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随机时滞网络控制系统的后退时域估计

李超超, 韩春艳†, 何 芳
(济南大学 自动化与电气工程学院, 山东 济南 250022)

摘要: 本文考虑具有随机观测时滞系统的后退时域估计问题. 首先, 针对随机时滞网络控制系统, 运用观测重组技术, 将带有时滞的观测方程转化为无时滞观测方程, 得到一组新的无时滞观测序列. 在此基础上, 运用线性最小方差无偏估计理论, 推导出后退时域估计器的批形式公式和迭代形式公式, 并给出稳定性分析. 通过具体的仿真实例, 对比现有卡尔曼滤波器, 验证了所提出的后退时域估计器具有更好的跟踪能力.

关键词: 后退时域估计; 稳定性分析; 随机时滞观测; 观测重组
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Receding horizon estimation for networked control systems with random transmission delays

LI Chao-chao, HAN Chun-yan†, HE Fang
(School of Electrical Engineering, University of Jinan, Jinan Shandong 250022, China)

Abstract: This paper is concerned with the receding horizon estimation problem for discrete -time systems with random delayed observations. Firstly, the random delay system is reconstructed as an equivalent delay-free one by measurement reorganization technique. Secondly, a batch form and a recursive form for receding horizon estimation are proposed on the basis of the new changed system and by minimizing a new cost function that includes two terminal weighting terms. Then based on the derived condition, the stability of the proposed receding horizon estimation is proved. Finally, a numerical example is given for illustration.

Key words: receding horizon estimation; stability analysis; random delayed observations; measurement reorganization
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1 Introduction

Recently, significant attention has been paid to networked control systems (NCSs) as they bring numerous benefits, such as reduced system wiring, lower cost in maintenance, increased system agility, ease of information sharing, etc. Along with the advantages, several challenging problems, such as bandwidth allocation, communication delays and packet dropouts, also emerged giving rise to many important research topics[1–4]. Transmission delay is now well known to be one of the most often occurred phenomena in NCSs, which may result in deterioration of system performance and even instability. Therefore, it is of great significance to study NCSs with transmission delays where the packet dropout incorporates naturally.

There is no doubt that state estimation is an important topic in both theoretical research and practical applications. In the past decade, a substantial body of literature has been devoted to state estimation for systems with transmission delays. There existed several techniques for dealing with time delay, such as the classical state augmentation method[5], the linear matrix inequality algorithm[6], the polynomial approach[7], and the reorganization innovation analysis method[8].

The transmission delay in NCSs may vary with time and is often modeled as a random process. Two stochastic processes: the Bernoulli process[9–13] and the Markov process[14–15], are commonly used to describe the characteristics of the random delays. In [10], the recursive estimation for linear and nonlinear systems with uncertain observations were considered. A binary switching sequence-the Bernoulli distribution process,

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†Corresponding author. E-mail: cyhan823@hotmail.com.
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was used to describe the uncertainty in the observations. An estimator was obtained by the covariance information method. Similar result was also given in [11]. In [14], the state estimation with missing measurements was considered, where the missing process was modeled as a Markov chain. A jump linear estimator was introduced to cope with the losses. Further in [12], an optimal filter problem with random delay and packet dropouts was studied, where the random received observations were stored in a possibly infinite-length buffer. In [13], the optimal and suboptimal linear estimators were designed for NCSs with random observation delays, where the random delay was modeled as a set of Bernoulli variables. The measurement reorganization method was employed for treating delay terms. In addition, the Markov type transmission delay was considered in [15] and three different types of filters were designed without state augmentation.

On the other hand, receding horizon estimation, also called moving horizon estimation, has become as an important research topic and gained much attention[16–19] in recent years. It explains the concept of full information estimation and introduces the moving horizon estimation as a computable approximation of full information. The basic design method for ensuring stability of moving horizon estimation was presented in [16]. Further, the moving horizon estimation algorithm was applied to the field of distributed estimation in [17–18]. In this paper, we will combine the receding horizon estimation algorithm and the observation reorganization technique to derive the estimator of the systems with random time delays, which reduce the calculation complexity for the design process.

Based on the aforementioned literature, we investigate the receding horizon estimation for discrete-time linear system with random observation delays. A set of Bernoulli variables are introduced to describe the characteristics of the random delay, and the measurement reorganization technique is employed for dealing with the delay terms. On the basis of the new system model without time-delay, both batch form and iterative form receding horizon estimation are derived afterward without state augmentation, and the stability analysis is supplied.

The contribution of this paper can be stated as: i) Compared with the Kalman-type estimator developed in [13], the receding horizon estimator developed in this paper, since based on a finite number of system measurements, can make more flexibility to tune weighting parameters and provide a higher estimator precision. The comparison has been shown in Section 4; ii) The Hadamard product is introduced in the derivation of the receding horizon estimator gains. This is the main difference between the receding horizon estimation developed in this paper and the Kalman-type estimator developed in [13]; iii) In the derivation of estimator gains, it is difficult to solve a global optimization problem. Then the decomposition method is employed, by which the receding horizon estimation subject to unbiasedness constraint is divided into N individual optimization problems. The independent optimization problem is solved by the optimality principle, and the individual estimation gains are obtained. This is one of the technique contribution of this paper.

The remainder of this paper is organized as follows. Problem description is given in Section 2. Section 3 mainly concerns with the design of the receding horizon estimation and the stability analysis of the proposed method. In Section 4, a simulation example is presented to illustrate the estimator's performance. Finally, conclusions are drawn in Section 5.

Notation: Throughout this paper, the superscripts $^{-1}$ and $^{\mathrm{T}}$ represent the inverse and transpose of the matrix. $\mathbb{R}^n$ represents the n-dimensional Euclidean space. Moreover, $\mathrm{E}\{\cdot\}$ means the mathematical expectation, $\odot$ is the Hadamard product, $\mathrm{col}\{\cdot\}$ indicates the column vector, $\mathrm{tr}\{\cdot\}$ means the trace of a matrix and $\mathrm{P}\{\cdot\}$ represents the occurrence probability of an event.

2 Problem description

Consider the following discrete-time linear system with random delay:

$$x(t+1) = Ax(t) + Cw(t), x(0) = x_0; \quad (1)$$

$$y_r(t) = Hx(t-r(t)) + v(t-r(t)), \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{R}^p$ is the input noise, $y_r(t) \in \mathbb{R}^n$ is the measurement and $v(t) \in \mathbb{R}^n$ is the measurement noise. Through the paper, it is assumed that the constant matrices A, C, H are known, $[C, A]$ is observable, A is nonsingular, and $r(t)$ means the random delay.

Assumption 1 $w(t)$ and $v(t)$ are white noises with covariance matrices $\mathrm{E}\{w(t)w^{\mathrm{T}}(s)\} = Q_w\delta_{ts}$, $\mathrm{E}\{v(t)v^{\mathrm{T}}(s)\} = R_v\delta_{ts}$, respectively. x_0, $w(t)$, and $v(t)$ are mutually independent.

Assumption 2 Measurements in (2) are time-stamped. As is well known, time-stamping of measurement information is necessary to reorder packets at the receiver side because there exist random delays in communication. The random delay $r(t)$ is bounded with $0 \leqslant r(t) \leqslant r$, where r is known as the length of memory buffer. If the received measurement is with a delay larger than r, it will be viewed as the lost packet. The probability distribution of $r(t)$ is $\mathrm{P}(r(t) = i) = \rho_i$, $i = 0, \cdots, r$. Obviously, $0 \leqslant \sum_{i=0}^{r} \rho_i \leqslant 1$. We assume that $r(t)$ is independent of x_0, $w(t)$, and $v(t)$.

Since formula (2) contains random delays which can't be treated directly by the reorganized observation technique, the original system needs to be transformed into a constant delay one first. Based on the above assumption, denote

$$y(t)=\begin{cases}\mathrm{col}\{y_0(t),\cdots,y_t(t),0,\cdots,0\}, & 0\leqslant t<r;\\ \mathrm{col}\{y_0(t),\cdots,y_r(t)\}, & t\geqslant r,\end{cases}$$

where

$$y_i(t)=\alpha_{i,t}Hx(t-i)+\alpha_{i,t}v(t-i),\\ i=0,1,\cdots,r \quad (3)$$

with $\alpha_{i,t}$ defined as a binary random variable indicating the arrival of the observation packet for state $x(t-i)$ at time t, that is

$$\alpha_{i,t}=\begin{cases}1, & \text{If the observation for the state } x(t-i)\\ & \text{is received with a delay } i \text{ at time } t;\\ 0, & \text{otherwise.}\end{cases} \quad (4)$$

Then $\alpha_{i,t}(i=0,1,\cdots,r)$ has the same stochastic probability as that of $r(t)$. That means $\mathrm{P}(\alpha_{i,t}=1)=\rho_i(i=0,1,\cdots,r)$, where $\rho_i(i=0,1,\cdots,r)$ is known. In the real-time control systems, the state $x(t)$ can only be observed at most one time, and thus the following assumption needs to be made.

Assumption 3 The stochastic variable $\alpha_{i,t}(i=0,1,\cdots,r)$ has the following property

$$\alpha_{i,t+i}\times\alpha_{j,t+j}=0,\ i\neq j.$$

Then the optimal filtering problem considered in this paper can be stated as follows:

Problem 1 (Optimal receding horizon estimation) Given the observation $\{y(s)|_{0\leqslant s\leqslant t}\}$, find a linear minimum mean square error receding horizon estimator $\hat{x}(t)$ of the state $x(t)$ with the finite horizon N, such that $E_{w,v}[\hat{x}(t)]=E_{w,v}[x(t)]$.

3 Construction of the receding horizon estimation

In this section, the random delayed system is transformed into a delay-free one by the reorganization observation method used for dealing with the random delay. Then, we will propose a new receding horizon estimator with deterministic gains by minimizing the mean square estimation error.

3.1 Observation reorganization

Because the state estimation for time-delay systems cannot be deduced directly, it needs to be transformed into a delay-free one by the reorganization observation method.

For the given time t, the received observations can be rearranged into a set of delay-free sequences as follows.

For $0\leqslant s\leqslant t-r$, define

$$\bar{y}_r(s)\triangleq\begin{bmatrix}y_0(s)\\ \vdots\\ y_r(s+r)\end{bmatrix}=\begin{bmatrix}\alpha_{0,s}H\\ \vdots\\ \alpha_{r,s+r}H\end{bmatrix}x(s)+\begin{bmatrix}\alpha_{0,s}v(s)\\ \vdots\\ \alpha_{r,s+r}v(s)\end{bmatrix}=\\ H_r(s)x(s)+v_r(s). \quad (5)$$

For $t-r<s\leqslant t$, define

$$\bar{y}_{t-s}(s)\triangleq\begin{bmatrix}y_0(s)\\ \vdots\\ y_{t-s}(t)\end{bmatrix}=\begin{bmatrix}\alpha_{0,s}H\\ \vdots\\ \alpha_{t-s,t}H\end{bmatrix}x(s)+\begin{bmatrix}\alpha_{0,s}v(s)\\ \vdots\\ \alpha_{t-s,t}v(s)\end{bmatrix}=\\ H_{t-s}(s)x(s)+v_{t-s}(s). \quad (6)$$

In addition, the covariance matrices of $v_r(s)$ and $v_{t-s}(s)$ are described as follows:

$$R_r=\mathrm{diag}\{\rho_0R_v,\cdots,\rho_rR_v\},\\ R_{t-s}=\mathrm{diag}\{\rho_0R_v,\cdots,\rho_{t-s}R_v\}.$$

For convenience, denote

$$E[H_r(s)]=\begin{bmatrix}\rho_0H\\ \vdots\\ \rho_rH\end{bmatrix}=H_r,$$

$$E[H_{t-s}(s)]=\begin{bmatrix}\rho_0H\\ \vdots\\ \rho_{t-s}H\end{bmatrix}=H_{t-s}.$$

3.2 Receding horizon estimator

The problem considered here is how to acquire a receding horizon estimate $\hat{x}(s|s-1)$ of the state vector $x(s)$ by using a finite number of measurements of the system output $\bar{y}(s)$ with weighted matrix. And two forms of receding horizon estimation are derived from the following two theorems.

In order to simplify the calculation, let us define in Step 1 as

$$Y_r(s-1)\triangleq\begin{bmatrix}\bar{y}_r(s-N)\\ \vdots\\ \bar{y}_r(s-1)\end{bmatrix},$$

$$\bar{H}_{r,N}(s-1)\triangleq\begin{bmatrix}H_r(s-N)A^{-N}\\ \vdots\\ H_r(s-1)A^{-1}\end{bmatrix},$$

$$\tilde{A}_{r,l}\triangleq\begin{bmatrix}HA^{-l}\\ \vdots\\ HA^{-l}\end{bmatrix},\ \bar{\rho}_r\triangleq\begin{bmatrix}\rho_0I & & \\ & \ddots & \\ & & \rho_rI\end{bmatrix},$$

$$\bar{\varepsilon}_{r,N}\triangleq\begin{bmatrix}\varepsilon_{r,N} & & \\ & \ddots & \\ & & \varepsilon_{r,1}\end{bmatrix},\ \bar{\beta}_{r,N}\triangleq\begin{bmatrix}\beta_{r,N} & & \\ & \ddots & \\ & & \beta_{r,1}\end{bmatrix},$$

$$\tilde{C}_{r,l} \triangleq \left[\tilde{H}_{r,1} \cdots \tilde{H}_{r,l}\right], \quad \tilde{H}_{r,l} \triangleq \begin{bmatrix} HA^{-l}C \\ \vdots \\ HA^{-l}C \end{bmatrix},$$

$$\bar{R}_{r,N} \triangleq \begin{bmatrix} R_r & & \\ & \ddots & \\ & & R_r \end{bmatrix},$$

$$\varepsilon_{r,l} \triangleq \bar{\rho}_r \odot (\tilde{A}_{r,l} X(s) \tilde{A}_{r,l}^{\mathrm{T}}),$$

$$\beta_{r,l} \triangleq \bar{\rho}_r \odot (\tilde{C}_{r,l} Q_l \tilde{C}_{r,l}^{\mathrm{T}}),$$

$$Q_l \triangleq \begin{bmatrix} Q_w & & \\ & \ddots & \\ & & Q_w \end{bmatrix}_{l\times l},$$

$$X(s) \triangleq \mathrm{E}[x(s)x^{\mathrm{T}}(s)],$$

where $\odot$ means Hadamard product and $X(s)$ satisfies

$$X(s) = AX(s-1)A^{\mathrm{T}} + CQ_wC^{\mathrm{T}}.$$

It is noted that some definitions of the algorithm for Step 2 are similar to those definitions above, which just need to replace the subscript r with $t-s$, and thus is omitted here.

For the given time t, we now develop a batch form receding horizon estimator $\hat{x}(t)$ in the following algorithm.

Algorithm 1 (Batch form receding horizon estimator)

Step 1 For $0 \leqslant s \leqslant t-r$, a receding horizon estimator $\hat{x}(s|s-1)$ is calculated by

$$\hat{x}(s|s-1) = F_r(s)Y_r(s-1), \tag{7}$$

where the optimal gain matrix $F_r(s)$ is determined by

$$F_r(s) = (\bar{H}_{r,N}^{\mathrm{T}} \varphi_{r,N}^{-1} \bar{H}_{r,N})^{-1} \bar{H}_{r,N}^{\mathrm{T}} \varphi_{r,N}^{-1}$$

with

$$\varphi_{r,N} = \bar{\varepsilon}_{r,N} + \bar{\beta}_{r,N} + \bar{R}_{r,N},$$

$$\bar{H}_{r,N} = \begin{bmatrix} H_rA^{-N} \\ \vdots \\ H_rA^{-1} \end{bmatrix}.$$

Step 2 For $t-r < s \leqslant t$, a receding horizon estimator $\hat{x}(s|s-1)$ is calculated by

$$\hat{x}(s|s-1) \triangleq F_{t-s}Y_{t-s}(s-1), \tag{8}$$

where the optimal gain matrix $F_{t-s}(s)$ is determined by

$$F_{t-s}(s) = (\bar{H}_{t-s,N}^{\mathrm{T}} \varphi_{t-s,N}^{-1} \bar{H}_{t-s,N})^{-1} \bar{H}_{t-s,N}^{\mathrm{T}} \varphi_{t-s,N}^{-1}$$

with

$$\varphi_{t-s,N} = \bar{\varepsilon}_{t-s,N} + \bar{\beta}_{t-s,N} + \bar{R}_{t-s,N},$$

$$\bar{H}_{t-s,N} = \begin{bmatrix} H_{t-s}A^{-N} \\ \vdots \\ H_{t-s}A^{-1} \end{bmatrix}.$$

Step 3 For $s = t$, set $\hat{x}(t) = \hat{x}(t|t-1)$ in Step 2.

In the following theorem, we will show that the estimator developed in Step 1–3 is the optimal solution to Problem 1.

Theorem 1 For systems (1) (4) and (5), when (C, A) is observable, the linear minimum mean square error receding horizon estimator $\hat{x}(t)$ with a batch form on the horizon $[t-N, t]$ can be derived by Algorithm 1, which satisfies the unbiased constraints.

Proof For $0 \leqslant s \leqslant t-r$, the finite number of measurements on the horizon $[s-N, s]$ can be expressed in terms of the state $x(s)$,

$$Y_r(s-1) = \bar{H}_{r,N}(s-1)x(s) - \bar{C}_{r,N}(s-1)W(s-1) + \bar{V}_r(s-1), \tag{9}$$

where

$$\bar{C}_{r,N}(s-1) = \begin{bmatrix} H_r(s-N)A^{-1}C & \cdots & H_r(s-N)A^{-N}C \\ & \ddots & \vdots \\ & & H_r(s-1)A^{-1}C \end{bmatrix},$$

$$W(s-1) = \begin{bmatrix} w(s-N) \\ \vdots \\ w(s-1) \end{bmatrix},$$

$$\bar{V}_r(s-1) = \begin{bmatrix} v_r(s-N) \\ \vdots \\ v_r(s-1) \end{bmatrix}.$$

$\hat{x}(s|s-1)$ can be indicated as a linear function of the finite measurements $Y_r(s-1)$ on the horizon $[s-N, s]$ as follows:

$$\hat{x}(s|s-1) = F_r(s)Y_r(s-1) = F_r(s)(\bar{H}_{r,N}(s-1)x(s) - \bar{C}_{r,N}(s-1)\times W(s-1) + \bar{V}_r(s-1)). \tag{10}$$

Taking expectation on both sides of (10), and to satisfy the unbiased condition, $\mathrm{E}\hat{x} = \mathrm{E}x$, the following relation is obtained

$$F_r(s)\bar{H}_{r,N} = I. \tag{11}$$

Based on the definition of estimation error, denote

$$\tilde{x}(s|s-1) = x(s) - \hat{x}(s|s-1) = [I - F_r(s)\bar{H}_{r,N}(s-1)]x(s) + F_r(s)\times [\bar{C}_{r,N}(s-1)W(s-1) - \bar{V}_r(s-1)]. \tag{12}$$

So, we can obtain the covariance of estimation error $\tilde{x}(s|s-1)$ as follows:

$$\mathrm{E}\{\tilde{x}(s|s-1)\tilde{x}^{\mathrm{T}}(s|s-1)\} = \mathrm{E}\{[I - F_r(s)\bar{H}_{r,N}(s-1)]x(s)x^{\mathrm{T}}(s)[I - F_r(s)\times \bar{H}_{r,N}(s-1)]^{\mathrm{T}}\} + F_r(s)\mathrm{E}\{[\bar{C}_{r,N}(s-1)\times W(s-1) - \bar{V}_r(s-1)][\bar{C}_{r,N}(s-1)W(s-1) - \bar{V}_r(s-1)]^{\mathrm{T}}\}F_r^{\mathrm{T}}(s) =$$

$$T_1 + T_2. \tag{13}$$

By the foregoing definitions, the following results can be drawn:

$$T_1 = F_r(s)\bar{\varepsilon}_{r,N}F_r^{\mathrm{T}}(s) - X(s), \tag{14}$$

$$T_2 = F_r(s)\bar{\beta}_{r,N}F_r^{\mathrm{T}}(s) + F_r(s)\bar{R}_{r,N}F_r^{\mathrm{T}}(s) = F_r(s)[\bar{\beta}_{r,N} + \bar{R}_{r,N}]F_r^{\mathrm{T}}(s). \tag{15}$$

From (13)–(14) and (15), we obtain

$$\begin{aligned}&\mathrm{E}\{\tilde{x}(s|s-1)\tilde{x}^{\mathrm{T}}(s|s-1)\} = \\ &F_r(s)[\bar{\varepsilon}_{r,N} + \bar{\beta}_{r,N} + \bar{R}_{r,N}]F_r^{\mathrm{T}}(s) - X(s) = \\ &F_r(s)\varphi_{r,N}F_r^{\mathrm{T}}(s) - X(s).\end{aligned} \tag{16}$$

The objective is to obtain the optimal gain matrix $F(s)$, subject to the unbiasedness constraint (11), in such a way that the error $\tilde{x}(s|s-1)$ of the estimate $\hat{x}(s|s-1)$ has minimum variance as follows:

$$\begin{aligned}F_r(s) &= \arg\min_{F_r(s)} \mathrm{E}[\tilde{x}(s|s-1)\tilde{x}^{\mathrm{T}}(s|s-1)] = \\ &\arg\min_{F_r(s)} \mathrm{E}[\mathrm{tr}(\tilde{x}^{\mathrm{T}}(s|s-1)\tilde{x}(s|s-1))] = \\ &\arg\min_{F_r(s)} \mathrm{tr}[F_r(s)\varphi_{r,N}F_r^{\mathrm{T}}(s) - X(s)].\end{aligned} \tag{17}$$

Before obtaining the solution to (17), we obtain the result on constraint optimization in the first instance. In order to simplify the calculation, using F_r for a temporary replacement $F_r(s)$. Now, suppose that the following trace optimization problem is given

$$\min_F\{\mathrm{tr}[F_r\varphi_{r,N}F_r^{\mathrm{T}} - X(s)]\}, \tag{18}$$

subject to

$$F_r\bar{H}_{r,N} = I. \tag{19}$$

For convenience, partition the matrix F_r in (11) as

$$F_r^{\mathrm{T}} = \left[\bar{f}_1 \cdots \bar{f}_j \cdots \bar{f}_N\right],\ 1 \leqslant j \leqslant N.$$

From (19), as a consequence, the s-th unbiasedness constraint can be written as

$$\bar{H}_{r,N}^{\mathrm{T}}\bar{f}_j = e_j. \tag{20}$$

In terms of the partitioned vector $\bar{f}_j$, the cost function (18) is represented as

$$\sum_{j=1}^{N} \bar{f}_j^{\mathrm{T}}\varphi_{r,N}\bar{f}_j - X(s).$$

Thus, the optimization problem (18) is reduced to N independent optimization problems

$$\min_{\bar{f}_j} \bar{f}_j^{\mathrm{T}}\varphi_{r,N}\bar{f}_j - X(s)/N, \tag{21}$$

subject to

$$\bar{H}_{r,N}^{\mathrm{T}}\bar{f}_j = e_j. \tag{22}$$

Obtaining the solutions to each optimization problem (21) and putting them together, we can finally obtain the solution to (17).

By solving the optimization problem (21), we can firstly establish the cost function

$$\Phi = \bar{f}_j^{\mathrm{T}}\varphi_{r,N}\bar{f}_j - X(s)/N + \lambda_j^{\mathrm{T}}(\bar{H}_{r,N}^{\mathrm{T}}\bar{f}_j - e_j),$$

where λ_j is the s-th vector of a Lagrange multiplier, which is associated with the s-th unbiased constraint.

In order to minimize Φ, two necessary conditions are obtained

$$\frac{\partial\Phi}{\partial f_j} = 0,\ \ \frac{\partial\Phi}{\partial\lambda_j} = 0,$$

which give

$$\bar{f}_j = -\frac{1}{2}\varphi_{r,N}^{-1}\bar{H}_{r,N}\lambda_j. \tag{23}$$

So

$$\bar{H}_{r,N}^{\mathrm{T}}\bar{f}_j = -\frac{1}{2}\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}\bar{H}_{r,N}\lambda_j = e_j$$

and

$$\lambda_j = -2(\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}\bar{H}_{r,N})^{-1}e_j. \tag{24}$$

Form (23) and (24), we have

$$\bar{f}_j = \varphi_{r,N}^{-1}\bar{H}_{r,N}(\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}\bar{H}_{r,N})^{-1}e_j$$

and

$$\bar{f}_j^{\mathrm{T}} = e_j^{\mathrm{T}}(\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}\bar{H}_{r,N})^{-1}\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}.$$

Putting them together, we can obtain

$$F_r = (\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}\bar{H}_{r,N})^{-1}\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}. \tag{25}$$

Bring (25) into (10), we can reach the batch form of receding horizon estimation

$$\hat{x}(s|s-1) = (\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}\bar{H}_{r,N})^{-1}\bar{H}_{r,N}^{\mathrm{T}}\varphi_{r,N}^{-1}Y_r(s-1). \tag{26}$$

The derivation of Step 2 is similar to that of Step 1. This completes the proof of Theorem 1. QED.

Remark 1 In the derivation of Theorem 1, the linear minimum mean square error receding horizon estimation subject to unbiasedness constraint is divided into N individual optimization problems. Then, by introducing the Lagrange multiplier, the independent optimization problem is solved, and the individual estimation gains are obtained. At last, the total gain is obtained by putting all the components together. The amount of computation meets our requirements. In addition, in (17), $F_r(s)$ should be updated over time.

In what follows, we will rewrite the batch form estimator in an iterative form for computational advantage. For the given time t, an iterative form receding horizon estimator $\hat{x}(t)$ is developed.

Algorithm 2 (Iterative form receding horizon estimator)

Step 1 For $0 \leqslant s \leqslant t-r$, an iterative form estimator $\hat{x}(s|s-1)$ with finite horizon N is given by

$$\hat{x}(s|s-1) = \Omega_{r,N}^{-1}\check{x}(s), \tag{27}$$

where

$$\check{x}(s-N+l) = \check{x}(s-N+l-1) + (A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} \times (\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1}\bar{y}_r(s-l),$$

and $\Omega_{r,N}$ can be obtained from

$$\Omega_{r,l} = \Omega_{r,l-1} + (A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} \times (\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1} H_r A^{-l}$$

with $0 \leqslant l \leqslant N$, $\check{x}(s-N-1) = \mathbf{0}$ and $\Omega_{r,0} = \mathbf{0}$.

Step 2 For $t-r < s \leqslant t$, an iterative form estimator $\hat{x}(s|s-1)$ with finite horizon N is given by

$$\hat{x}(s|s-1) = \Omega_{t-s,N}^{-1}\check{x}(s),$$

where

$$\check{x}(s-N+l) = \check{x}(s-N+l-1) + (A^{-l})^{\mathrm{T}} H_{t-s}^{\mathrm{T}} \times (\varepsilon_{t-s,l}+\beta_{t-s,l}+R_{t-s})^{-1}\bar{y}_{t-s}(s-l),$$

and $\Omega_{t-s,N}$ can be obtained from

$$\Omega_{t-s,l} = \Omega_{t-s,l-1} + (A^{-l})^{\mathrm{T}} H_{t-s}^{\mathrm{T}} \times (\varepsilon_{t-s,l}+\beta_{t-s,l}+R_{t-s})^{-1} H_{t-s} A^{-l}$$

with $0 \leqslant l \leqslant N$, $\check{x}(s-N-1) = \mathbf{0}$ and $\Omega_{t-s,0} = \mathbf{0}$.

Step 3 For $s = t$, set $\hat{x}(t) = \hat{x}(t|t-1)$ in Step 2.

It will be shown in Theorem 2 that the iterative estimator developed in Algorithm 2 is the optimal solution to Problem 1 subject to unbiased constraints.

Theorem 2 Assume that (C, A) is observable. Then the linear minimum mean square error receding horizon estimator $\hat{x}(t)$ with an iterative form on the horizon $[t-N, t]$ is given by Algorithm 2, which satisfies the unbiased constraints.

Proof Firstly, for $0 \leqslant s \leqslant t-r$, define

$$\Omega_{r,l} = \bar{H}_{r,l}^{\mathrm{T}} \varphi_{r,l}^{-1} \bar{H}_{r,l}.$$

So it can be represented in the following Riccati Equation for $0 \leqslant l \leqslant N$:

$$\begin{aligned}
&\Omega_{r,l} = \\
&\bar{H}_{r,l}^{\mathrm{T}} \varphi_{r,l}^{-1} \bar{H}_{r,l} = \\
&[(A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} \ \ \bar{H}_{r,l-1}^{\mathrm{T}}] \times \\
&\begin{bmatrix} (\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1} & 0 \\ 0 & \varphi_{r,l-1}^{-1} \end{bmatrix} \times \\
&\begin{bmatrix} H_r A^{-l} \\ \bar{H}_{r,l-1} \end{bmatrix} = \\
&\bar{H}_{r,l-1}^{\mathrm{T}} \varphi_{r,l-1}^{-1} \bar{H}_{r,l-1} + (A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} \times \\
&(\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1} H_r A^{-l} = \\
&\Omega_{r,l-1} + (A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} (\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1} \times \\
&H_r A^{-l}.
\end{aligned} \tag{28}$$

Similarly, it is available for $0 \leqslant l \leqslant N$ that

$$\begin{aligned}
&\check{x}(s-N+l) = \\
&\bar{H}_{r,l}^{\mathrm{T}} \varphi_{r,l}^{-1} Y_{r,l}(s-1) = \\
&[(A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} \ \ \bar{H}_{r,l-1}^{\mathrm{T}}] \times \\
&\begin{bmatrix} (\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1} & 0 \\ 0 & \varphi_{r,l-1}^{-1} \end{bmatrix} \times \\
&\begin{bmatrix} \bar{y}_r(s-l) \\ Y_{r,l-1}(s-1) \end{bmatrix} = \\
&\bar{H}_{r,l-1}^{\mathrm{T}} \varphi_{r,l-1}^{-1} Y_{r,l-1}(s-1) + (A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} \times \\
&(\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1}\bar{y}_r(s-l) = \\
&\check{x}(s-N+l-1) + (A^{-l})^{\mathrm{T}} H_r^{\mathrm{T}} (\varepsilon_{r,l}+\beta_{r,l}+R_r)^{-1} \times \\
&\bar{y}_r(s-l).
\end{aligned} \tag{29}$$

From (28) and (29), an iterative form for receding horizon estimation is obtained

$$\hat{x}(s|s-1) = \Omega_{r,N}^{-1}\check{x}(s).$$

Similarly, We are able to get an iterative form of receding horizon estimation in Step 2. This completes the proof of Theorem 2. QED.

3.3 Stability analysis

The stability of the receding-horizon estimator will be investigated below. Thus we just need to analyze the stability of the filter developed in Theorem 2. It needs to require consideration of the filter's transfer matrix. From Theorem 2, we define the transfer matrix for $0 \leqslant s \leqslant t-r$ as

$$\begin{aligned}
&\Gamma_N = \\
&I - \Omega_{r,N-1}^{-1}(A^{-N})^{\mathrm{T}} H_r^{\mathrm{T}} [H_r A^{-N} \times \\
&\Omega_{r,N-1}^{-1}(A^{-N})^{\mathrm{T}} H_r^{\mathrm{T}} + (\varepsilon_{r,N}+\beta_{r,N}+R_r)]^{-1} \times \\
&H_r A^{-N}.
\end{aligned} \tag{30}$$

Under the given assumption, the necessary and sufficient condition subject to asymptotical stability of the proposed filter is that the transfer matrix Γ_N of the estimator is one stability matrix. It means that all of its eigenvalues are located in the unit circle. The stability of the observer is ensured by the following theorem.

Theorem 3 If (C, A) is observable, and A nonsingular, then the matrix Γ_N has all its eigenvalues strictly within the unit circle for all finite $N \geqslant n-1$ where n is the dimension of the state vector.

Proof For $0 \leqslant s \leqslant t-r$, define[20]

$$\hat{x}(s) = \Gamma_N \hat{x}(s-1) + \Phi(s),$$

where

$$\hat{x}(s) = \Omega_{r,N}^{-1}\check{x}(s), \ \hat{x}(s-1) = \Omega_{r,N-1}^{-1}\check{x}(s-1).$$

In view of (28) and (29), we can obtain (30) immediately. This completes the proof of Theorem 3. QED.

Remark 2 Conditions for the stability of the proposed moving horizon estimation is proposed for time-invariant systems. The advantage of this estimation algorithm is that it is easy to implement since the gains can be performed off-line.

4 Simulation example

In this section, a simulation example is given to illustrate the efficiency of the proposed receding horizon estimation for random delay system (1) and (2). In this part, we define the time horizon $0 \leqslant t \leqslant 100$, the estimator horizon size $N = 5$, and the random delay horizon $0 \leqslant r(t) \leqslant 2$. The other parameters of the system are as follows

$$x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix},\ A=\begin{bmatrix}1.45 & 1\\-1.2 & -0.22\end{bmatrix},\ C=\begin{bmatrix}0.3\\0.45\end{bmatrix},$$

$$H=\begin{bmatrix}1.8 & 0.5\\0 & 1.8\end{bmatrix},\ \rho_i=\frac{1}{3},\ i=0,1,2.$$

Based on the design procedures of Theorem 2 in this paper and Kalman filter in [13], the simulation results are obtained as follows. Fig.1 shows the trace of the real value $x_1(t)$ and its estimate. Fig.2 shows the trace of the real value $x_2(t)$ and its estimate. Fig.3 shows the root of the mean square estimation errors (RMSEEs) of $x_1(t)$ according to the two algorithms, while Fig.4 shows the RMSEEs of $x_2(t)$ according to the two algorithms. Fig.5 shows the summation of the RMSEEs of $x_1(t)$ of the two algorithms. Fig.6 shows the summation of the RMSEEs of $x_2(t)$ of the two algorithms. It can be seen from Figs.3–6 that the obtained receding horizon estimation for systems with observation delays track better than Kalman filter and the estimation scheme produces better performance. On the other hand, it can be seen from Fig.7 and Fig.8 that the tracking performance for the case of $N = 5$ is better than that of $N = 2$. It is a suitable choice for the estimator horizon size $N = 5$.

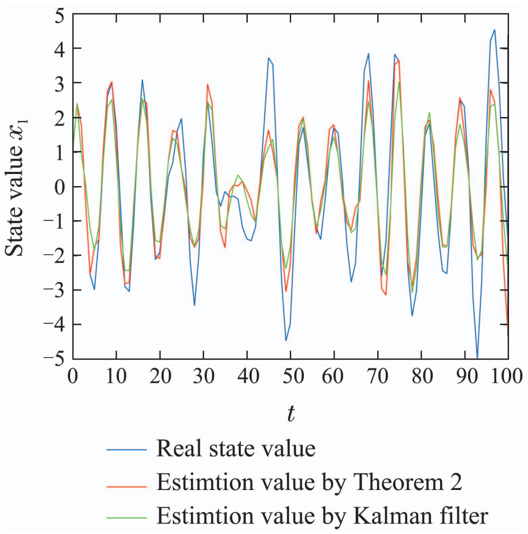

Fig. 1 State trajectories of $x_1(t)$

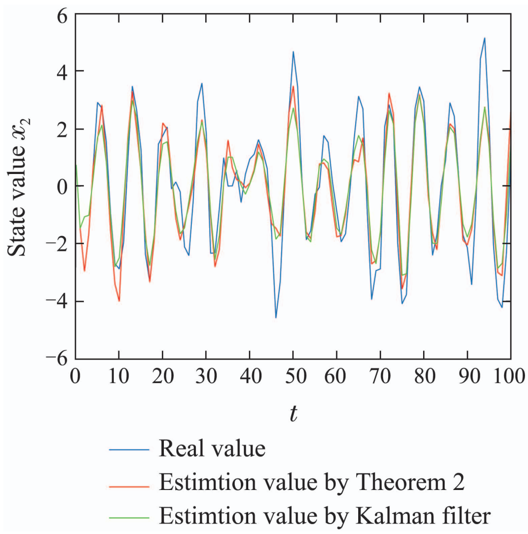

Fig. 2 State trajectories of $x_2(t)$

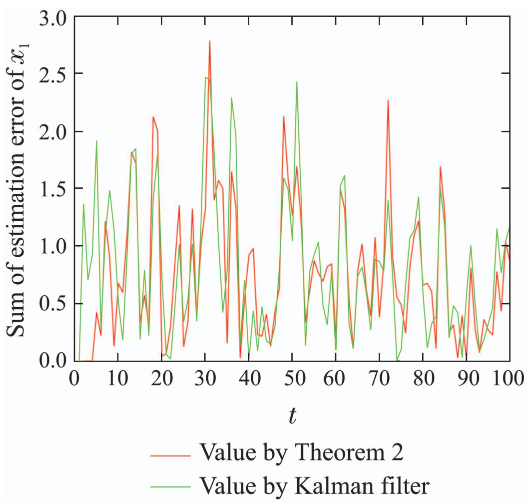

Fig. 3 The RMSEEs of $x_1(t)$

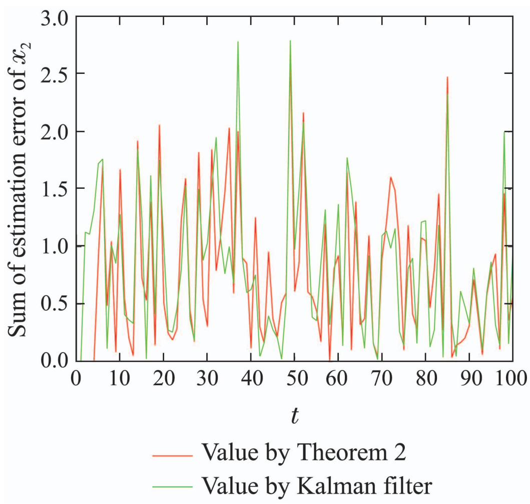

Fig. 4 The RMSEEs of $x_2(t)$

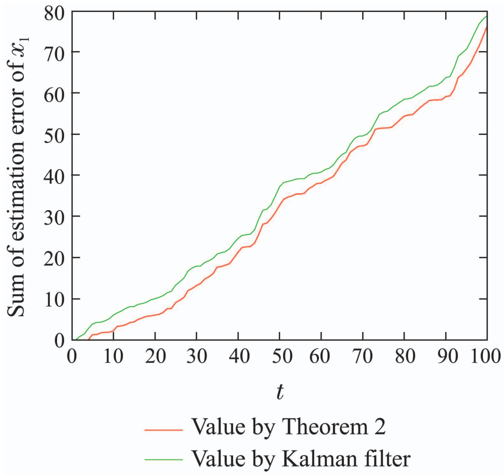

Fig. 5 Summation of RMSEE trajectories of $x_1(t)$

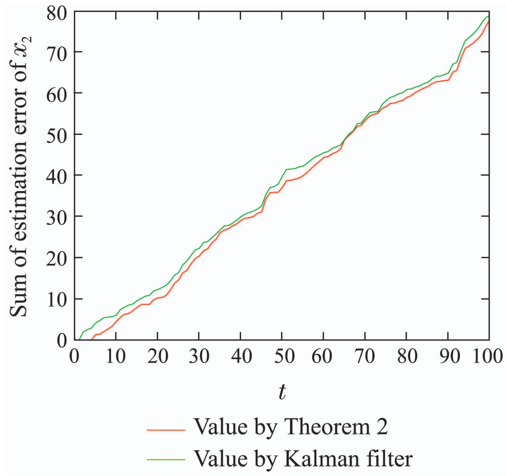

Fig. 6 Summation of RMSEE trajectories of $x_2(t)$

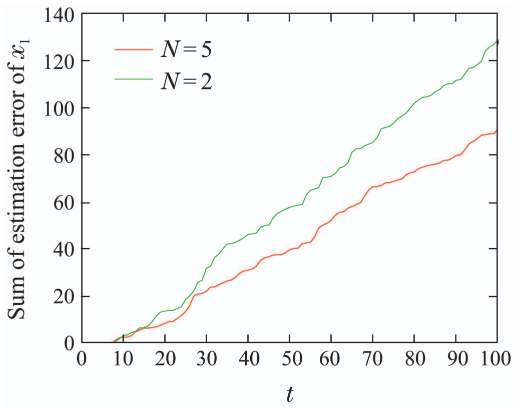

Fig. 7 Summation of RMSEE trajectories of $x_1(t)$ for RHE estimation: $N = 5, 2$

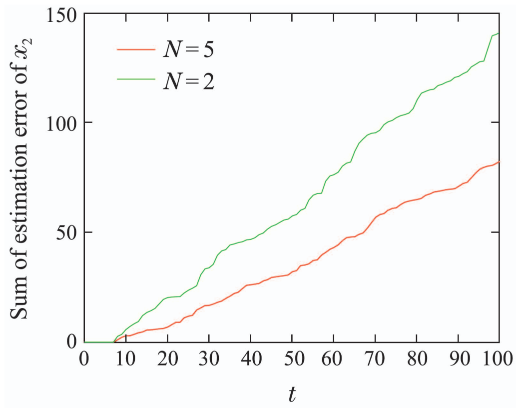

Fig. 8 Summation of RMSEE trajectories of $x_2(t)$ for RHE estimation: $N = 5, 2$

5 Conclusion

In this paper, the receding horizon estimators were proposed for discrete-time linear system with random observation delay. The random delay system was transformed into a delay-free one by the reorganization observation method. On the basis of the new observation equation, a batch form and an iterative form for receding horizon estimation were designed. The observation reorganization technique is firstly applied to the receding horizon estimation for discrete-time systems with random delays. It is obvious that this method simplifies the computation compared to state augmentation method for dealing with random delays. This is the main technique novelty of this paper. The stability analysis was supplied and the theoretical results were illustrated by a numerical example.

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作者简介:

李超超 硕士研究生, 目前研究方向为不可靠网络控制系统、后退时域估计、分布式估计, E-mail: 307791649@qq.com;

韩春艳 教授, 硕士生导师, 目前研究方向为时滞系统、Markov跳变系统、最优估计与控制, E-mail: cyhan823@hotmail.com;

何 芳 教授, 硕士生导师, 目前研究方向为网络控制系统、驱动控制系统、张力控制系统, E-mail: cse_hf@163.com.