquad b^i_{\lambda_i}(x(t)^{[1,i-1]}; x(t)^i_{[1,\lambda_i]}, t), \\ \qquad t \in [t^i_k + \tau^i_k + d^i_k, t^i_{k+1} + \tau^i_{k+1} + d^i_{k+1}). \end{cases} \tag{7}$$

Remark 1 Usually, the transmission delays τ^i_k and d^i_k are unknown. However, we can obtain the time instants that the inputs $u_i(t)$ $(i = 1, \cdots, m)$ arrive. In another word, the nonlinear system (1) is updated automatically once $u_i(t)$ $(i = 1, \cdots, m)$ are available.

Remark 2 For the system (5), when $t \in [t^i_k + \tau^i_k, t^i_{k+1} + \tau^i_{k+1})$, $y_i(t^i_k) - \hat{x}^i_1(t^i_k)$ and $u_i(t)$ are constants, thus the system (5) is continuous on $[t^i_k + \tau^i_k, t^i_{k+1} + \tau^i_{k+1})$. Moreover, note that $\hat{x}^i_j(t^i_{k+1} + \tau^i_{k+1}) = \lim\limits_{t\to t^i_{k+1}+\tau^i_{k+1}{}^-} \hat{x}^i_j(t)$, then the system (5) is continuous. For the same reason, we can obtain that the system (7) is also continuous.

Note that the systems (5)–(6) is updated at the time instant $t^i_k + \tau^i_k$, and the system (7) is updated at the different time instant $t^i_k + \tau^i_k + d^i_k$. In order to analyze the stability of the closed-loop system, we assume that the time delays d^i_k and τ^i_k satisfy $d^i_k < \bar{m}_1 T^i$ and $\tau^i_k < \bar{m}_2 T^i$ for all $k > 0$, where $\bar{m}_1$ and $\bar{m}_2$ are two positive integers. Then, $t^i_k + \tau^i_k + d^i_k < t^i_k + (\bar{m}_1 + \bar{m}_2)T^i$ and

$t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i < t_k^i + (\bar{m}_1 + \bar{m}_2 + 1)T^i$. Therefore, there exist two positive integers $m_{k_1} < \bar{m}_1 + \bar{m}_2$ and $m_{k_2} < \bar{m}_1 + \bar{m}_2 + 1$ such that

$$t_k^i + \tau_k^i + d_k^i \in [t_{k+m_{k_1}}^i + \tau_{k+m_{k_1}}^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i)$$

and

$$t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i \in [t_{k+m_{k_2}}^i + \tau_{k+m_{k_2}}^i, t_{k+m_{k_2}+1}^i + \tau_{k+m_{k_2}+1}^i).$$

Without loss of generality, let $m_{k_1} < m_{k_2}$. Then,

$$\begin{aligned}&[t_k^i + \tau_k^i + d_k^i, t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i) = \\ &\bigcup_{s=m_{k_1}+1}^{m_{k_2}-1} [t_{k+s}^i + \tau_{k+s}^i, t_{k+s+1}^i + \tau_{k+s+1}^i) \bigcup \\ &[t_k^i + \tau_k^i + d_k^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i) \bigcup \\ &[t_{k+m_{k_2}}^i + \tau_{k+m_{k_2}}^i, t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i).\end{aligned} \tag{8}$$

With the interval decomposition (8), we can consider the systems (5)–(6) and the system (7) on the same interval. When $t \in [t_k^i + \tau_k^i + d_k^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i)$, the output feedback control law is given as

$$\begin{cases}\dot{\hat{x}}_j^i(t) = \\ \hat{x}_{j+1}^i(t) + L_i^j a_j^i (y_i(t_{k+m_{k_1}}^i) - \hat{x}_1^i(t_{k+m_{k_1}}^i)), \\ \quad j = 1, \cdots, \lambda_i - 1, \\ \dot{\hat{x}}_{\lambda_i}^i(t) = \\ u_i(t) + L_i^{\lambda_i} a_{\lambda_i}^i (y_i(t_{k+m_{k_1}}^i) - \hat{x}_1^i(t_{k+m_{k_1}}^i)), \\ \quad t \in [t_k^i + \tau_k^i + d_k^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i), \\ \quad i = 1, 2, \cdots, m,\end{cases} \tag{9}$$

and

$$\begin{aligned}&u_i(t) = \\ &-[L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_{k+m_{k_1}}^i) + \cdots + L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_{k+m_{k_1}}^i)], \\ &t \in [t_k^i + \tau_k^i + d_k^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i).\end{aligned} \tag{10}$$

From (7)(9)–(10), we have

$$\begin{cases}\dot{e}_j^i(t) = e_{j+1}^i(t) + L_i^j a_j^i (x_1^i(t_{k+m_{k_1}}^i) - \\ \qquad h_i(x_1^i(t_{k+m_{k_1}}^i))) - L_i^j a_j^i e_1^i(t_{k+m_{k_1}}^i) + b_j^i, \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{e}_{\lambda_i}^i(t) = [L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_{k+m_{k_1}}^i) + \cdots + \\ \qquad L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_{k+m_{k_1}}^i)] - [L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_k^i) + \\ \qquad \cdots + L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_k^i)] + \\ \qquad L_i^{\lambda_i} a_{\lambda_i}^i (x_1^i(t_{k+m_{k_1}}^i) - h_i(x_1^i(t_{k+m_{k_1}}^i))) - \\ \qquad L_i^{\lambda_i} a_{\lambda_i}^i e_1^i(t_{k+m_{k_1}}^i) + b_{\lambda_i}^i, \\ \qquad t \in [t_k^i + \tau_k^i + d_k^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i), \\ \qquad i = 1, 2, \cdots, m,\end{cases} \tag{11}$$

and

$$\begin{cases}\dot{\hat{x}}_j^i(t) = \hat{x}_{j+1}^i(t) + L_i^j a_j^i (y_i(t_{k+m_{k_1}}^i) - \hat{x}_1^i(t_{k+m_{k_1}}^i)), \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{\hat{x}}_{\lambda_i}^i(t) = \\ -[L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_{k+m_{k_1}}^i) + \cdots + L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_{k+m_{k_1}}^i)] + \\ L_i^{\lambda_i} a_{\lambda_i}^i (y_i(t_{k+m_{k_1}}^i) - \hat{x}_1^i(t_{k+m_{k_1}}^i)), \\ \qquad t \in [t_k^i + \tau_k^i + d_k^i, t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i), \\ \qquad i = 1, 2, \cdots, m,\end{cases} \tag{12}$$

where $e_j^i(t) = x_j^i(t) - \hat{x}_j^i(t)$ $(1 \leqslant i \leqslant m, 1 \leqslant j \leqslant \lambda_i)$ and $e(t) = [e^1(t)^{\mathrm{T}} \ e^2(t)^{\mathrm{T}} \ \cdots \ e^m(t)^{\mathrm{T}}]^{\mathrm{T}}$, $e^i(t) = [e_1^i(t) \ \cdots \ e_{\lambda_i}^i(t)]^{\mathrm{T}}$.

Using the same method, when $t \in [t_{k+s}^i + \tau_{k+s}^i, t_{k+s+1}^i + \tau_{k+s+1}^i)$, $s = m_{k_1} + 1, \cdots, m_{k_2} - 1$, we have

$$\begin{cases}\dot{e}_j^i(t) = e_{j+1}^i(t) + L_i^j a_j^i (x_1^i(t_{k+s}^i) - h_i(x_1^i(t_{k+s}^i))) - \\ \qquad L_i^j a_j^i e_1^i(t_{k+s}^i) + b_j^i, j = 1, \cdots, \lambda_i - 1, \\ \dot{e}_{\lambda_i}^i(t) = [L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_{k+s}^i) + L_i^{\lambda_i - 1} k_2^i \hat{x}_2^i(t_{k+s}^i) + \cdots + \\ \qquad L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_{k+s}^i)] - [L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_k^i) + \cdots + \\ \qquad L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_k^i)] + L_i^{\lambda_i} a_{\lambda_i}^i (x_1^i(t_{k+s}^i) - \\ \qquad h_i(x_1^i(t_{k+s}^i))) - L_i^{\lambda_i} a_{\lambda_i}^i e_1^i(t_{k+s}^i) + b_{\lambda_i}^i, \\ \qquad t \in [t_{k+s}^i + \tau_{k+s}^i, t_{k+s+1}^i + \tau_{k+s+1}^i), \\ \qquad i = 1, \cdots, m, \ s = m_{k_1} + 1, \cdots, m_{k_2} - 1,\end{cases} \tag{13}$$

and

$$\begin{cases}\dot{\hat{x}}_j^i(t) = \hat{x}_{j+1}^i(t) + L_i^j a_j^i (y_i(t_{k+s}^i) - \hat{x}_1^i(t_{k+s}^i)), \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{\hat{x}}_{\lambda_i}^i(t) = \\ -[L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_{k+s}^i) + \cdots + L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_{k+s}^i)] + \\ L_i^{\lambda_i} a_{\lambda_i}^i (y_i(t_{k+s}^i) - \hat{x}_1^i(t_{k+s}^i)), \\ \qquad t \in [t_{k+s}^i + \tau_{k+s}^i, t_{k+s+1}^i + \tau_{k+s+1}^i), \\ \qquad i = 1, \cdots, m, \ s = m_{k_1} + 1, \cdots, m_{k_2} - 1.\end{cases} \tag{14}$$

When $t \in [t_{k+m_{k_2}}^i + \tau_{k+m_{k_2}}^i, t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i)$, we have

$$\begin{cases}\dot{e}_j^i(t) = e_{j+1}^i(t) + L_i^j a_j^i (x_1^i(t_{k+m_{k_2}}^i) - \\ \qquad h_i(x_1^i(t_{k+m_{k_2}}^i))) - L_i^j a_j^i e_1^i(t_{k+m_{k_2}}^i) + b_j^i, \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{e}_{\lambda_i}^i(t) = [L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_{k+m_{k_2}}^i) + \cdots + \\ \qquad L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_{k+m_{k_2}}^i)] - [L_i^{\lambda_i} k_1^i \hat{x}_1^i(t_k^i) + \cdots + \\ \qquad L_i k_{\lambda_i}^i \hat{x}_{\lambda_i}^i(t_k^i)] + L_i^{\lambda_i} a_{\lambda_i}^i (x_1^i(t_{k+m_{k_2}}^i) - \\ \qquad h_i(x_1^i(t_{k+m_{k_2}}^i))) - L_i^{\lambda_i} a_{\lambda_i}^i e_1^i(t_{k+m_{k_2}}^i) + b_{\lambda_i}^i, \\ \qquad t \in [t_{k+m_{k_2}}^i + \tau_{k+m_{k_2}}^i, t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i), \\ \qquad i = 1, 2, \cdots, m,\end{cases} \tag{15}$$

and

$$\begin{cases}\dot{\hat{x}}_j^i(t)=\hat{x}_{j+1}^i(t)+L_i^ja_j^i(y_i(t_{k+m_{k_2}}^i)-\hat{x}_1^i(t_{k+m_{k_2}}^i)),\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{\hat{x}}_{\lambda_i}^i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_1^i(t_{k+m_{k_2}}^i)+\\ \qquad L_i^{\lambda_i-1}k_2^i\hat{x}_2^i(t_{k+m_{k_2}}^i)+\cdots+\\ \qquad L_ik_{\lambda_i}^i\hat{x}_{\lambda_i}^i(t_{k+m_{k_2}}^i)]+\\ \qquad L_i^{\lambda_i}a_{\lambda_i}^i(y_i(t_{k+m_{k_2}}^i)-\hat{x}_1^i(t_{k+m_{k_2}}^i)),\\ \qquad t\in[t_{k+m_{k_2}}^i+\tau_{k+m_{k_2}}^i,t_{k+1}^i+\tau_{k+1}^i+d_{k+1}^i),\\ \qquad i=1,2,\cdots,m.\end{cases}\tag{16}$$

During the time periods $[t_0,t_0+\tau_0^i)$ $(i=1,\cdots,m)$, we set

$$\begin{cases}\dot{\hat{x}}_j^i(t)=\hat{x}_{j+1}^i(t)+L_i^ja_j^i(y_i(t_0)-\hat{x}_1^i(t_0)),\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{\hat{x}}_{\lambda_i}^i(t)=u_i(t)+L_i^{\lambda_i}a_{\lambda_i}^i(y_i(t_0)-\hat{x}_1^i(t_0)),\\ \qquad t\in[t_0,t_0+\tau_0^i),\\ \hat{x}_j^i(t_0+\tau_0^i)=\lim\limits_{t\to t_0+\tau_0^{i-}}\hat{x}_j^i(t),\\ \qquad i=1,2,\cdots,m,\ j=1,\cdots,\lambda_i,\end{cases}\tag{17}$$

and

$$u_i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_{1_0}^i+L_i^{\lambda_i-1}k_2^i\hat{x}_{2_0}^i+\cdots+ L_ik_{\lambda_i}^i\hat{x}_{\lambda_{i0}}^i],\ t\in[t_0,t_0+\tau_0^i),\tag{18}$$

where $\hat{x}_{j_0}^i$ $(i=1,\cdots,m,j=1,\cdots,\lambda_i)$ are the initial values of $\hat{x}_j^i(t)$ at the time instants t_0. For the nonlinear system (2), we also set $u_i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_{1_0}^i+\cdots+L_ik_{\lambda_i}^i\hat{x}_{\lambda_{i0}}^i],\ t\in[t_0,t_0+\tau_0^i+d_0^i)$. Then,

$$\begin{cases}\dot{x}_j^i(t)=x_{j+1}^i(t)+b_j^i(x(t)^{[1,i-1]};x_{[1,j]}^i(t),t),\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{x}_{\lambda_i}^i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_{1_0}^i+\cdots+L_ik_{\lambda_i}^i\hat{x}_{\lambda_{i0}}^i]+\\ \qquad b_{\lambda_i}^i(x(t)^{[1,i-1]};x(t)_{[1,\lambda_i]}^i,t),\\ \qquad t\in[t_0,t_0+\tau_0^i+d_0^i).\end{cases}\tag{19}$$

There also exists a positive integer $m_{0_1}<\bar{m}_1+\bar{m}_2$ such that $t_0+\tau_0^i+d_0^i\in[t_{m_{0_1}}^i+\tau_{m_{0_1}}^i,t_{m_{0_1}+1}^i+\tau_{m_{0_1}+1}^i)$. Then we have

$$\begin{aligned}&[t_0,t_0+\tau_0^i+d_0^i)=\\ &[t_0,t_0+\tau_0^i)\bigcup_{s=0}^{m_{0_1}-1}[t_s^i+\tau_s^i,t_{s+1}^i+\tau_{s+1}^i)\\ &\bigcup[t_{m_{0_1}}^i+\tau_{m_{0_1}}^i,t_0+\tau_0^i+d_0^i).\end{aligned}\tag{20}$$

By using the same method, when $t\in[t_0,t_0+\tau_0^i)$, from (17), (18) and (19), we have

$$\begin{cases}\dot{e}_j^i(t)=e_{j+1}^i(t)+L_i^ja_j^i(x_1^i(t_0)-h_i(x_1^i(t_0)))-\\ \qquad L_i^ja_j^ie_1^i(t_0)+b_j^i,\ j=1,\cdots,\lambda_i-1,\\ \dot{e}_{\lambda_i}^i(t)=L_i^{\lambda_i}a_{\lambda_i}^i(x_1^i(t_0)-h_i(x_1^i(t_0)))-\\ \qquad L_i^{\lambda_i}a_{\lambda_i}^ie_1^i(t_0)+b_{\lambda_i}^i,\\ \qquad t\in[t_0,t_0+\tau_0^i),\ i=1,2,\cdots,m,\end{cases}\tag{21}$$

and

$$\begin{cases}\dot{\hat{x}}_j^i(t)=\hat{x}_{j+1}^i(t)+L_i^ja_j^i(y_i(t_0)-\hat{x}_1^i(t_0)),\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{\hat{x}}_{\lambda_i}^i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_{1_0}^i+\cdots+L_ik_{\lambda_i}^i\hat{x}_{\lambda_{i0}}^i]+\\ \qquad L_i^{\lambda_i}a_{\lambda_i}^i(y_i(t_0)-\hat{x}_1^i(t_0)),\\ \qquad t\in[t_0,t_0+\tau_0^i),\ i=1,2,\cdots,m.\end{cases}\tag{22}$$

When $t\in[t_s^i+\tau_s^i,\ t_{s+1}^i+\tau_{s+1}^i)$, $s=0,\cdots,m_{0_1}-1$, we have

$$\begin{cases}\dot{e}_j^i(t)=e_{j+1}^i(t)+L_i^ja_j^i(x_1^i(t_s^i)-h_i(x_1^i(t_s^i)))-\\ \qquad L_i^ja_j^ie_1^i(t_s^i)+b_j^i,\ j=1,\cdots,\lambda_i-1,\\ \dot{e}_{\lambda_i}^i(t)=[L_i^{\lambda_i}k_1^i\hat{x}_1^i(t_s^i)+L_i^{\lambda_i-1}k_2^i\hat{x}_2^i(t_s^i)+\cdots+\\ \qquad L_ik_{\lambda_i}^i\hat{x}_{\lambda_i}^i(t_s^i)]-[L_i^{\lambda_i}k_1^i\hat{x}_{1_0}^i+\cdots+\\ \qquad L_ik_{\lambda_i}^i\hat{x}_{\lambda_{i0}}^i]+L_i^{\lambda_i}a_{\lambda_i}^i(x_1^i(t_s^i)-\\ \qquad h_i(x_1^i(t_s^i)))-L_i^{\lambda_i}a_{\lambda_i}^ie_1^i(t_s^i)+b_{\lambda_i}^i,\\ \qquad t\in[t_s^i+\tau_s^i,t_{s+1}^i+\tau_{s+1}^i),\\ \qquad i=1,2,\cdots,m,\ s=0,\cdots,m_{0_1}-1,\end{cases}\tag{23}$$

and

$$\begin{cases}\dot{\hat{x}}_j^i(t)=\hat{x}_{j+1}^i(t)+L_i^ja_j^i(y_i(t_s^i)-\hat{x}_1^i(t_s^i)),\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{\hat{x}}_{\lambda_i}^i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_1^i(t_s^i)+L_i^{\lambda_i-1}k_2^i\hat{x}_2^i(t_s^i)+\cdots+\\ \qquad L_ik_{\lambda_i}^i\hat{x}_{\lambda_i}^i(t_s^i)]+L_i^{\lambda_i}a_{\lambda_i}^i(y_i(t_s^i)-\hat{x}_1^i(t_s^i)),\\ \qquad t\in[t_s^i+\tau_s^i,t_{s+1}^i+\tau_{s+1}^i),\\ \qquad i=1,2,\cdots,m,\ s=0,\cdots,m_{0_1}-1.\end{cases}\tag{24}$$

When $t\in[t_{m_{0_1}}^i+\tau_{m_{0_1}}^i,t_0+\tau_0^i+d_0^i)$, we have

$$\begin{cases}\dot{e}_j^i(t)=e_{j+1}^i(t)+L_i^ja_j^i(x_1^i(t_{m_{0_1}}^i)-\\ \qquad h_i(x_1^i(t_{m_{0_1}}^i)))-L_i^ja_j^ie_1^i(t_{m_{0_1}}^i)+b_j^i,\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{e}_{\lambda_i}^i(t)=[L_i^{\lambda_i}k_1^i\hat{x}_1^i(t_{m_{0_1}}^i)+L_i^{\lambda_i-1}k_2^i\hat{x}_2^i(t_{m_{0_1}}^i)+\\ \qquad \cdots+L_ik_{\lambda_i}^i\hat{x}_{\lambda_i}^i(t_{m_{0_1}}^i)]-[L_i^{\lambda_i}k_1^i\hat{x}_{1_0}^i+\\ \qquad \cdots+L_ik_{\lambda_i}^i\hat{x}_{\lambda_{i0}}^i]+L_i^{\lambda_i}a_{\lambda_i}^i(x_1^i(t_{m_{0_1}}^i)-\\ \qquad h_i(x_1^i(t_{m_{0_1}}^i)))-L_i^{\lambda_i}a_{\lambda_i}^ie_1^i(t_{k+m_{k_2}}^i)+b_{\lambda_i}^i,\\ \qquad t\in[t_{m_{0_1}}^i+\tau_{m_{0_1}}^i,t_0+\tau_0^i+d_0^i),\\ \qquad i=1,2,\cdots,m,\end{cases}\tag{25}$$

and

$$\begin{cases}\dot{\hat{x}}_j^i(t)=\hat{x}_{j+1}^i(t)+L_i^ja_j^i(y_i(t_{m_{0_1}}^i)-\hat{x}_1^i(t_{m_{0_1}}^i)),\\ \qquad j=1,\cdots,\lambda_i-1,\\ \dot{\hat{x}}_{\lambda_i}^i(t)=-[L_i^{\lambda_i}k_1^i\hat{x}_1^i(t_{m_{0_1}}^i)+L_i^{\lambda_i-1}k_2^i\hat{x}_2^i(t_{m_{0_1}}^i)+\\ \qquad \cdots+L_ik_{\lambda_i}^i\hat{x}_{\lambda_i}^i(t_{m_{0_1}}^i)]+\\ \qquad L_i^{\lambda_i}a_{\lambda_i}^i(y_i(t_{m_{0_1}}^i)-\hat{x}_1^i(t_{m_{0_1}}^i)),\\ \qquad t\in[t_{m_{0_1}}^i+\tau_{m_{0_1}}^i,t_0+\tau_0^i+d_0^i),\\ \qquad i=1,2,\cdots,m.\end{cases}\tag{26}$$

Definition 2 We say that the system (2) under the condition (4) is uniformly ultimately stabilizable, or the closed-loop system (2)(5)–(6) is uniformly ultimately bounded, if there exists an n dimensional dynamic system (5)–(6) and two positive constants $\delta, T'(e_0, \delta)$ such that

$$\|\hat{x}(t)\| \leqslant \delta,\ \|\hat{x}(t) - x(t)\| \leqslant \delta,\ \forall t > t_0 + T'(e_0, \delta), \quad (27)$$

where $e_0 = x_0 - \hat{x}_0 \in \mathbb{R}^n$. Moreover, if for any $x_0 \in \mathbb{R}^n$ and $\hat{x}_0 \in \mathbb{R}^n$, there exists an n dimensional dynamic system (5)–(6) and constants δ, $T'(e_0, \delta)$ such that (27) holds, then the system (2) under the condition (4) is globally uniformly ultimately stabilizable, or the closed-loop system (4)–(6) is globally uniformly ultimately bounded.

Let $\eta_i(t) = t - t_k^i$, $\gamma_i(t) = t - t_{\tilde{k}}^i$ and $\theta_i = x_1^i(t_{\tilde{k}}^i) - h_i(x_1^i(t_{\tilde{k}}^i))$, where $\tilde{k}$ is given by

$$\tilde{k} = \begin{cases} 0, & t \in [t_0, t_0 + \tau_0^i), \\ s, & t \in [t_s^i + \tau_s^i, t_{s+1}^i + \tau_{s+1}^i), \\ & s = 0, 1, \cdots, m_{0_1} - 1, \\ m_{0_1}, & t \in [t_{m_{0_1}}^i + \tau_{m_{0_1}}^i, t_0 + \tau_0^i + d_0^i), \\ k + m_{k_1}, & t \in [t_k^i + \tau_k^i + d_k^i, \\ & \quad t_{k+m_{k_1}+1}^i + \tau_{k+m_{k_1}+1}^i), \\ k + s, & t \in [t_{k+s}^i + \tau_{k+s}^i, t_{k+s+1}^i + \tau_{k+s+1}^i), \\ & s = m_{k_1} + 1, \cdots, m_{k_2} - 1, \\ k + m_{k_2}, & t \in [t_{k+m_{k_2}}^i + \tau_{k+m_{k_2}}^i, \\ & \quad t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i). \end{cases}$$

Then t_k^i and $t_{\tilde{k}}^i$ can be expressed by

$$t_k^i = t - \eta_i(t),\ t_{\tilde{k}}^i = t - \gamma_i(t), \quad (28)$$

where

$$\eta_i(t) = t - t_k^i \leqslant t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i - t_k^i \leqslant (\bar{m}_1 + \bar{m}_2 + 1)T^i,$$

$$\gamma_i(t) = t - t_{\tilde{k}}^i \leqslant t - t_k^i \leqslant (\bar{m}_1 + \bar{m}_2 + 1)T^i,\ |\theta_i| \leqslant c_1^i.$$

Denote $\varpi_i = (\bar{m}_1 + \bar{m}_2 + 1)T^i, i = 1, 2, \cdots, m$.

Consider a change of coordinates as follows: $\varepsilon_j^i = e_j^i / L^{\lambda_j^i - 1}$, $z_j^i = \hat{x}_j^i / L^{\lambda_j^i - 1}$, $1 \leqslant i \leqslant m, 1 \leqslant j \leqslant \lambda_i$, where $\lambda_j^i = \sum_{k=1}^{i-1} \lambda_k + j, (1 \leqslant i \leqslant m, 1 \leqslant j \leqslant \lambda_i)$. Then the error systems can be rewritten by

$$\begin{cases} \dot{\varepsilon}_j^i(t) = L_i \varepsilon_{j+1}^i(t) - L_i a_j^i \varepsilon_1^i(t) + L_i a_j^i \theta_i + \\ \qquad L_i a_j^i (\varepsilon_1^i(t) - \varepsilon_1^i(t - \gamma_i(t))) + \dfrac{b_j^i}{L_i^{\lambda_j^i - 1}}, \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{\varepsilon}_{\lambda_i}^i(t) = \\ -L_i a_{\lambda_i}^i \varepsilon_1^i(t) + L_i a_{\lambda_i}^i \theta_i + \\ L_i a_{\lambda_i}^i (\varepsilon_1^i(t) - \varepsilon_1^i(t - \gamma_i(t))) + \dfrac{b_{\lambda_i}^i}{L_i^{\lambda_{\lambda_i}^i - 1}} + \\ L_i[k_1^i z_1^i(t - \gamma_i(t)) + \cdots + k_{\lambda_i}^i z_{\lambda_i}^i(t - \gamma_i(t))] - \\ L_i[k_1^i z_1^i(t - \eta_i(t)) + \cdots + k_{\lambda_i}^i z_{\lambda_i}^i(t - \eta_i(t))], \\ \qquad i = 1, 2, \cdots, m,\ t \geqslant t_0, \end{cases} \quad (29)$$

and (26) can be written by

$$\begin{cases} \dot{z}_j^i(t) = L_i z_{j+1}^i(t) - L_i a_j^i \theta_i + L_i a_j^i \varepsilon_1^i(t) - \\ \qquad L_i a_j^i (\varepsilon_1^i(t) - \varepsilon_1^i(t - \gamma_i(t))), \\ \qquad j = 1, \cdots, \lambda_i - 1, \\ \dot{z}_{\lambda_i}^i(t) = -L_i a_{\lambda_i}^i \theta_i - L_i[k_1^i z_1^i(t - \gamma_i(t)) + \cdots + \\ \qquad k_{\lambda_i}^i z_{\lambda_i}^i(t - \gamma_i(t))] + L_i a_{\lambda_i}^i \varepsilon_1^i(t) - \\ \qquad L_i a_{\lambda_i}^i (\varepsilon_1^i(t) - \varepsilon_1^i(t - \gamma_i(t))), \\ \qquad i = 1, 2, \cdots, m,\ t \geqslant t_0. \end{cases} \quad (30)$$

Remark 3 Note that the systems (5) and (7) are updated based on the updated information $y_i(t_k^i)$ and $\hat{x}^i(t_k^i)$ on intervals $[t_k^i + \tau_k^i, t_{k+1}^i + \tau_{k+1}^i)$ and $[t_k^i + \tau_k^i + d_k^i, t_{k+1}^i + \tau_{k+1}^i + d_{k+1}^i)$, respectively. In order to facilitate stability analysis, the interval decomposition (8) is introduced to take the system (5) and the system (7) into consideration on the same interval. Firstly, based on the interval decomposition (8), we obtain three kinds of closed-loop system on three different time intervals. Then based on the transformations (28) and the change of coordinates, the unified closed-loop system (29)–(30) is derived, which is convenient for stability analysis.

Our main results can be stated as follows.

Theorem 1 Consider the system (2) with the condition (3) and the data truncation (4), there exists an output feedback control law of the form (5)–(6) with some suitable parameters L_i, a_j^i, and k_j^i, which globally uniformly ultimately stabilizes the system (2).

Proof Select $a_j^i > 0$ and $k_j^i > 0$ $(1 \leqslant i \leqslant m, 1 \leqslant j \leqslant \lambda_i)$ such that there exist two symmetric positive definite matrices P and Q such that

$$\bar{A}^{\mathrm{T}} P + P \bar{A} \leqslant -I, \quad (31)$$

$$\bar{B}^{\mathrm{T}} Q + Q \bar{B} \leqslant -2I, \quad (32)$$

where

$$\bar{A} = \mathrm{diag}\{\bar{A}_1, \bar{A}_2, \cdots, \bar{A}_m\},$$
$$\bar{B} = \mathrm{diag}\{\bar{B}_1, \bar{B}_2, \cdots, \bar{B}_m\},$$
$$P = \mathrm{diag}\{P_1, P_2, \cdots, P_m\},$$
$$Q = \mathrm{diag}\{Q_1, Q_2, \cdots, Q_m\},$$

$$\bar{A}_i = \begin{bmatrix} -a_1^i & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{\lambda_i - 1}^i & 0 & \cdots & 1 \\ -a_{\lambda_i}^i & 0 & \cdots & 0 \end{bmatrix},$$

$$\bar{B}_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -k_1^i & -k_2^i & \cdots & -k_{\lambda_i}^i \end{bmatrix}.$$

Let

$$\lambda_{m1}^i = \lambda_{\max}(P_i),\ \lambda_{m2}^i = \lambda_{\min}(P_i),$$
$$\lambda_{m3}^i = \lambda_{\max}(Q_i),\ \lambda_{m4}^i = \lambda_{\min}(Q_i),$$
$$\bar{a}_i = \max_{1 \leqslant j \leqslant \lambda_i} \{(a_j^i)^2\},\ \beta_i = 2\lambda_i \bar{a}_i (\lambda_{m3}^i)^2,$$

$$\bar{k}_i = \max_{1\leqslant j\leqslant\lambda_i}\{(k_j^i)^2\},$$
$$\varXi_1 = (3+2\lambda_1^i)(a_1^i)^2L_i^2+\lambda_i(2\lambda_i+4)L_i^2\bar{a}_i,$$
$$\varXi_2 = 8(\beta_i+1)L_i\lambda_i\bar{a}_i(\lambda_{m1}^i)^2+ 8L_i\lambda_i\bar{a}_i(\lambda_{m3}^i)^2+\alpha_iL_i,$$
$$\varXi_3 = \lambda_i(2\lambda_i+4)(\bar{k}_i+1)L_i^2,$$
$$\varXi_4 = 32(\beta_i+1)L_i\lambda_i\bar{k}_i(\lambda_{m1}^i)^2+ 8L_i\lambda_i\bar{k}_i(\lambda_{m3}^i)^2+\alpha_iL_i.$$

$P_{j,r}^i$ is the element of P_i at the jth line and rth column, $\bar{p}_2^i=\max\{|P_{j,r}^i|\}$, $1\leqslant i\leqslant m$, $1\leqslant j\leqslant\lambda_i$, $1\leqslant r\leqslant\lambda_i$.

Now, consider the following Lyapunov-Krasovskii function

$$V(t)=\sum_{i=1}^{4}V_i(t)= \sum_{i=1}^{m}(V_1^i(t)+V_2^i(t)+V_3^i(t)+V_4^i(t)), \tag{33}$$

where

$$V_1(t)=\sum_{i=1}^{m}V_1^i(t)=\sum_{i=1}^{m}(\beta_i+1)\varepsilon^i(t)^{\mathrm{T}}P_i\varepsilon^i(t),$$
$$V_2(t)=\sum_{i=1}^{m}V_2^i(t)=\sum_{i=1}^{m}z^i(t)^{\mathrm{T}}Q_iz^i(t),$$
$$V_3(t)=\sum_{i=1}^{m}V_3^i(t)=\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\int_{\rho}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s\mathrm{d}\rho,$$
$$V_4(t)=\sum_{i=1}^{m}V_4^i(t)=\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\int_{\rho}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s\mathrm{d}\rho,$$

and supplement definitions of $\varepsilon_1^i(t)$ and $z^i(t)$ are given as $\dot{\varepsilon}_1^i(t)=0$ and $\dot{z}^i(t)=0$, for $t\in[t_0-\varpi_i,t_0)$, $i=1,\cdots,m$.

$$\varepsilon(t)=[\varepsilon^1(t)^{\mathrm{T}}\ \cdots\ \varepsilon^m(t)^{\mathrm{T}}]^{\mathrm{T}},$$
$$z(t)=[z^1(t)^{\mathrm{T}}\ \cdots\ z^m(t)^{\mathrm{T}}]^{\mathrm{T}},$$
$$\varepsilon^i(t)=[\varepsilon_1^i(t)\ \cdots\ \varepsilon_{\lambda_i}^i(t)]^{\mathrm{T}},$$
$$z^i(t)=[z_1^i(t)\ \cdots\ z_{\lambda_i}^i(t)]^{\mathrm{T}}.$$

Then the derivative of $V_1(t)$ along the system (29) is given by

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}V_1(t)|_{(29)}=\\
&\sum_{i=1}^{m}(\beta_i+1)[\dot{\varepsilon}^i(t)^{\mathrm{T}}P_i\varepsilon^i(t)+\varepsilon^i(t)^{\mathrm{T}}P_i\dot{\varepsilon}^i(t)]\leqslant\\
&-\frac{5}{8}\sum_{i=1}^{m}(\beta_i+1)L_i\varepsilon^i(t)^{\mathrm{T}}\varepsilon^i(t)+\\
&2l_1\sum_{i=1}^{m}\sum_{r=1}^{\lambda_i}\sum_{j=1}^{\lambda_i}(\beta_i+1)|\varepsilon_j^i(t)P_{j,r}^i|\times\\
&(\sum_{a=1}^{\lambda_b}\sum_{b=1}^{i-1}(|\varepsilon_a^b|+|z_a^b|)+\sum_{k=1}^{j}(|\varepsilon_k^i|+|z_k^i|))+\\
&8\sum_{i=1}^{m}(\beta_i+1)L_i(a_1^i,a_2^i,\cdots,a_{\lambda_i}^i)\times\\
&(\varepsilon_1^i(t)-\varepsilon_1^i(t-\gamma_i(t)))P_iP_i\times\\
&(a_1^i,a_2^i,\cdots,a_{\lambda_i}^i)^{\mathrm{T}}(\varepsilon_1^i(t)-\varepsilon_1^i(t-\gamma_i(t)))+\\
&16\sum_{i=1}^{m}\sum_{j=1}^{\lambda_i}(\beta_i+1)L_i\lambda_i\bar{k}_i(\lambda_{m1}^i)^2\times\\
&((z_j^i(t)-z_j^i(t-\gamma_i(t)))^2+\\
&(z_j^i(t)-z_j^i(t-\eta_i(t)))^2)+\\
&8\sum_{i=1}^{m}(\beta_i+1)L_i(a_1^i,a_2^i,\cdots,a_{\lambda_i}^i)\theta_iP_i\times\\
&P_i(a_1^i,a_2^i,\cdots,a_{\lambda_i}^i)^{\mathrm{T}}\theta_i\leqslant\\
&-\sum_{i=1}^{m}(\frac{5}{8}L_i-2\lambda_il_1\bar{p}_2^i)(\beta_i+1)\varepsilon^i(t)^{\mathrm{T}}\varepsilon^i(t)+\\
&16\sum_{i=1}^{m}\sum_{j=1}^{\lambda_i}(\beta_i+1)L_i\lambda_i\bar{k}_i(\lambda_{m1}^i)^2\times\\
&((z_j^i(t)-z_j^i(t-\gamma_i(t)))^2+\\
&(z_j^i(t)-z_j^i(t-\eta_i(t)))^2)+\\
&2l_1\sum_{i=1}^{m}\lambda_i\bar{p}_2^i(\beta_i+1)z^i(t)^{\mathrm{T}}z^i(t))+\\
&8\sum_{i=1}^{m}(\beta_i+1)L_i\lambda_i\bar{a}_i(\lambda_{m1}^i)^2\times\\
&((\varepsilon_1^i(t)-\varepsilon_1^i(t-\gamma_i(t)))^2+(c_1^i)^2).
\end{aligned}\tag{34}$$

The derivative of $V_2(t)$ along the system (30) is given by

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}V_2(t)|_{(30)}=\\
&\sum_{i=1}^{m}[\dot{z}^i(t)^{\mathrm{T}}Q_iz^i(t)+z^i(t)^{\mathrm{T}}Q_i\dot{z}^i(t)]\leqslant\\
&-\frac{5}{8}\sum_{i=1}^{m}L_iz^i(t)^{\mathrm{T}}z^i(t)+\sum_{i=1}^{m}L_i\lambda_i\bar{a}_i(\lambda_{m3}^i)^2\varepsilon_1^i(t)^2+\\
&8\sum_{i=1}^{m}L_i\lambda_i\bar{a}_i(\lambda_{m3}^i)^2((\varepsilon_1^i(t)-\varepsilon_1^i(t-\gamma_i(t)))^2+(c_1^i)^2)+\\
&8\sum_{i=1}^{m}\sum_{j=1}^{\lambda_i}L_i\lambda_i\bar{k}_i(\lambda_{m3}^i)^2(z_j^i(t)-z_j^i(t-\gamma_i(t)))^2.
\end{aligned}\tag{35}$$

By using the Leibniz integration formula and Lemma 1, we have

$$\begin{cases}
|\varepsilon_1^i(t)-\varepsilon_1^i(t-\gamma_i(t))|^2=\\
|\int_{t-\gamma_i(t)}^{T}\dot{\varepsilon}_1^i(s)\mathrm{d}s|^2\leqslant\varpi_i\int_{t-\varpi_i}^{T}|\dot{\varepsilon}_1^i(s)|^2\mathrm{d}s,\\
|z_j^i(t)-z_j^i(t-\gamma_i(t))|^2=\\
|\int_{t-\gamma_i(t)}^{T}\dot{z}_j^i(s)\mathrm{d}s|^2\leqslant\varpi_i\int_{t-\varpi_i}^{T}|\dot{z}_j^i(s)|^2\mathrm{d}s,\\
|z_j^i(t)-z_j^i(t-\eta_i(t))|^2=\\
|\int_{t-\eta_i(t)}^{T}\dot{z}_j^i(s)\mathrm{d}s|^2\leqslant\varpi_i\int_{t-\varpi_i}^{T}|\dot{z}_j^i(s)|^2\mathrm{d}s.
\end{cases}\tag{36}$$

Choose L_i such that $L_i\geqslant\max\{1,l_1,16(\beta_i+1)\times\lambda_il_1\bar{p}_2^i,2(\beta_i+1)\lambda_{m1}^i,2\lambda_{m3}^i\}$. It follows from (34)–(36) that

$$\frac{\mathrm{d}}{\mathrm{d}t}V_1(t)|_{(29)}+\frac{\mathrm{d}}{\mathrm{d}t}V_2(t)|_{(30)}\leqslant$$

$$-\frac{1}{2}\sum_{i=1}^{m}L_i\varepsilon^i(t)^{\mathrm{T}}\varepsilon^i(t)-\frac{1}{2}\sum_{i=1}^{m}L_iz^i(t)^{\mathrm{T}}z^i(t)+$$
$$8\sum_{i=1}^{m}(\beta_i+1)L_i\lambda_i\bar{a}_i(\lambda_{m1}^i)^2\varpi_i\int_{t-\varpi_i}^{T}|\dot{\varepsilon}_1^i(s)|^2\mathrm{d}s+$$
$$8\sum_{i=1}^{m}L_i\lambda_i\bar{a}_i(\lambda_{m3}^i)^2\varpi_i\int_{t-\varpi_i}^{T}|\dot{\varepsilon}_1^i(s)|^2\mathrm{d}s+$$
$$8\sum_{i=1}^{m}L_i\lambda_i\bar{a}_i((\beta_i+1)(\lambda_{m1}^i)^2+(\lambda_{m3}^i)^2)(c_1^i)^2+$$
$$8\sum_{i=1}^{m}L_i\lambda_i\bar{k}_i(\lambda_{m3}^i)^2\varpi_i\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s+$$
$$32\sum_{i=1}^{m}(\beta_i+1)L_i\lambda_i\bar{k}_i(\lambda_{m1}^i)^2\varpi_i\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s. \tag{37}$$

The derivatives of $V_3(t)$ and $V_4(t)$ are given by

$$\frac{\mathrm{d}}{\mathrm{d}t}V_3(t)=$$
$$\sum_{i=1}^{m}\varpi_i\dot{\varepsilon}_1^i(t)^2-\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s\leqslant$$
$$\sum_{i=1}^{m}\varpi_i(3+2\lambda_1^i)(1+(a_1^i)^2)L_i^2\varepsilon^i(t)^{\mathrm{T}}\varepsilon^i(t)+$$
$$\sum_{i=1}^{m}\varpi_i(3+2\lambda_1^i)L_i^2z^i(t)^{\mathrm{T}}z^i(t)+$$
$$\sum_{i=1}^{m}\varpi_i(3+2\lambda_1^i)(a_1^i)^2L_i^2(c_1^i)^2+$$
$$\sum_{i=1}^{m}\varpi_i^2(3+2\lambda_1^i)(a_1^i)^2L_i^2\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s-$$
$$\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s, \tag{38}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}V_4(t)=$$
$$\sum_{i=1}^{m}\varpi_i\dot{z}^i(t)^{\mathrm{T}}\dot{z}^i(t)-\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s\leqslant$$
$$\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)(\bar{k}_i+1)\varpi_iL_i^2z^i(t)^{\mathrm{T}}z^i(t)-$$
$$\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s+\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)\varpi_iL_i^2\bar{a}_i\times$$
$$\varepsilon^i(t)^{\mathrm{T}}\varepsilon^i(t)+\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)L_i^2\bar{a}_i\varpi_i^2\times$$
$$\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s+\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)\bar{a}_iL_i^2\varpi_i(c_1^i)^2+$$
$$\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)(\bar{k}_i+1)L_i^2\varpi_i^2\times$$
$$\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s. \tag{39}$$

We also have

$$V_3(t)\leqslant\sum_{i=1}^{m}\varpi_i\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s,$$
$$V_4(t)\leqslant\sum_{i=1}^{m}\varpi_i\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s.$$

Then from (37)–(39), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t)|_{(29),(30)}\leqslant$$
$$-\sum_{i=1}^{m}(\frac{1}{2}-\varpi_i(3+2\lambda_1^i)(1+(a_1^i)^2)L_i-$$
$$\lambda_i(2\lambda_i+4)\varpi_iL_i\bar{a}_i)L_i\varepsilon^i(t)^{\mathrm{T}}\varepsilon^i(t)-$$
$$\sum_{i=1}^{m}(\frac{1}{2}-\varpi_i(3+2\lambda_1^i)L_i-$$
$$\lambda_i(2\lambda_i+4)(\bar{k}_i+1)\varpi_iL_i)L_iz^i(t)^{\mathrm{T}}z^i(t)+$$
$$8\sum_{i=1}^{m}(\beta_i+1)L_i\lambda_i\bar{a}_i(\lambda_{m1}^i)^2\varpi_i\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s+$$
$$\sum_{i=1}^{m}\varpi_i^2(3+2\lambda_1^i)(a_1^i)^2L_i^2\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s+$$
$$8\sum_{i=1}^{m}L_i\lambda_i\bar{a}_i(\lambda_{m3}^i)^2\varpi_i\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s+$$
$$\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)L_i^2\bar{a}_i\varpi_i^2\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s+$$
$$32\sum_{i=1}^{m}(\beta_i+1)L_i\lambda_i\bar{k}_i(\lambda_{m1}^i)^2\varpi_i\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s+$$
$$8\sum_{i=1}^{m}L_i\lambda_i\bar{a}_i((\beta_i+1)(\lambda_{m1}^i)^2+(\lambda_{m3}^i)^2)(c_1^i)^2-$$
$$\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\dot{\varepsilon}_1^i(s)^2\mathrm{d}s+\sum_{i=1}^{m}\varpi_i(3+2\lambda_1^i)(a_1^i)^2\times$$
$$L_i^2(c_1^i)^2-\sum_{i=1}^{m}\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s+$$
$$\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)\bar{a}_iL_i^2\varpi_i(c_1^i)^2+$$
$$8\sum_{i=1}^{m}L_i\lambda_i\bar{k}_i(\lambda_{m3}^i)^2\varpi_i\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s+$$
$$\sum_{i=1}^{m}\lambda_i(2\lambda_i+4)(\bar{k}_i+1)L_i^2\varpi_i^2\int_{t-\varpi_i}^{T}\dot{z}^i(s)^{\mathrm{T}}\dot{z}^i(s)\mathrm{d}s. \tag{40}$$

If the bounds T^i satisfy

$$T^i<\frac{1}{\bar{m}_1+\bar{m}_2+1}\times$$
$$\min\{\frac{1}{8\lambda_i(\lambda_i+4)(\bar{k}_i+1)L_i+4(3+2\lambda_1^i)L_i},$$
$$\frac{1}{8\lambda_i(\lambda_i+4)\bar{a}_iL_i+4(3+2\lambda_1^i)(1+(a_1^i)^2)L_i},$$
$$\frac{\sqrt{\varXi_2^2+4\varXi_1}-\varXi_2}{2\varXi_1},\frac{\sqrt{\varXi_4^2+4\varXi_3}-\varXi_4}{2\varXi_3}\}. \tag{41}$$

It follows from (40) that

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t)|_{(29),(30)}\leqslant-\sum_{i=1}^{m}(\alpha_iL_iV^i(t)+L_i\xi_i), \tag{42}$$

where

$$\alpha_i=\min\{\frac{1}{4(\beta_i+1)\lambda_{m1}^i},\frac{1}{4\lambda_{m3}^i}\},$$
$$\xi_i=8\lambda_i\bar{a}_i((\beta_i+1)(\lambda_{m1}^i)^2+(\lambda_{m3}^i)^2)(c_1^i)^2+$$
$$\varpi_i(3+2\lambda_1^i)(a_1^i)^2L_i(c_1^i)^2+$$

$$\lambda_i(2\lambda_i+4)\bar{a}_i L_i \varpi_i (c_1^i)^2.$$

Let $\mathcal{V}_1(c_1)=\{(\varepsilon(t),z(t)):V(t)\leqslant 2\sum\limits_{i=1}^{m}\dfrac{\xi_i}{\alpha_i}\}$, where $c_1=(c_1^1,c_2^1,\cdots,c_m^1)^{\mathrm{T}}$. When $(\varepsilon(t),z(t))\in\mathbb{R}^n\times\mathbb{R}^n/\mathcal{V}_1(c_1)$, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V(t)\leqslant-\frac{1}{2}\sum_{i=1}^{m}\alpha_i L_i V^i(t),\ t\geqslant t_0. \tag{43}$$

Thus,

$$V(\varepsilon(t),z(t))\leqslant\sum_{i=1}^{m}e^{-\alpha_i L_i(t-t_0)/2}V^i(t_0)\leqslant$$
$$e^{-\alpha Lt/2}\sum_{i=1}^{m}e^{\alpha_i L_i/2(t_0)}V^i(t_0), \tag{44}$$

where $L=\min\limits_{1\leqslant i\leqslant m}\{L_i\},\ \alpha=\min\limits_{1\leqslant i\leqslant m}\{\alpha_i\}$. Let

$$\pi_1=2\sum_{i=1}^{m}\frac{\xi_i}{\alpha_i},\ \pi_2=\sum_{i=1}^{m}e^{\alpha_i L_i t_0/2}V^i(t_0),$$
$$T'=4(\ln\pi_2-\ln 4\pi_1).$$

For any $(x_0,\hat{x}_0)\in\mathbb{R}^n\times\mathbb{R}^n/\mathcal{V}_1(c_1)$, when $t>T'$, we have $V(\varepsilon(t),z(t))<4\pi_1$, or $(\varepsilon(t),z(t))\in\mathcal{V}_1(2c_1)$. Thus, it is easy to find that there exist a constant $\delta_1(\pi_1)>0$ such that $||\hat{x}(t)-x(t)||\leqslant\delta_1$ and $||\hat{x}(t)||\leqslant\delta_1$. On the other hand, if $(x_0,\hat{x}_0)\in\mathcal{V}_1(c_1)$, $||\hat{x}(t)-x(t)||\leqslant\delta_1$ and $||\hat{x}(t)||\leqslant\delta_1$ hold all the time. Therefore, from Definition 1, the closed-loop systems (2)(5)–(6) is globally uniformly ultimately bounded. The proof is completed.

Remark 4 If $m=1$, the multi-output system (1) is a single output system. For the closed-loop systems (1) and (5)–(6), if $\tau_k=0$, some similar results on output feedback stabilization have been obtained without considering transmission delay in [19]. If $\tau_k<T$, T denotes the sampling period, sample-data observer design and sample-data output feedback stabilization have been studied for the SISO systems in [18,24] and [20], respectively. In [21], the authors investigated sample-data output feedback stabilization for the SISO systems. The time delay may be larger than the sampling period. But, the sampled and delayed outputs aren't applied to update the stabilizer immediately even if they are available. Whereas, in this paper, although the transmission delays are unknown, the time instants that the sampled data arrive are known. Once they are available, they are used to design the stabilizer immediately.

4 Numerical simulations

In this section, we use an example to show the effectiveness of our proposed methods. Consider a multi-output nonlinear system in the form of (2):

$$\dot{x}_1(t)=x_2(t)+lx_1(t)\cos t,\ \dot{x}_2(t)=u_1(t),$$
$$\dot{x}_3(t)=x_4(t)+l(x_2(t)+x_3(t))\sin t,$$
$$\dot{x}_4(t)=u_2(t),\ \tilde{y}_1(t)=x_1(t),\ \tilde{y}_2(t)=x_3(t),$$

with $x(0)=(0.55,-0.54,0.7,0.5)$. It is easy to observe that this nonlinear system consists of two blocks, the first one $(\tilde{y}_1)$ consists of x_1 and x_2, the second one $(\tilde{y}_2)$ consists of the rest. From (5)–(6), an output feedback stabilizer can be constructed by

$$\dot{\hat{x}}_1(t)=\hat{x}_2(t)+a_1^1L_1(y_1(t_k^1)-\hat{x}_1(t_k^1)),$$
$$\dot{\hat{x}}_2(t)=u_1+a_2^1L_1^2(y_1(t_k^1)-\hat{x}_1(t_k^1)),$$
$$u_1=-[L_1^2k_1^1\hat{x}_1(t_k^1)+L_1k_2^1\hat{x}_2(t_k^1)],$$
$$t\in[t_k^1+\tau_k^1,t_{k+1}^1+\tau_{k+1}^1),\ k\geqslant 0,$$
$$\dot{\hat{x}}_3(t)=\hat{x}_4(t)+a_1^2L_2(y_2(t_k^2)-\hat{x}_3(t_k^2)),$$
$$\dot{\hat{x}}_4(t)=u_2+a_2^2L_2^2(y_2(t_k^2)-\hat{x}_3(t_k^2)),$$
$$u_2=-[L_2^2k_1^2\hat{x}_3(t_k^2)+L_2k_2^2\hat{x}_4(t_k^2)],$$
$$t\in[t_k^2+\tau_k^2,t_{k+1}^2+\tau_{k+1}^2),\ k\geqslant 0,$$

with $\hat{x}(0)=(-0.1,0.3,-0.4,0.2)$ and the parameters are given as $(a_1^1,a_2^1,a_1^2,a_2^2)=(0.3,0.3,0.2,0.2)$, $(k_1^1,k_2^1,k_1^2,k_2^2)=(2,3,2,1)$ such that matrices $\bar{A}$ and $\bar{B}$ are Hurwitz. Figs. 2–3 show the simulation results.

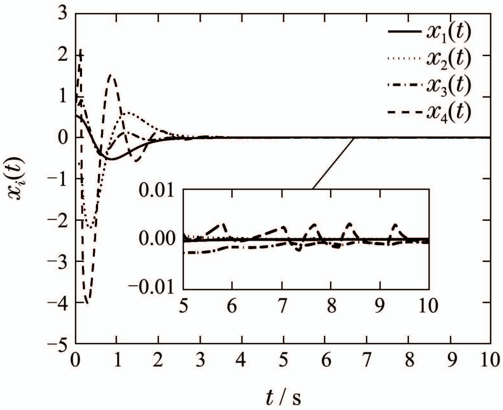

Fig. 2 Trajectories of the states $x_i(t)$ $(1\leqslant i\leqslant 4)$ with $T^1=T^2=0.01$ s

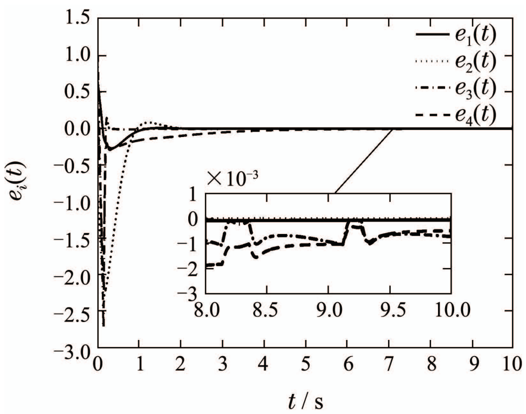

Fig. 3 Trajectories of the errors $e_i(t)$ $(1\leqslant i\leqslant 4)$ with $T^1=T^2=0.01$ s

In the simulation, we set the sampling period $T^1=T^2=0.01$ s. τ_k^1 and d_k^1 denote the unknown transmission delays and are simulated by random numbers in the intervals $[0,1.5T^1]$ and $[0,1.7T^1]$, respectively. τ_k^2 and d_k^2 are simulated by random numbers in the intervals $[0,1.6T^2]$ and $[0,1.9T^2]$, respectively. Then the upper bounds $\bar{\tau}_1=1.5T^1>T^1$, $\bar{d}_1=1.7T^1>T^1$, $\bar{\tau}_2=1.6T^2>T^2$, $\bar{d}_2=1.9T^2>T^2$, whereas, it is required that the maximum delay is strictly less than the the sampling period, that is, $\bar{\tau}<T$ in [17–18, 20, 24, 27]. The

other parameters are selected as: $l = 0.1$, $L_1 = 2$, $L_2 = 4$, $c_1^1 = 10^{-4}$ and $c_1^2 = 10^{-3}$. For simplicity, the following algorithms are used to truncate the sampled data, $h_1(x) = [10^4 x]/10^4$ and $h_2(x) = [10^3 x]/10^3$, where $[x]$ denotes the integer part of x.

5 Conclusions

In this paper, we addressed the output feedback stabilization for a class of multi-output nonlinear systems under static data truncation protocol. The system outputs were sampled in a fully asynchronous way, transmitted through communication network, and used to construct the output feedback stabilizer immediately when they were available. The nonlinear system could be updated by the system inputs once they were available. There also existed two kinds of transmission delays, i.e., transmission delays from the nonlinear system to the output feedback stabilizer, and transmission delays from the output feedback stabilizer to the nonlinear system. Therefore, the output feedback stabilizer and the nonlinear system were updated at different time instants. Based on interval decomposition, we put them on the same interval and obtain the closed-loop system. Sufficient condition was presented to guarantee that the closed-loop system was globally uniformly ultimately bounded. Numerical simulation was provided to illustrate the efficiency of the proposed methods.

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