A New Indentification Algorithm for Deterministic ARMA Models by Using Incremental Matrices

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Abstract: In this paper, a new method is developed which gives unbiased estimates of the parameters and the order for a deterministic ARMA model. Being different from any other identification algorithms, this method employs the incremental matrices of the open-loop step response of the systems. several algebraic properties of the sub-matrix of incremental matrix are analysed and a simple method to estimate the order of system is deduced. Simulation results show the correctness of the theory proposed.

Key works: system indentification; incremental matrix; simulation

1 Introduction

So far, many methods[1] have been developed which are used to identify the unknown parameters, time delay as well as the order of deterministic systems expressed by

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})u(t), \tag{1.1}$$

where $A(q^{-1})=1+\sum_i a_i q^{-i}$ and $B(q^{-1})=\sum_j b_j q^{-j}$, and $\{y(t)\}$, $\{u(t)\}$ are output and input sequences respectively, d is the time delay and q^{-1} is the unit delay operator (i. e. $q^{-1}y(t)=y(t-1)$ for any t).

In the identification of unknown parameters $a_i(i=1,2,\cdots,n)$ and $b_j(j=0,1,\cdots,m)$ in the case of known n, m and d, the most representative method was the Recursive Least Square[1] (RLS) algorithm which uses the input-output data to recursively estimate the values of a_i and b_j. However, in the practical point of view, a persistent excitation[2,3] input signal is needed in order to obtain an unbiased estimate in most cases, even though this requirement was not necessary. This would lead to some undesirable behaviours for many practical systems. As we know, in many physical systems, a finite number of data of open loop step response can be obtained whether the open-loop system is stable or not. So the first problem is to look for an off-line unbiased estimation method for unknown parameters a_i and b_j by using only finite number or a minimum number data of open-loop step response? This problem will be solved in section Ⅱ of this paper.

To identify the orders of system (1.1), a widely used method is first apply the recursive es-

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timation algorthm, e. g. RLS, to identify parameters with different pre-specified orders and then the estimates of orders n and m are obtained by optimizing some functional index. However, this methos is time consuming. So it is also necessary to study the unbiased order estimation by using the finite number of data of the output step response.

2 Incremental Matrix and the Identification Algorithm with known Orders

As for system (1,1) the initial conditions are as follows

$$y(t) = u(t) = 0, \qquad t < 0. \tag{2.1}$$

To obtain the output data of the system (1. 1), a step input signal $u(t)=u(0)=$ Constant, $(t \geqslant 0)$ is applied to the system and the response sequence of the system (1. 1) is measured, which is described by the finite set $I_y(N)=\{0,0,\cdots,y(d),y(d+1),\cdots,y(N)\}$ where $N>n+m+d-1$. By system equation (1. 1), we have

$$y(d) = b_0u(0), \quad y(d) \neq 0. \tag{2.2}$$

From this equation, the unbiased estimate for b_0 is

$$b_0 = y(d)/u(0). \tag{2.3}$$

and the unbiased estimate of d is

$$\hat{d} = \{t_0 \,|\, y(t_0) \neq 0 \quad \forall\, t \in (-\infty, t_0 - 1), y(t) = 0\} = d. \tag{2.4}$$

Also, let $t=d+1$, we have

$$y(d+1) + a_1y(d) = b_0u(1) + b_1u(0). \tag{2.5}$$

since $u(1)=u(0)$ and $u(0)b_0=y(d)$, the above equation becomes

$$a_1y(d) - b_1u(0) = -\Delta y(d). \tag{2.6}$$

where $\Delta y(t) \triangleq y(t+1)-y(t)$, for any t. Let $t=d+2$, we can also get

$$a_1\Delta y(d) + a_2y(d) - b_2u(0) = -\Delta y(d+1). \tag{2.7}$$

Generally, the following algebraic equations

$$\begin{aligned} &a_1\Delta y(d+j) + a_2\Delta y(d+j-1) + \cdots + a_{j+1}\Delta y(d) + a_{j+2}y(d) - b_{j+2}u(0) \\ &= -\Delta y(d+j+1); \quad 0 \leqslant j \leqslant m-2; \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} &a_1\Delta y(d+j) + a_2\Delta y(d+j-1) + \cdots + a_{j+1}\Delta y(d) + a_{j+2}y(d) \\ &= -\Delta y(d+j+1) \qquad m-1 \leqslant j \leqslant n-2; \end{aligned} \tag{2.9}$$

$$\begin{aligned} &a_1\Delta y(d+j) + a_2\Delta y(d+j-1) + \cdots + a_{j+1}\Delta y(d+j-n+1) \\ &= -\Delta y(d+j+1); \qquad j \geqslant n-1, \end{aligned} \tag{2.10}$$

can be obtained. write equations (2. 8)-(2. 10) in matrix form as

$$A\theta = \beta, \tag{2.11}$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}, \tag{2.12}$$

$$A_{12} = \mathrm{diag}(-u(0), -u(0), \cdots, -u(0)) \in R^{m\times m}, \tag{2.13}$$

$$A_{11} = \begin{bmatrix} y(d) & 0 & 0 & 0 & 0 & 0 \\ \Delta y(d) & y(d) & 0 & 0 & 0 & 0 \\ \Delta y(d+1) & \Delta y(d) & y(d) & 0 & 0 & 0 \\ \vdots & & & & & \vdots \\ \Delta y(d+m-2) & \cdots & \Delta y(d) & y(d) & & 0 \end{bmatrix} \in R^{m\times n}, \tag{2.14}$$

$$A_{21} = \begin{bmatrix} \Delta y(d+m-1) & \Delta y(d+m-2) & \cdots & \Delta y(d) & y(d) & 0 & 0\cdots 0 \\ \Delta y(d+m) & \Delta y(d+m-1) & \cdots & \Delta y(d+1) & \Delta y(d) & y(d) & \cdots 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta y(d+n-2) & \Delta y(d+n-3) & \cdots & \cdots & \cdots & \cdots & y(d) \\ \Delta y(d+n-1) & \Delta y(d+n-2) & \cdots & \cdots & \cdots & \cdots & \Delta y(d) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta y(d+n+m-2) & \Delta y(d+n+m-3) & \cdots & \cdots & \cdots & \cdots & \Delta y(d+m-1) \end{bmatrix},$$

$$\theta_1 = [a_1, a_2, \cdots, a_n]^T \in R^n, \theta_2 = [b_1, b_2, \cdots b_m]^T \in R^m, \tag{2.15}$$

$$\beta_1 = [-\Delta y(d), -\Delta y(d+1), \cdots, -\Delta y(d+m-1)]^T \in R^m, \tag{2.16}$$

$$\beta_2 = [-\Delta y(d+m), -\Delta y(d+m+1), \cdots, -\Delta y(d+n+m-1)]^T \in R^n. \tag{2.17}$$

Denote $\hat{\theta}_1$ and $\hat{\theta}_2$ $(\hat{\theta}_1 = [\hat{a}_1, \hat{a}_2, \cdots, \hat{a}_n] \in R^n, \hat{\theta}_2 = [\hat{b}_1, \hat{b}_2, \cdots, \hat{b}_m] \in R^m$ where, a_i, b_j are estimated values of a_i and b_j) as the estimates of θ_1 and θ_2, and solve equation (2.11), we can obtain the unbiased estimates $\hat{\theta}_1$ and $\hat{\theta}_2$ as

$$\hat{\theta}_1 = A_{21}^{-1}\beta_2, \tag{2.18}$$

$$\hat{\theta}_2 = A_{12}^{-1}[\beta_1 - A_{11}\hat{\theta}_1] \tag{2.19}$$

From notation (2.13), it can be seen that A_{12}^{-1} always exists since $u(0) \neq 0$. Thus the existence of unique solution of the equation (2.11) depends on the non-singularty of matrix A_{21}. So it is necessary to prove that A_{21}^{-1} exists. This leads to the following theorem.

Theorem 2.1 As for the system (1.1), if $a_n \neq 0$ and the polynomials $A(q^{-1})$ and $B(q^{-1})$ are prime, then

(i) $\det A_{21} \neq 0$;

(ii) $\hat{\theta}_1 = \theta_1, \hat{\theta}_2 = \theta_2$.

Proof Denote

$$A_{21}^T = [\sigma_1, \sigma_2, \cdots, \sigma_n], \quad \sigma_i \in R^n, i = 1, 2, \cdots, n, \tag{2.20}$$

where σ_i is the i-th row of matrix A_{21}. At first, we will show that $\sigma_i \neq 0$ for any i from 1 to n. If this is not true, we assume that there exists i_o such that

$$\sigma_{i_o} = 0,$$

$$i_o = \min\{i \mid \sigma_i = 0\}. \tag{2.21}$$

Obviously, $i_o > n - m$. Moreover, by the structure of matrix A_{21} we have $\sigma_i = 0$ for any $i \geqslant i_o$. Rewriting matrix A_{21} as

$$
A_{21}=\left[\begin{array}{cccccc}
\Delta y(d+m-1) & \Delta y(d+m-2)\cdots\ \Delta y(d) & y(d) & 0\cdots 0 & & 0 \\
\Delta y(d+m) & \Delta y(d+m-1)\cdots\Delta y(d+1) & \Delta y(d) & y(d) & & \cdots 0 \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
\Delta y(d+n-2) & \cdots & \cdots & \cdots & \cdots & y(d) \\
\cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\
\Delta y(d+m+i_o-3) & \cdots & \cdots & & & \Delta y(d+m+i_o-n-2)) \\
0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 0
\end{array}\right],
$$

we can see that $\Delta y(d+m+i_o-n-2)\neq 0$, otherwise $\sigma_{i_o-1}=0$ and which contradicts to the definition of i_o. From equation (2. 9) we have

$$
\begin{aligned}
-\Delta y(d+m+i_o-2) = & a_1\Delta y(d+m+i_o-3)+a_2\Delta y(d+m+i_o-4)+\cdots \\
& +a_n\Delta y(d+m-n+i_o-2).
\end{aligned} \tag{2. 22}
$$

Because $\sigma_i=0$ when $i>i_o$ and by (2. 22), we have

$$
a_n\Delta y(d+m+i_o-n-2)=0, \tag{2. 23}
$$

and since $\Delta y(d+m+i_o-n-2)\neq 0$, we can conclude that $a_n=0$, this also contradicts to the condition of the theorem.

Secondly, we want to show that the n rows of matrix A_{21} are linear independent. To show this, we also assume that there exist α_i with $i=1,2,\cdots,n$ and $\sum_{i=1}^{n}\alpha_i^2\neq 0$, such that

$$
\sum_{i=1}^{n}\alpha_i\sigma_i=0. \tag{2. 24}
$$

Without loss of generality, let $\alpha_n\neq 0$ and from (2. 24), $-\sigma_n$ can be expressed as

$$
-\sigma_n=\sum_{i=1}^{n-1}\bar{\alpha}_i\sigma_i,\quad \bar{\alpha}_i=\alpha_i/\alpha_n,\qquad i=1,2,\cdots,n. \tag{2. 25}
$$

Construct matrix $\bar{A}_{21}$ as

$$
\bar{A}_{21}=[A_{21}\,|\,\beta_2], \tag{2. 26}
$$

and let the last row of matrix A_{21} be added by $\sum_{i=1}^{n-1}\bar{\alpha}_i\sigma_i$ and the last term of β_2 be added by $\bar{\alpha}_{n-1}\Delta y(d+m+n-2)+\bar{\alpha}_{n-2}\Delta y(d+m+n-3)+\cdots+-\bar{\alpha}_1\Delta y(d+m)$. Because both (2. 9) and (2. 25) are satisfied, so we can see that

$$
\begin{aligned}
-\Delta y(d+m+n-1) = & \bar{\alpha}_1\Delta y(d+m)+\bar{\alpha}_2\Delta y(d+m+1)+\cdots \\
& +\bar{\alpha}_{n-1}\Delta y(d+n+m-2).
\end{aligned} \tag{2. 27}
$$

By the similar way it can be proved that the following equation

$$
\begin{aligned}
-\Delta y(d+j) = & \bar{\alpha}_{n-1}\Delta y(d+j-1)+\bar{\alpha}_{n-2}\Delta y(d+j-2)+\cdots \\
& +\bar{\alpha}_1\Delta y(d+j-n+1),
\end{aligned}
$$

holds for $\forall\ j\geqslant n+m-1$. This means that the system (1. 1) can also be expressed by $n-1$ order ARMA model, which also contradicts with the condition of the theorem. Thus (i) holds and there exists an unique solution for equation (2. 11). Q. E. D.

3 The Identification of Orders n and m

In the above section, we have shown that the matrix A_{21} is nonsingular if n and m are known exactly. It is natural that there must be some relations among the determinant of matrix A_{21} and the orders of polynomials $A(q^{-1})$ and $B(q^{-1})$. In this section we will give a theorem which reveals these relations. Denote $A_{21}^{(n,m)} \triangleq A_{21}$ and $A_{21}^{(n+p,m+r)}$ as

$$\begin{bmatrix} \Delta y(d+m+r-1) & \Delta y(d+m+r-2) & \cdots & \Delta y(d) & y(d) & 0\cdots 0 & 0 \\ \Delta y(d+m+r) & \Delta y(d+m+r-2) & \cdots & \Delta y(d+1) & \Delta y(d) & y(d) & \cdots 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta y(d+n+p-2) & \Delta y(d+n+p-3) & \cdots & \cdots & \cdots & \cdots & y(d) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \Delta y(d+n+p+m+r-2) & \cdots & \cdots & \cdots & \cdots & \cdots & \Delta y(d+m+r-1) \end{bmatrix},$$

then we can obtain the following theorem.

Theorem 3. 1 As for system (1. 1), the relations

i) $\det A_{21}^{(n+p,m)} = \det A_{21}^{(n,m)} b_m^p u^p(0)$ for $\forall\ p \geqslant 1$;

ii) $\det A_{21}^{(n,m+r)} = (-1)^{n \times r} a_n^r \det A_{21}^{(n,m)}$ for $\forall\ r \geqslant 1$;

iii) $\det A_{21}^{(n+r,m+r)} = 0$ for $\forall\ r \geqslant 1$;

hold.

In order to identify the orders n and m by using theorem 3. 1, the normalized expression of system (1. 1) is introduced by

$$(1 + a_1 q^{-1} + a_2 q^{-2} + \cdots + a_n q^{-n}) y(t)$$
$$= q^{-d}(b_0 + b_1 q^{-1} + \cdots b_m q^{-m} + \underbrace{0 \cdot q^{-m-1} + \cdots 0 \cdot q^{-n+1}}_{n-m-2}) u(t)$$

and the following algorithm is obtained.

Algorithm

1° Let M is a number which is larger than the true order n^*, for example $M = 100$;

2° Calculate $\det A_{21}^{(n,n-1)}$ with the order n from 2 to M;

3° Choose n^* which satisfies

$$n^* = \min\{n \mid \det A_{21}^{(n,n-1)} \neq 0 \quad \text{and} \quad \det A_{21}^{(p,p-1)} = 0, \quad p > n\}$$

and this n^* is the true order of polynomial $A(q^{-1})$;

4° The correct order m^* is determined from $m^* = \max\{j \mid b_j = 0\}$.

4 Simulation Examples

Example 1 (Stable system) Consider the system

$$(1 - 1.6q^{-1} + 0.63q^{-2}) y(t) = q^{-3}(1 + 0.23q^{-1}) u(t),$$

with $u(t) = 1.0$ when $t \geqslant 0$, The output of the step response is shown in Table 1, and the matrix $A_{21}^{(2,1)}$ is

$$A_{21}^{(2,1)} = \begin{bmatrix} 1.8300 & 1.0000 \\ 2.2980 & 1.8300 \end{bmatrix}.$$

The estimated parameters are $\hat{a}_1 = -1.6000$, $\hat{a}_2 = 0.6299$, $\hat{b}_0 = 1.0000$, $\hat{b}_1 = 0.2300$, $\hat{d} = 3$.

Example 2 Let the system be

$$(1 + 0.9q^{-1} + 0.3q^{-2+q^{-3}}) y(t) = q^{-3}(0.3 + 0.6q^{-1} + 0.2q^{-2}) u(t),$$

and also with $u(t) = 1.0$ when $t \geqslant 0$. By using the algorithm of section III, the $\max\{n, m\} = 3$ is obtained with $M=7$ and the relation between $f_1(n)=\det A_{21}^{(n,n-1)}/\det A_{21}^{(n-1,n-2)}$ and n is shown in Fig. 1. And also we can get the $m^*=2$ and the estimates of the parameters are $\hat{a}_1=0.8999$, $\hat{a}_2=0.3000$, $\hat{a}_3=0.9999$; $\hat{d}=3$; $\hat{b}_0=0.3000$, $\hat{b}_1=0.6000$, $\hat{b}_2=0.2000$. In order to illustrate the correctness of theorem 3.1, simulation with n fixed to 3 is also carried and the corresponding result is shown in Fig. 2, where $f_2(m)=\det A_{21}^{(3,m)}/\det A_{21}^{(3,m-1)}$. Fig. 3 shows the simulation result with $m=2$ and n ranking from 2-7, where $f_3(n)=\det A_{21}^{(n,2)}/\det A_{21}^{(n-1,2)}$. From these simulations we can see that the theoretical results of the previous two sections are correct.

Table 1 Step response data and their incremental values

t	0	1	2	3	4	5
$y(t)$	0	0	0	1	2.83	5.128
$\Delta y(t)$	0	0	1	1.83	2.298	2.523

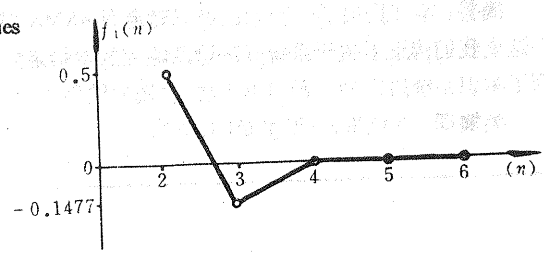

Fig. 1 The relation between $f_1(n)$ and n

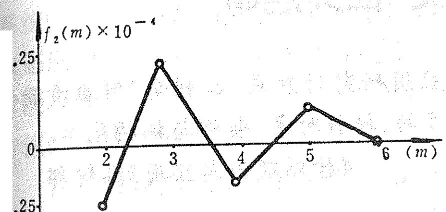

Fig. 2 The relation between $f_2(m)$ and m

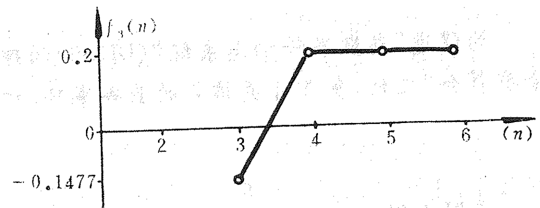

Fig. 3 The relation between $f_3(n)$ and n

5 Conclusions

As for the identification of both parameters and the orders of unknown ARMA model by only using the step response data, an incremental matrix can be constructed by using the incremental values of the step response. The orders as well as the parameters can thus be identified by simply algebraic calculations. The main advantages of this incremental method are

1° Only minimum (or finite) information of the systems is needed;

2° The input signals $\{u(t)\}$ need not to be persistent excited;

3° There is no requirement on the stability of open-loop systems.

Thus this method is simple in calculation and can be easily applied in real systems.

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利用增量矩阵辨识确定型 ARMA 模型的新算法

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摘要：本文提出了一种无偏辨识确定型 ARMA 模型参数及阶次的新方法. 和其他辨识算法不同的是在这里我们构造了关于系统开环阶跃响应的增量矩阵，并且通过分析该增量矩阵子矩阵的代数性质而得到了辨识系统阶次的一种简单方法. 仿真的结果说明了本文理论的正确性.

关键词：系统辨识；增量矩阵；仿真

"何潘清漪优秀论文奖"征文启事

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