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一类广泛的高阶非线性系统的自适应实际输出追踪控制

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摘要: 研究了一类带有不确定控制系数和不可测零动态的高阶非线性系统的自适应实际输出追踪控制问题. 与现有文献相比较, 所研究的系统更一般化, 并且零动态的约束条件得到进一步放宽. 通过运用增加幂次积分方法和自适应技术, 设计了连续的自适应追踪控制器. 最后, 给出一个仿真算例验证控制设计方案的有效性.

关键词: 实际输出追踪; 高阶非线性系统; 零动态; 增加幂次积分

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Adaptive practical output tracking for a general class of higher-order nonlinear systems

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Abstract: The problem of adaptive practical output tracking has been further investigated for a class of higher-order nonlinear systems with uncertain control coefficients and unmeasurable zero dynamics. Compared with the existing results, the restriction on zero dynamics is relaxed and the system to be studied is more general. The design procedures of the continuous adaptive tracking controller are provided by flexibly incorporating the method of adding a power integrator with the related adaptive technique. Finally, a numerical example is given to demonstrate the effectiveness of the control scheme.

Key words: practical output tracking; higher-order nonlinear systems; zero dynamics; adding a power integrator

1 Introduction

This paper considers a class of higher-order nonlinear systems with uncertain control coefficients and zero dynamics described by

$$\begin{cases}\dot{\eta}(t)=f_0(x(t),\eta(t),\theta),\\ \dot{x}_i(t)=d_i(x(t),\eta(t),\theta)x_{i+1}^{p_i}+f_i(x(t),\eta(t),\theta),\\ \dot{x}_n(t)=d_n(x(t),\eta(t),\theta)u^{p_n}+f_n(x(t),\eta(t),\theta),\\ y(t)=x_1(t),\end{cases} \tag{1}$$

where $i=1,\cdots,n-1$; $x(t)=(x_1(t),\cdots,x_n(t))^{\mathrm T}\in\mathbb{R}^n$ is the measurable state and $\eta(t)\in\mathbb{R}^m$ is the unmeasurable state; $u(t)\in\mathbb{R}$ and $y(t)\in\mathbb{R}$ are the input and the output; the system initial condition is $(x(0),\eta(0))^{\mathrm T}=(x_0,\eta_0)^{\mathrm T}$; system power $p_i\in\mathbb{R}_{\text{odd}}^{\geqslant 1}:=\{\frac{p}{q}|p$ and q are odd positive integers, and $p\geqslant q\}$; $\theta\in\mathbb{R}^s$ represents parameter uncertainty of the system; $f_0:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^s\rightarrow\mathbb{R}^m$, $d_i:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^s\rightarrow\mathbb{R}$ and $f_i:\mathbb{R}^n\times\mathbb{R}^m\times\mathbb{R}^s\rightarrow\mathbb{R}$ are unknown continuous functions satisfying $f_0(0,0,\theta)\equiv 0$, $f_i(0,0,\theta)\equiv 0$, $d_i(0,0,\theta)\neq 0$.

During the last decades, higher-order nonlinear systems have been received considerable attention and lots of efforts[1–5] have been acquired based on the method of adding a power integrator[6–8] which is a new technique of the control design and can be regarded as the latest development of the traditional backstepping approach. Tracking control is one of the most important problems of higher-order nonlinear systems which was firstly presented by Qian and Lin in [9], with the help of the nonlinear output regulator theory[10–15] and the method of adding a power integrator, they successfully solved this problem. However there is no control coefficients and zero dynamics in their works. Generally speaking, the control design of higher-order nonlinear systems can be classified into two types, that is, smooth and continuous control design. It is worth pointing out that some assumptions imposed on system nonlinearities are almost same to some extent, however, uncertain control coefficients and zero dynamics are not in the situation like system nonlinearities. In the subsequent works, restrictions on the assumptions of systems are relaxed more or less, but the most of existing work just aimed at control coefficients, and there is no zero dynamics in those literatures, or else, zero dynamics still meet a strong input-to-state stability-type (ISS-type) property. For

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example, [4, 8, 16] demand that the zero dynamics satisfies a very strong ISS-type property. This paper continues the investigation of the tracking control problem, and restriction on the zero dynamics is successfully reduced to the more general case(see Assumption 1). Based on the idea of continuous stabilization[4,17–19], this paper designs a continuous adaptive tracking controller.

Differential equations (1) can be seen as the broadest possible form of the higher-order nonlinear systems, and some technical difficulties will be encountered in the control design mainly due to the presence of the unknown system nonlinearities and the unknown control coefficients together with zero dynamics. The main contributions of the paper are briefly characterized by the following specific features:

i) The system studied in this paper is more general than those existing systems, such as [9, 20–21] have not zero dynamics, zero dynamics satisfy a special ISS-type property in [16, 18] requires the lower bound of unknown control coefficients is known, and so on.

ii) From some existing results of higher-order nonlinear systems, it is not hard to see that the management of the nonlinearities will bring many difficulties and complexities. Undoubtedly, the appearance of unmeasurable zero dynamics will produce much more nonlinear terms, and how to deal with these terms is the main difficulty of the paper.

2 Adaptive continuous control

2.1 Problem statement

Let $y_{\rm r}(t)$ be a continuously differentiable reference signal. For any given positive real number ε, the purpose of this paper is to design a continuous partial-state adaptive controller for system (1),

$$\begin{cases}u(t)=u(x(t),\,y_{\rm r}(t),K(t)),\\ \dot K(t)=\varOmega(x(t),y_{\rm r}(t)),\ K(0)\geqslant 1,\end{cases}\tag{2}$$

such that

i) The state of the closed-loop systems (1)–(2) is well-defined on $[0,+\infty)$ and globally bounded;

ii) For every $(x_0,\eta_0)^{\rm T}\in\mathbb{R}^{n+m}$, there exists a finite time $T>0$, such that the output of the closed-loop system satisfies $|y(t)-y_{\rm r}(t)|\leqslant\varepsilon,\forall t\geqslant T$.

In the remainder of the paper, the arguments of the functions will be omitted or simplified. For instance, we sometimes denote a function $f(x(t))$ by simply $f(x)$, $f(\cdot)$ or f. In order to solve the previous problem, we make the following assumptions:

Assumption 1 There exists a continuously differentiable function $U_0:\mathbb{R}^m\to\mathbb{R}^+$, such that

$$\underline{\alpha}(\|\eta\|)\leqslant U_0(\eta)\leqslant\bar\alpha(\|\eta\|),$$

$$\frac{\partial U_0(\eta)}{\partial\eta}f_0(x,\eta,\theta)\leqslant -M(\eta)+\xi(x_1,\theta),$$

where $\underline{\alpha}:\mathbb{R}^+\to\mathbb{R}^+$ and $\bar\alpha:\mathbb{R}^+\to\mathbb{R}^+$ are $\mathcal{K}_\infty$ functions, $M:\mathbb{R}^m\to\mathbb{R}^+$ is a continuous positive definite function and $\xi:\mathbb{R}\times\mathbb{R}^s\to\mathbb{R}^+$ is a smooth function.

Assumption 2 For each $i=1,\cdots,n$, there exist continuously differentiable functions $b_{ij}:\mathbb{R}^i\times\mathbb{R}^m\times\mathbb{R}^s\to\mathbb{R}^+$ satisfying $b_{ij}(0,\eta(t),\theta)\equiv 0$, such that

$$|f_i(x,\eta,\theta)|\leqslant\sum_{j=1}^{l_i}b_{ij}(x_{[i]},\eta,\theta)|x_{i+1}|^{q_{ij}},$$

where l_i is a finite positive integer, q_{ij}'s are real numbers satisfying $0\leqslant q_{i1}<\cdots<q_{il_i}<p_i$, and $x_{n+1}(t):=u(t)$.

Assumption 3 For each $i=1,\cdots,n$, there exist an unknown positive constant a, known smooth functions $\lambda_i:\mathbb{R}^i\to\mathbb{R}^+$, $\mu_i:\mathbb{R}^{i+1}\times\mathbb{R}^m\times\mathbb{R}^s\to\mathbb{R}^+$, such that

$$a\lambda_i(x_{[i]})\leqslant|d_i(x,\eta,\theta)|\leqslant\mu_i(x_{[i+1]},\eta,\theta),$$

where $x_{[n+1]}=(x^{\rm T},u)^{\rm T}$.

Assumption 4 For any continuous differential reference signal $y_{\rm r}(t)$, there exists an unknown positive number M, such that $|y_{\rm r}(t)|\leqslant M$, $|\dot y_{\rm r}(t)|\leqslant M$ for all $t\geqslant 0$.

Remark 1 Assumption 1 shows that zero dynamics meet a more weaker ISS-type property than those in [8, 21]. In fact, by taking $M(\eta(t))=\|\eta(t)\|^2$, Assumption 1 reduces to Assumption 2 in [21] and Assumption 1.1 in [18]. By Lemma 1 given in the next section, it is not hard to find that Assumption 2 includes Assumption 1 in [16] as a special case even if there are no zero dynamics. Assumption 3 implies that all the sign of d_i remain invariable, thereby without loss of generality, we suppose $d_i>0$ in the later control design procedure.

2.2 Preliminary results

Now, we introduce four technical lemmas which will play a key role in the control design and the theoretical analysis.

Lemma 1[8] For any continuous function $f(x,y)$ where $x\in\mathbb{R}^m$, $y\in\mathbb{R}^n$, there are smooth scalar-value functions $a(x)\geqslant 0$, $b(y)\geqslant 0$, $c(x)\geqslant 1$, $d(y)\geqslant 1$ such that$|f(x,\,y)|\leqslant a(x)+b(y)$, $|f(x,\,y)|\leqslant c(x)d(y)$.

Lemma 2[7] For a given $p\in\mathbb{R}^{\geqslant 1}_{\rm odd}$, and $\forall x\in\mathbb{R}$, $\forall y\in\mathbb{R}$, there hold

$$|x+y|^{\frac{1}{p}}\leqslant|x|^{\frac{1}{p}}+|y|^{\frac{1}{p}},\ |x-y|\leqslant 2^{\frac{p-1}{p}}|x^p-y^p|^{\frac{1}{p}}.$$

Lemma 3[22] For given positive integers m, n and any real valued function $\gamma(x,y)>0$, the following inequality holds:

$$|x|^m|y|^n\leqslant\frac{m}{m+n}\gamma|x|^{m+n}+\frac{n}{m+n}\gamma^{-\frac{m}{n}}|y|^{m+n}.$$

Lemma 4[23] Let $\phi(t)$ be an uniformly continuous function on $[0,\,\infty)$. If $\int_0^\infty\phi(s){\rm d}s$ exists and is finite, then $\lim\limits_{t\to\infty}\phi(t)=0$.

To proceed the control design, according to the properties of $\mathcal{K}$ functions[23], it is easy to find an appropriate $\mathcal{K}$ function $\varPsi(\|\eta(t)\|)$ defined on $[0,\infty)$ and satisfies $\varPsi(\|\eta(t)\|)\leqslant M(\eta(t))$, where $M(\eta(t))$ is defined in Assumption 1. Then, using Assumptions 2 and 3, we can get the following proposition, which more clearly characterizes the increasing properties of the nonlinearities f_i's and implements the separation between the partial state $x_{i+1}(t)$ and the other partial states $x_j(t)$'s together with $\eta(t)$, and whose similar proof can be found in work [19].

Proposition 1 For each $i=1,\cdots,n$, there exist smooth functions $\gamma_i:\mathbb{R}^i\to\mathbb{R}^+$, $\Phi_1:\mathbb{R}^+\to[1,\infty)$ and an unknown constant $\bar{\Theta}>1$, such that

$$|f_i|\leqslant\frac{d_i}{2}|x_{i+1}^{p_i}|+\bar{\Theta}(\Phi_1(\cdot)\sqrt{\Psi(\cdot)}+\gamma_i(x_{[i]})).\quad(3)$$

For any continuous function $\mu_i(\cdot)$ defined in Assumption 3, there exists the following proposition:

Proposition 2 For each $i=1,\cdots,n$, there exist smooth function $\nu_i:\mathbb{R}^{i+1}\to[1,\infty)$, $\Phi_2:\mathbb{R}^+\to[1,\infty)$ and an unknown constant N, such that

$$\mu_i(x_{[i+1]},\eta,\theta)\leqslant N(\nu_i(x_{[i+1]})+\Phi_2(\cdot)\sqrt{\Psi(\cdot)}).$$

Proof By the well-known mean value theorem of multivariate function, for each $i=1,\cdots,n$, there exists a number $\sigma\in[0,1]$ such that

$$\mu_i(\cdot)=\mu_i(x_{[i+1]},0,\theta)+\eta^{\rm T}\frac{\partial}{\partial\eta}\mu_i(x_{[i+1]},\sigma\eta,\theta).\quad(4)$$

By Lemma 1, there exist smooth functions $m_i:\mathbb{R}^s\to[1,\infty)$, $n_i:\mathbb{R}^s\to[1,\infty)$, $\rho_i:\mathbb{R}^+\to[1,\infty)$, $\bar{m}_i:\mathbb{R}^{i+1}\to[1,\infty)$, $\bar{n}_i:\mathbb{R}^{i+1}\to[1,\infty)$ such that

$$\mu_i(x_{[i+1]},0,\theta)\leqslant m_i(\theta)\cdot\bar{m}_i(x_{[i+1]}),$$

$$\eta^{\rm T}\frac{\partial}{\partial\eta}\mu_i(x_{[i+1]},\sigma\eta,\theta)\leqslant n_i(\theta)\cdot\rho_i(\|\eta\|)\cdot\bar{n}_i(x_{[i+1]}).$$

Since $\Psi(\|\eta\|)$ is a $\mathcal{K}$ function, we have

$$\rho_i(\|\eta\|)\leqslant(\frac{\Psi(\|\eta\|)}{\Psi(1)})^{\frac{1}{4}}\rho_i(\|\eta\|)+\max_{\|\eta\|\leqslant1}\rho_i(\|\eta\|),$$

by which and using Lemma 3, we can further get

$$\mu_i(\cdot)\leqslant N(\nu_i(x_{[i+1]})+\Phi_2(\|\eta\|)\sqrt{\Psi(\|\eta\|)}),$$

where $N=\max\{m_i(\theta),n_i(\theta)(\Psi(1))^{-\frac{1}{2}},n_i(\theta)\cdot\max_{\|\eta\|\leqslant1}\rho_i(\|\eta\|)\}$, $\Phi_2(\|\eta\|)=\rho_i^2(\|\eta\|)$, $\nu(x_{[i+1]})=\bar{m}_i(x_{[i+1]})+\bar{n}_i^2(x_{[i+1]})+\bar{n}_i(x_{[i+1]})$. This completes the proof.

Then, the following transformation is presented:

$$\begin{cases}z_1=x_1-y_{\rm r},\\ z_i=x_i^{p_1\cdots p_{i-1}}-\alpha_{i-1}^{p_1\cdots p_{i-1}}(x_{[i-1]},y_{\rm r},K),\end{cases}\quad(5)$$

where $i=2,3,\cdots,n$, $\alpha_i(x_{[i]},y_{\rm r},K)$ is called a virtual controller and $u=\alpha_n(x,y_{\rm r},K)$ is an actual controller, all of them will be specified later.

Now, we introduce $W_1(x_1,y_{\rm r})=\frac{1}{2}z_1^2$, W_k: $\mathbb{R}^k\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}$, $k=2,3,\cdots,n$ as follows:

$$W_k=\int_{\alpha_{k-1}}^{x_k}(s^{p_1\cdots p_{k-1}}-\alpha_{k-1}^{p_1\cdots p_{k-1}})^{2-\frac{1}{p_1\cdots p_{k-1}}}{\rm d}s,$$

whose properties are characterized by the following proposition.

Proposition 3[7] W_k, $k=2,3,\cdots,n$ are continuously differentiable and satisfy

$$\begin{cases}\dfrac{\partial W_k}{\partial x_k}=z_k^{2-\frac{1}{p_1\cdots p_{k-1}}},\\ \dfrac{\partial W_k}{\partial\chi_i}=-(2-\dfrac{1}{p_1\cdots p_{k-1}})\cdot\dfrac{\partial x_{k-1}^{*p_1\cdots p_{k-1}}}{\partial\chi_i}\cdot\\ \qquad\displaystyle\int_{x_{k-1}^*}^{x_k}(s^{p_1\cdots p_{k-1}}-\alpha_{k-1}^{p_1\cdots p_{k-1}})^{1-\frac{1}{p_1\cdots p_{k-1}}}{\rm d}s,\end{cases}$$

where $\chi_i=x_i$ for $i=1,\cdots,k-1$, $\chi_k=y_{\rm r}$, $\chi_{k+1}=K$. Furthermore, there holds

$$\frac{2^{-p_1\cdots p_{k-1}}}{p_1\cdots p_{k-1}}(x_k-\alpha_{k-1})^{2p_1\cdots p_{k-1}}\leqslant W_k\leqslant2z_k^2.$$

Next, we address the following proposition, whose proof can be completed by taking the same manipulations as Proposition A.1 in [4].

Proposition 4 For each $l=1,\cdots,k-1$ with $k=2,3,\cdots,n$, one can find a smooth positive function $\Upsilon_{k-1,l}:\mathbb{R}^{k-1}\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}^+$ and an unknown constant $\tilde{\Theta}$, such that

$$\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial x_l}(d_lx_{l+1}^{p_l}+f_l)\leqslant$$
$$\tilde{\Theta}\Upsilon_{k-1,l}(\Phi_3(\|\eta\|)\cdot\sqrt{\Psi(\|\eta\|)}+\bar{\gamma}_{k-1}(x_{[k-1]})).$$

At the end of this subsection, to deal with zero dynamics effectively, we need the next proposition which can be shown straightforwardly by the idea of changing supply functions.

Proposition 5[19] Let $V_0(\eta)=\int_0^{U_0(\eta)}4L(s){\rm d}s$, where $U_0(\eta)$ is defined in Assumption 1 and $L:\mathbb{R}^+\to[1,\infty)$ is continuous and monotone nondecreasing, then the following properties hold:

i) $V_0(\eta)$ is continuous differentiable, positive definite and radially unbounded.

ii) There exists a smooth function $\pi(x_1,\theta)\geqslant0$, such that

$$\frac{\partial V_0}{\partial\eta}f_0(\cdot)\leqslant-2L(U_0(\eta))\Psi(\|\eta\|)+\pi(x_1,\theta).\quad(6)$$

Remark 2 To proceed the control design, we should find an appropriate function $L(\cdot)$ satisfying $L(U_0(\eta))\geqslant n\Phi^2(\|\eta\|)$, where $\Phi(\|\eta\|)$ is an appropriate smooth upper function of $\Phi_i(\|\eta\|)$, $i=1,2,3$. For instance, we can select $L(\cdot)=nP(\underline{\alpha}^{-1}(\cdot))$ with $P(\|\eta\|)=\max_{\|\eta\|\geqslant|s|}\Phi^2(s)$ being a continuous monotone nondecreasing function.

2.3 Design procedure

In this subsection, we shall construct a continuous partial-state adaptive controller for the system (1), which is addressed in a step-by-step manner.

Before the control design, it is necessary to define an unknown positive constant λ as

$$\lambda=1+\frac{2a\Theta}{\varepsilon_1^2}+\frac{a^2}{2\varepsilon_1^2}\int_0^{\bar{\alpha}(\Psi^{-1}(2\Theta))}4L(s){\rm d}s,$$

where $\varepsilon_1=\frac{a\varepsilon}{4}$, and Θ will be specified later.

Step 1 Choose $V_1(x_1,\eta,y_{\rm r})=\frac{V_0}{\lambda}+W_1$. Obviously, V_1 is continuous differentiable, positive definite and radially unbounded. Taking the time derivative of V_1 along the solution of Eqs.(1), using Assumption 4, Propositions 1 and 5, noticing $L(\cdot)\geqslant n\Phi^2(\cdot)$, $\Phi(\cdot)\geqslant\Phi_1(\cdot)$, we have

$$\dot{V}_1\leqslant-\frac{\Psi(\cdot)}{\lambda}-\frac{n-1}{\lambda}\Phi^2(\cdot)\Psi(\cdot)+\frac{3}{2}d_1z_1x_2^{p_1}+$$
$$\frac{\pi(x_1,\theta)}{\lambda}+\lambda\bar{\Theta}^2z_1^2+|z_1|(\gamma_1\bar{\Theta}+M).\quad(7)$$

To attain the first virtual controller α_1, we have to find the appropriate upper bound estimates of the last three terms on the right-hand side of inequality (7).

Firstly, Lemma 1, Assumption 4 and the transformation (5) show that there exist a smooth function $\hat{\pi}(z_1) \geqslant 1$ and a constant $r(M,\theta)$ such that $\dfrac{\pi(x_1,\theta)}{\lambda} \leqslant \dfrac{1}{\lambda} r(M,\theta)(\hat{\pi}(z_1)z_1 + \hat{\pi}(0))$. With this in hand and using Lemma 3, we can further get

$$\begin{aligned}&\frac{\pi(x_1,\theta)}{\lambda} \leqslant \\ &aK\hat{\pi}^2(\cdot)z_1^2 + \frac{r^2(M,\theta)}{aK\lambda^2} + \frac{r(M,\theta)\hat{\pi}(0)}{\lambda} =: \\ &a\rho_{11}(x_1,y_\mathrm{r},K)z_1^2 + \frac{\Theta_{11}}{K} + \frac{\Theta}{\lambda},\end{aligned} \tag{8}$$

where $\rho_{11} : \mathbb{R}\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}^+$ is positive and smooth.

Secondly, according to Lemma 3, we can obtain

$$\begin{aligned}&\bar{\Theta}^2\lambda z_1^2 + z_1(\gamma_1(x_1)\bar{\Theta} + M) \leqslant \\ &aK\left(z_1^2+\gamma_1^2+1\right)z_1^2 + \frac{\bar{\Theta}^4\lambda^2 + M^2 + \bar{\Theta}^2}{aK} =: \\ &a\rho_{12}(x_1,y_\mathrm{r},K)z_1^2 + \frac{\Theta_{12}}{K},\end{aligned} \tag{9}$$

where $\rho_{12} : \mathbb{R}\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}^+$ is a positive smooth function. Now, in view of the transformation (5), and substituting inequalities (8)–(9) into inequality (7), we can deduce

$$\begin{aligned}\dot{V}_1 \leqslant & -\frac{1}{\lambda}\Psi(\cdot) - \frac{n-1}{\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{3}{2}d_1z_1z_2 + \frac{\Theta}{\lambda} + \\ &\frac{3}{2}d_1z_1\alpha_1^{p_1} + a\rho_1(x_1,y_\mathrm{r},K)z_1^2 + \frac{\Theta_1}{K},\end{aligned} \tag{10}$$

where $\rho_1 = \rho_{11} + \rho_{12}$ is a positive smooth function and $\Theta_1 = \Theta_{11} + \Theta_{12}$ is an unknown positive constant. Clearly, for the stabilization objective, it is necessary to choose α_1 such that $d_1z_1\alpha_1^{p_1} \leqslant 0$. By this and Assumption 3, one can choose the first virtual controller α_1 as

$$\alpha_1^{p_1} = -\frac{\rho_1(x_1,y_\mathrm{r},K)+1}{\lambda_1}z_1 =: -g_1(\cdot)z_1, \tag{11}$$

with the gain-update law

$$\dot{K} = \begin{cases} |z_1| - \dfrac{\varepsilon}{2}, & |z_1| \geqslant \dfrac{\varepsilon}{2}, \\ 0, & |z_1| < \dfrac{\varepsilon}{2}, \end{cases} \tag{12}$$

where the initial value $K(0) \geqslant 1$. Substituting Eq.(11) into inequality (10), we finally get

$$\begin{aligned}\dot{V}_1 \leqslant & -\frac{1}{\lambda}\Psi(\cdot) - \frac{n-1}{\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{3}{2}\mu_1z_1z_2 + \\ &\frac{\Theta_1}{K} + \frac{\Theta}{\lambda} - az_1^2.\end{aligned} \tag{13}$$

This completes Step 1. The first step can be viewed as the initialization of the whole recursive design procedure. From Step 2, we turn to the recursive steps.

Step k $(k = 2,3,\cdots,n)$ Suppose V_{k-1} for step $k-1$ satisfies

$$\begin{aligned}&\dot{V}_{k-1} \leqslant \\ &-\frac{1}{\lambda}\Psi - \frac{n-k+1}{\lambda}\Phi^2\Psi + \frac{\Theta_{k-1}}{K} - a\sum_{i=1}^{k-1} z_i^2 + \\ &\frac{\Theta}{\lambda} + c_{k-1}\mu_{k-1}|z_{k-1}|^{2-\frac{1}{p_1\cdots p_{k-2}}}|z_k|^{\frac{1}{p_1\cdots p_{k-2}}},\end{aligned} \tag{14}$$

where $c_{k-1} = \dfrac{1}{2} + 2^{1-\frac{1}{p_1\cdots p_{k-2}}}$.

In the following, we need prove that inequality (14) still holds for Step k. For this aim, choose $V_k(x_{[k]},\eta,y_\mathrm{r},K) = V_{k-1} + W_k(x_{[k]},y_\mathrm{r},K)$. Taking the time derivative of V_k along solutions to Eqs.(1), using Proposition 3 and substituting inequality (14) into it, we get

$$\begin{aligned}&\dot{V}_k \leqslant \\ &-\frac{\Psi}{\lambda} - \frac{n-k+1}{\lambda}\Phi^2\Psi - \omega_k\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial K}\dot{K} - \\ &\omega_k\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial y_\mathrm{r}}\dot{y}_\mathrm{r} - \omega_k\sum_{l=1}^{k-1}\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial x_l}(d_lx_{l+1}^{p_l} + f_l) + \\ &c_{k-1}\mu_{k-1}|z_{k-1}|^{2-\frac{1}{p_1\cdots p_{k-2}}}|z_k|^{\frac{1}{p_1\cdots p_{k-2}}} + \frac{\Theta_{k-1}}{K} + \\ &\frac{\Theta}{\lambda} + z_k^{2-\frac{1}{(p_1\cdots p_{k-1})}}(d_kx_{k+1}^{p_k} + f_k) - a\sum_{i=1}^{k-1}z_i^2,\end{aligned} \tag{15}$$

where $\omega_k(\cdot) = (2 - \dfrac{1}{p_1\cdots p_{k-1}})\displaystyle\int_{\alpha_{k-1}}^{x_k}(s^{p_1\cdots p_{k-1}} - \alpha_{k-1}^{p_1\cdots p_{k-1}})^{1-\frac{1}{p_1\cdots p_{k-1}}}\mathrm{d}s$.

Firstly, using Lemma 3, Proposition 2, and noting $\Phi(\cdot) \geqslant \Phi_2(\cdot)$, we deduce

$$\begin{aligned}&c_{k-1}\mu_{k-1}|z_{k-1}|^{2-\frac{1}{p_1\cdots p_{k-2}}}|z_k|^{\frac{1}{p_1\cdots p_{k-2}}} \leqslant \\ &aK(c_{k-1}^2\nu_{k-1}^2(1+z_{k-1}^2)^{2-\frac{1}{p_1\cdots p_{k-2}}}(1+z_k^2)^{\frac{1}{p_1\cdots p_{k-2}}-1} + \\ &c_{k-1}^4(1+z_{k-1}^4)^{2-\frac{1}{p_1\cdots p_{k-2}}}(1+z_k^2)^{\frac{2}{p_1\cdots p_{k-2}}-1})z_k^2 + \\ &\frac{1}{4\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{N^2+\lambda^2N^4}{aK} =: \\ &a\rho_{k1}(x_{[k]},y_\mathrm{r},K)z_k^2 + \frac{1}{4\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{\Theta_{k1}}{K},\end{aligned} \tag{16}$$

where $\rho_{k1} : \mathbb{R}^k\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}^+$ is positive.

Secondly, by Lemma 2, we know

$$\begin{aligned}&|x_k - \alpha_{k-1}|^{p_1\cdots p_{k-1}} \leqslant \\ &2^{p_1\cdots p_{k-1}-1}|x_k^{p_1\cdots p_{k-1}} - \alpha_{k-1}^{p_1\cdots p_{k-1}}| = 2^{p_1\cdots p_{k-1}-1}|z_k|.\end{aligned}$$

With this in hand, we have

$$\begin{aligned}\omega_k \leqslant\ & 2^{1-\frac{1}{p_1\cdots p_{k-1}}}(2 - \frac{1}{p_1\cdots p_{k-1}})|z_k|^{\frac{1}{p_1\cdots p_{k-1}}}\cdot \\ &|x_k^{p_1\cdots p_{k-1}} - \alpha_{k-1}^{p_1\cdots p_{k-1}}|^{1-\frac{1}{p_1\cdots p_{k-1}}} = \bar{c}_k|z_k|,\end{aligned}$$

where $\bar{c}_k = 2^{1-\frac{1}{p_1\cdots p_{k-1}}}(2 - \dfrac{1}{p_1\cdots p_{k-1}})$. In view of above inequality, the following estimate is obtained:

$$\begin{aligned}&-\omega_k(\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial K}\dot{K} + \frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial y_\mathrm{r}}\dot{y}_\mathrm{r}) \leqslant \\ &a\bar{c}_k^2K(2 + (\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial y_\mathrm{r}})^2 + (\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial K})^2 + \\ &(\frac{\partial\alpha_{k-1}^{p_1\cdots p_{k-1}}}{\partial K})^2x_1^2)z_k^2 + \frac{2M^2+1}{aK} =: \\ &a\rho_{k2}(x_{[k-1]},y_\mathrm{r},K)z_k^2 + \frac{\Theta_{k2}}{K},\end{aligned} \tag{17}$$

where $\rho_{k2} : \mathbb{R}^{k-1}\times\mathbb{R}\times\mathbb{R} \to \mathbb{R}^+$ is positive.

Then, according to Lemma 3 and Proposition 4, meanwhile noting $\Phi(\cdot) \geqslant \Phi_3(\cdot)$, we get

$$-\omega_k\sum_{l=1}^{k-1}\frac{\partial x_{k-1}^{p_1\cdots p_{k-1}}}{\partial x_l}(d_lx_{l+1}^{p_l} + f_l) \leqslant$$

$$aK(\bar{c}_k^4(\sum_{l=1}^{k-1}\Upsilon_{k-1,l})^4 z_k^2 + \bar{c}_k^2(\sum_{l=1}^{k-1}\Upsilon_{k-1,l})^2 \cdot \bar{\gamma}_{k-1}^i)z_k^2 + \frac{1}{4\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{\tilde{\Theta}^2+\lambda^2\tilde{\Theta}^4}{aK} =: a\rho_{k3}(x_{[k]},y_{\rm r},K)z_k^2 + \frac{1}{4\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{\Theta_{k3}}{K}, \tag{18}$$

where $\rho_{k3}:\mathbb{R}^k\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}^+$ is positive and smooth.

Finally, in view of Proposition 1, it is not hard to get

$$z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}f_k \leqslant (\gamma_k^2(1+z_k^2)^{\frac{p_1\cdots p_{k-1}-1}{p_1\cdots p_{k-1}}} + \frac{(1+z_k^2)^{\frac{3p_1\cdots p_{k-1}-2}{p_1\cdots p_{k-1}}}}{4})\cdot aKz_k^2 + \frac{1}{2\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{1}{2}d_k|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}\cdot (|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}} + |\alpha_k|^{p_k}) + \frac{\lambda^2\bar{\Theta}^4+\bar{\Theta}^2}{aK} =: a\rho_{k4}(x_{[k]},y_{\rm r},K)z_k^2 + \frac{d_k}{2}|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}(|\alpha_k|^{p_k} + |z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}}) + \frac{1}{2\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{\Theta_{k4}}{K}, \tag{19}$$

where $\rho_{k4}:\mathbb{R}^k\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}^+$ is a positive smooth function. In addition

$$d_k z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}x_{k+1}^{p_k} \leqslant 2^{1-\frac{1}{p_1\cdots p_{k-1}}}d_k|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}\times |z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}} + d_k z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}\alpha_k^{p_k}.$$

By which and in view of inequality (19), we can further get

$$z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}(d_k x_{k+1}^{p_2} + f_k) \leqslant c_k\mu_k|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}} + a\lambda_k z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}\cdot \alpha_k^{p_k} + \frac{1}{2\lambda}\Phi^2\Psi + a\rho_{k4}(x_{[k]},y_{\rm r},K)z_k^2 + \frac{\Theta_{k4}}{K}, \tag{20}$$

where $c_k = 2^{\frac{p_1\cdots p_{k-1}-1}{p_1\cdots p_{k-1}}} + \frac{1}{2}$. Substituting inequalities (16)–(20) into inequality (15), the following inequality can be obtained:

$$\dot{V}_k \leqslant -\frac{\Psi(\cdot)}{\lambda} - \frac{n-k}{\lambda}\Phi^2(\cdot)\Psi(\cdot) - a\sum_{i=1}^{k-1}z_i^2 + \frac{\Theta}{\lambda} + c_k\mu_k|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}} + \frac{\Theta_k}{K} + \frac{a}{2}\lambda_k z_k^{2-\frac{1}{p_1\cdots p_{k-1}}}\alpha_k^{p_k} + a\rho_k(x_{[k]},y_{\rm r},K)z_k^2, \tag{21}$$

where $\rho_k = \sum_{i=1}^{4}\rho_{ki}$ is a positive smooth function, $\Theta_k = \sum_{i=1}^{4}\Theta_{ki}$ is an unknown positive constant.

Now, we choose the virtual controller α_k satisfying

$$\alpha_k^{p_1\cdots p_k} = -(\frac{\rho_k(x_{[k]},y_{\rm r},K)+1}{\lambda_k})^{p_1\cdots p_{k-1}}z_k =: -g_k(x_{[k]},y_{\rm r},K)z_k. \tag{22}$$

Substituting Eq.(22) into inequality (21), we finally obtain

$$\dot{V}_k \leqslant -\frac{\Psi(\cdot)}{\lambda} - \sum_{i=1}^{k}a_i z_i^2 - \frac{n-k}{\lambda}\Phi^2(\cdot)\Psi(\cdot) + \frac{\Theta_k}{K} + \frac{\Theta}{\lambda} + c_k\mu_k|z_k|^{2-\frac{1}{p_1\cdots p_{k-1}}}|z_{k+1}|^{\frac{1}{p_1\cdots p_{k-1}}}. \tag{23}$$

This completes Step k. When $k=n$, we choose the Lyapunov function as

$$V_n(x,\eta,y_{\rm r},K) = \frac{V_0}{\lambda} + \sum_{k=1}^{n}W_k.$$

Under the actual adaptive control

$$u = \alpha_n(x,y_{\rm r},K), \tag{24}$$

its time derivative satisfies

$$\dot{V}_n \leqslant -\frac{\Psi(\cdot)}{\lambda} - a\sum_{i=1}^{n}z_i^2 + \frac{\Theta_n}{K} + \frac{\Theta}{\lambda}. \tag{25}$$

Up to now, the recursive design procedure is finished.

3 Main results

Proposition 6 For any initial value $K(0)\geqslant 1$, $K(t)$ defined by inequality (13) is bounded on $[0,\infty)$.

Proof If the monotone nondecreasing, continuous function $K(t)$ given by Eq.(12) is unbounded, there must exists a finite time T_1, such that $K(t)\geqslant \frac{\lambda\Theta_n}{\Theta}$, $\forall t\geqslant T_1$. Therefore, from inequality (25), we can get

$$\dot{V}_n \leqslant -\frac{\Psi(\|\eta\|)}{\lambda} - a\sum_{i=1}^{n}z_i^2(t) + \frac{2\Theta}{\lambda},\ \forall t\geqslant T_1.$$

Now we define two compact sets:

$$N_1 = \{(z,\eta): \frac{\Psi(\|\eta\|)}{\lambda} + a\sum_{1}^{n}z_i^2 \leqslant \frac{\varepsilon_1^2}{a}\},$$

$$N_2 = \{(z,\eta): V_n \leqslant \frac{4\varepsilon_1^2}{a^2}\},$$

where $z(t)= (z_1(t),\cdots,z_n(t))^{\rm T}\in\mathbb{R}^n$. Then, for all $(z(t),\eta(t))^{\rm T}\in N_1$, we have

$$a(z_1^2(t)+\cdots+z_n^2(t)) \leqslant \frac{\varepsilon_1^2}{a}.$$

Since $\Psi(\|\eta\|)$ is a $\mathcal{K}$ function, using the monotone property of $\Psi(\|\eta\|)$, we can further get

$$\|\eta\| \leqslant \Psi^{-1}(2\Theta).$$

By above two inequalities, Assumption 1, Proposition 3 and the definition of λ, we know

$$V_n \leqslant \frac{1}{\lambda}\int_0^{\bar{\alpha}(\Psi^{-1}(2\Theta))}4L(s){\rm d}s + \frac{2\varepsilon_1^2}{a^2} = \frac{4\varepsilon_1^2}{a^2}.$$

This shows that $N_1\subseteq N_2$. Therefore, from inequality (25), we know $\dot{V}_n<0$ for all $(z(t),\eta(t))^{\rm T}\in\mathbb{R}^{n+m}-N_2$. In other words, the state $(z(t),\eta(t))^{\rm T}$ enters N_2 in a finite time. Consequently, there exits a finite T_2, such that

$$z_1^2(t) \leqslant V_n \leqslant \frac{4\varepsilon_1^2}{a^2}.$$

We can immediately get $|z_1(t)|\leqslant \frac{2\varepsilon_1}{a} = \frac{\varepsilon}{2}$. According to the definition of $K(t)$, we can finally get $\dot{K}(t)=0$, $\forall t\geqslant T$, $T=\max\{T_1,T_2\}$. This implies that $K(t)$ is bounded, which is a contradiction. Therefore, the monotone nondecreasing function K is bounded.

Now, we address the main results of this paper, which are summarized by the following theorem.

Theorem 1 For the higher-order nonlinear system (1) under Assumptions 1–4, the continuous partial-state adaptive controller (24) guarantees that

i) The closed-loop system state $(x(t),\eta(t),K(t))^{\mathrm{T}}$ is well-defined on $[0,+\infty)$ and globally bounded.

ii) $\forall\varepsilon>0$, there exits a finite time $T>0$, such that $|y(t)-y_{\mathrm{r}}(t)|\leqslant\varepsilon, \forall\, t\geqslant T$.

Proof We define two compact sets:

$$\varOmega_1=\{(z,\eta):\frac{\varPsi(\|\eta\|)}{\lambda}+a\sum_1^n z_i^2\leqslant\varTheta_n+\varTheta\},$$

$$\varOmega_2=\{(z,\eta):V_n\leqslant\int_0^{\bar\alpha(\varPsi^{-1}(\lambda(\varTheta_n+\varTheta)))}4L(s)\mathrm{d}s+\frac{2(\varTheta_n+\varTheta)}{a}\},$$

where $\bar\alpha(\cdot)$ is defined in Assumption 1. Then, for all $(z(t),\eta(t))^{\mathrm{T}}\in\varOmega_1$, we have

$$a(z_1^2(t)+\cdots+z_n^2(t))\leqslant\varTheta_n+\varTheta.$$

Using the monotone property of $\varPsi(\|\eta\|)$, we deduce

$$\|\eta(t)\|\leqslant\varPsi^{-1}(\lambda(\varTheta_n+\varTheta)).$$

By the definition of V_0 and noting $\lambda>1$, one can get

$$\frac{V_0}{\lambda}\leqslant\int_0^{\bar\alpha(\varPsi^{-1}(\lambda(\varTheta_n+\varTheta)))}4L(s)\mathrm{d}s.$$

From Proposition 3, we can conclude

$$\sum_{k=1}^n W_k\leqslant 2\sum_{k=1}^n z_k^2(t).$$

Using above inequalities, the following inequality can be obtained

$$V_n\leqslant\int_0^{\bar\alpha(\varPsi^{-1}(\lambda(\varTheta_n+\varTheta)))}4L(s)\mathrm{d}s+\frac{2(\varTheta_n+\varTheta)}{a},$$

which implies $\varOmega_1\subseteq\varOmega_2$. As a result, it is immediate to deduce from inequality (25) that $\dot V_n<0$ for all $(z(t),\eta(t))^{\mathrm{T}}\in\mathbb{R}^{n+m}-\varOmega_2$. Therefore, the state trajectories $(z(t),\eta(t))^{\mathrm{T}}$ of the closed-loop system enter $\varOmega_2$ in a finite time and stay in $\varOmega_2$ thereafter, in other words, the state $(z(t),\eta(t))^{\mathrm{T}}$ is globally bounded. Then by the transforming relationship between $x(t)$ and $z(t)$, we can immediately deduce all the state $(x(t),\eta(t),K(t))^{\mathrm{T}}$ is globally bounded.

Finally, it is not difficult to prove that $\dot K(t)$ is uniformly continuous with respect to t, since $z_1(t)$ is uniformly continuous. Moreover,

$$\lim_{t\to\infty}\int_0^t\dot K(\tau)\mathrm{d}\tau\ =\ K(\infty)-K(0)<+\infty.$$

By Lemma 4, we can finally get $\lim\limits_{t\to\infty}\dot K(t)=0$. This and Eq.(12) imply the existence of a finite $T>0$, such that $|y(t)-y_{\mathrm{r}}(t)|\leqslant\varepsilon,\forall t\geqslant T$.

Remark 3 It is necessary to emphasize that λ greater than 1 is mainly used to dominate unknown zero dynamics and all unknown parameters coming from the system to be investigated. Specifically, compared with the reference [20], a remarkable feature of this paper is the existence of zero dynamics $\eta(t)$. To effectively deal with them, we first separate $\eta(t)$ from $x(t)$ by the idea of changing supply functions. Then, in light of the delicate choice of $L(\cdot)$, and including the term '$\frac{a^2}{2\varepsilon_1^2}\int_0^{\bar\alpha(\varPsi^{-1}(2\varTheta))}4L(s)\mathrm{d}s$' in the expression of λ, one can implement the domination of $\varPsi(\|\eta\|)$. On the other hand, the actual control $u(t)$ can't stabilize some positive terms composed of unknown parameters, and one can see Step k for details. To overcome this obstacle, we introduce the term '$\frac{2a\varTheta}{\varepsilon_1^2}$' into λ.

4 Simulation

Consider the following higher-order uncertain nonlinear system:

$$\begin{cases}\dot\eta=-\eta^3+\theta_1\eta\sin(1+x_1x_2),\\ \dot x_1=\theta_2(2-0.7\sin t)x_2^3+x_1,\\ \dot x_2=\theta_3u+\theta_4x_1x_2,\end{cases}$$

where θ_i, $i=1,\cdots,4$ are unknown positive constants. It is easy to verify that the system satisfies Assumptions 1–3 with $1.3\theta_2\leqslant d_1=\theta_2(2-0.7\sin t)\leqslant 2.7\theta_2$, $d_2=\theta_3$, $\lambda_1=\mu_1=\lambda_2=\mu_2=1$. Choose $a=\min\{1.3\theta_1,\theta_3\}$, $y_{\mathrm{r}}=\sin t$. Based on the above design procedure, we get the actual controller

$$u=-K((1+z_2^4)^{\frac13}x_1x_2+2.7^2z_1^2+\frac{25}{9}4^{\frac13}(z_1^2(1+4z_1^2)+4K^2+(1+4K^2)(z_2^2+4K^2z_1^2))+2)z_2,$$

where $z_1=x_1-\sin t$, $z_2=x_2^3+2Kz_1$.

In simulation, choose $\theta_1=0.3$, $\theta_2=1$, $\theta_3=0.2$, $\theta_4=1$, $\varepsilon=0.2$ and set the initial conditions as $\eta(0)=1$, $x_1(0)=0.4$, $x_2(0)=0.7$, $K(0)=1$. Figs.1 and 2 demonstrate the effectiveness of the adaptive practical tracking controller, i.e., the tracking error satisfies $|y(t)-y_{\mathrm{r}}(t)|\leqslant 0.2$, and the gain $K(t)$ is monotone nondecreasing and bounded on $[0,\infty)$.

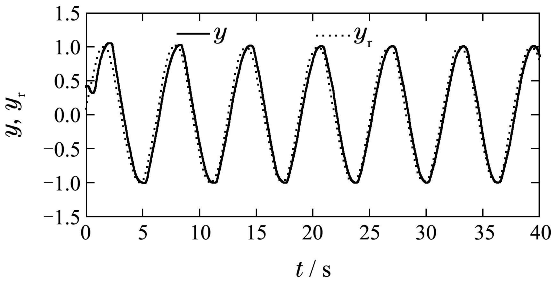

Fig. 1 The trajectories of $y(t)$ and $y_{\mathrm{r}}(t)$

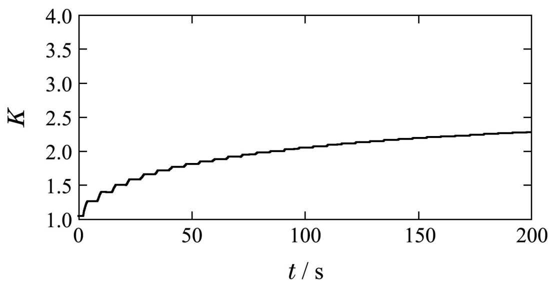

Fig. 2 The trajectory of $K(t)$

5 Conclusions

In this paper, a systematic approach has been developed to design a continuous partial-state adaptive controller for a class of higher-order nonlinear system with uncertain control coefficients and zero dynamics.

References:

[1] QIAN C J, LIN W. Recursive observer design, homogeneous approximation, and nonsmooth output feedback stabilization of nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 2006, 51(9): 1457 – 1471.

[2] XIE X J, TIAN J. Adaptive state-feedback stabilization of high-order stochastic systems with nonlinear parameterization [J]. *Automatica*, 2009, 45(1): 126 – 133.

[3] SUN Z Y, LIU Y G. Adaptive state-feedback stabilization for a class of high-order nonlinear uncertain systems [J]. *Automatica*, 2007, 43(10): 1772 – 1783.

[4] SUN Z Y, LIU Y G. Adaptive stabilization for a large class of high-order uncertain nonlinear systems [J]. *International Journal of Control*, 2009, 82(7): 1275 – 1287.

[5] ZHANG J, LIU Y. A new approach to adaptive control design without overparametrization for a class of uncertain nonlinear systems [J]. *Science China Information Sciences*, 2011, 54(7): 1419 – 1429.

[6] LIN W, QIAN C J. Adding one power integrator: a tool for global stabilization of high-order lower-triangular systems [J]. *Systems & Control Letters*, 2000, 39(5): 339 – 351.

[7] QIAN C J, LIN W. A continuous feedback approach to global strong stabilization of nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 2001, 46(7): 1061 – 1079.

[8] LIN W, QIAN C J. Adaptive control of nonlinearly parameterized systems: a nonsmooth feedback framework [J]. *IEEE Transactions on Automatic Control*, 2002, 47(5): 757 – 774.

[9] QIAN C J, LIN W. Practical output tracking of nonlinear systems with uncontrollable unstable linearization [J]. *IEEE Transactions on Automatic Control*, 2002, 47(1): 21 – 36.

[10] BARTOLINI G, FERRARA A, USAI E. Output tracking control of uncertain nonlinear second-order systems [J]. *Automatica*, 1997, 33(12): 2203 – 2212.

[11] HUANG J. Asymptotic tracking and disturbance rejection in uncertain nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 1995, 40(6): 1118 – 1122.

[12] KHALIL H K. Robust servomechanism output feedback controller for feedback linearizable system [J]. *Automatica*, 1994, 30(10): 1587 – 1599.

[13] ISIDORI A, BYRNES C I. Output regulation of nonlinear system [J]. *IEEE Transactions on Automatic Control*, 1990, 35(2): 131 – 140.

[14] MARCONI L, ISIDORI A. Mixed internal model-based and feedforward control for robust tracking in nonlinear systems [J]. *Automatica*, 2000, 36(7): 993 – 1000.

[15] BYRNES C I, PSICOLI F D, ISIDORI A. *Output Regulation of Uncertain Nonlinear Systems* [M]. Boston: Birkhauser, 1997.

[16] LIN W, PONGVUTHITHUM R. Adaptive output tracking of inherently nonlinear systems with nonlinear parameterization [J]. *IEEE Transactions on Automatic Control*, 2003, 48(10): 1737 – 1749.

[17] QIAN C J, LIN W. A continuous feedback approach to global strong stabilization of nonlinear systems [J]. *IEEE Transactions on Automatic Control*, 2001, 46(7): 1061 – 1079.

[18] LIN W, PONGVUTHITHUM R. Nonsmooth adaptive stabilization of cascade systems with nonlinear parameterization via partial-state feedback [J]. *IEEE Transactions on Automatic Control*, 2003, 48(10): 1809 – 1816.

[19] SUN Z Y, SUN W. Global adaptive stabilization of high-order nonlinear systems with zero dynamics [J]. *Acta Automatica Sinica*, 2012, 38(1): 64 – 71.

[20] SUN Z Y, LIU Y G. Adaptive practical output tracking control for high-order nonlinear systems [J]. *Acta Automatica Sinica*, 2008, 34(8): 984 – 988.

[21] SUN Z Y, LIU Y G, XIE X J. Global stabilization for a class of high-order time-delay nonlinear systems [J]. *International Journal of Innovative Computing, Information and Control*, 2011, 7(12): 7119 – 7130.

[22] LIN W, QIAN C J. Addingonepower integrator: a tool for global stabilization of high-order lower-triangular systems [J]. *Systems Control & Letters*, 2000, 39(5): 339 – 351.

[23] KHALIL H K. *Nonlinear Systems* [M]. 3rd edition. New Jersey: Prentice Hall, 2002.

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