Predictive State Feedback Control of Real-Time Discrete Event Systems

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Abstract: In this paper we extend the results of Predictive Supervisory Control(PSC) to the case when the control objective is to maintain some predicates invariant in the closed loop system. Conditions for the existence as well as synthesis procedure of Predictive State Feedback Controller(PSFC) are discussed. For a special class of Discrete Event Systems(DES), we show that an optimal solution of traditional State Feedback Control is also optimal for PSFC, thus renders our approach attractive. Finally, we give an example to illustrate the results.

Key words: discrete event systems; feedback control; invariance

1 Introduction

Great achievements have been made on control of Discrete Event Systems(DES) modeled as controlled automata after the pioneering work of Ramadge & Wonham[1]. Results include modular synthesis, decentralized and hierarchical control, etc[2][3][4][5][6]. Extension on control form of the above mentioned results has been made from event-based feedback control to state-based feedback control[7][8].

Although the results are complete theoretically, however, as is well known[9], the Ramadge-Wonham Theory(RWT) is based on two idealized assumptions on controlled plant, namely:

(1) Communications between the plant G and supervisor F take place in zero time delay. In particular a new control pattern is imposed on controllable event just as soon as F makes the appropriate state transition in response to an input label from G.

(2) In case the plant G is defined as the shuffle of component generators $G_1, G_2, \cdots, G_N$, say, events in distinct G_i occur in an interleaving fashion. That is, simultaneity of events in distinct G_i is ruled out.

These assumptions greatly obstacled the practical applicability of RWT.

Li & Wonham[9] solved the problem by introducing the concept of well-posedness with respect to(wrt) time delay. It is shown that there is a complete and well-posed supervisor F s. t. $L(F/G)=K$ if and only if K is closed, controllable and well-posed.

However, as pointed out in[10], there are two drawbacks in [9]:

(1) In order to disable some controllable events, disablement signals should be sent at least one state "earlier" than the actual system state, say q''(correspondingly, x'' in the supervisor) of

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Fig 1. 1. Such an "earlier" control action will affect the legal occurence of the same controllable event in other states following q' (i. e. , $\beta \in \Sigma(q_3)$ in Fig. 1).

(2) No optimal solution exists for Well-Posed Supervisory Control Problem (WPSCP). And until now no suboptimal solution is proposed.

In connection with timing aspect of events, for some systems (e. g. , database management system[11]), controllable event should not be disabled while it is executing. This is quite different as stated in [9].

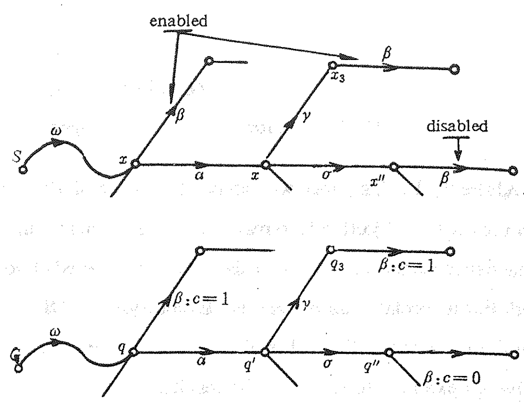

Fig. 1 Illustrating example for drawback (1) of [9]

Based on the above mentioned facts , we proposed in [10] the Predictive Supervisory Control scheme to treat the first problem arised in [1]. The term "predictive" implies that the control action of the supervisor takes "one step earlier" than the actual system state. Some interesting synthesis as well as language aspects of predictive supervisory control problem were discussed.

The main motivation of this paper is to extend the results of [10] to the case when a (static) state feedback is employed. We will show when a predictive state feedback controller exist and how it is constructed.

This paper is organised as following. In section 2 we review some basic results of PSC. In section 3 existence conditions and construction procedure of PSFC are discussed. In section 4 we consider PSFC of a special class of DES. Section 5 gives an illustrating example while section 6 is a conclusion.

2 Predictive Supervisory Control: a brief review

From related theories on DES, we know that an event is, actually, a process[12]. Control signals can affect the occurence of a controllable event only at the very begining of the execution of that event. For example, a repair action in a manufacturing unit, a write action in a database management system[11], etc. When a system is carrying out one controllable event, any change in the status of the corresponding control channel is irrelevant to the occurence of the event.

In [10] two assumptions were made:

Assumption 1 Whether or not a controllable event could be prevented from occuring is decided by the control status of that event at the instant when its carrying state occurs.

Assumption 2 Let $T(\sigma, q)$ be the occurence duration of σ at q. Define[9]

$$T_{\min} = \min_{\sigma, q}\{T(\sigma, q)\}.$$

Then $T_a + T_b \leqslant T_{\min}$.

Assumption 2 is to guarantee the effectiveness of predictive control.

Throughout the paper these assumptions still hold for the considered systems.

With these assumptions and several definitions, we have

Proposition 2. 1[10] There is a complete & conflict-free predictive supervisor F s. t. $L(F/G)=K$ if and only if K is closed, controllable and predictable.

Define the following classes of languages

$C'(L)=\{K:K\subset L, K$ is closed, controllable and predictable$\}$

$C(L)=\{K:K\supset L, K$ is closed, controllable and predictable$\}$

We have

Proposition 2. 2[10] (1). $C'(L)$ is not closed under arbitrary union; (2). $C(L)$ is closed under arbitrary intersection.

Following proposition 2. 2, it is clear that no optimal solution to PSCP is guaranteed to exist in general. However, for a special class of DES, an optimal solution does exist. This topic will be reconsidered in section 4.

3 Predictive State Feedback Control

In some cases, control objects are expressed in terms of predicates, i. e. , to maintain some predicates invariant by employing control mechanisms. It is shown that for such control tasks, a state feedback control police is more convenient than an event-feedback one[16].

Let $G=\{\Sigma, Q, \delta, C_a, q_0\}$ be the controlled plant[7]. For a given predicate $P\subset 2^Q$, suppose $q_0\in P$, we define recursively the state set $\mathrm{Re}\{G,P\}$ as following[7]:

(1) $q^0\in Re\{G,P\}$; (2). if $q\in \mathrm{Re}(G,P)$, $q\in P$ and $q\in C_a$ for some $a\in\Sigma$, then $\delta\{a,q\}\in \mathrm{Re}\{G,P\}$; (3). Every state in $\mathrm{Re}(G,P)$ is obtained in (1) or (2).

Next, we define predicate transformation $wlp_a: 2^Q\rightarrow 2^Q$ as following:

$$wlp_a(P)(q)=\begin{cases}1, \text{if } C_a(q)=1, \delta(a,q)\in P \text{ or } C_a(q)=0,\\ 0, \text{otherwise.}\end{cases}$$

P is called control-invariant[2][7] if $P\leqslant wlp_a(P)$ $\forall$ $a\in\Sigma_u$. P is called controllable[7] if $P\leqslant wlp_a(P)\cap \mathrm{Re}\{G,P\}$ $\forall$ $a\in\Sigma_u$.

With these terminologies, it is now ready for us to discuss Predictive State Feedback Control (PSFC).

Definition 3. 1 Let $P\subset 2^Q$. P is called predictable if

(1) $q_0\in P$;

(2) $\forall$ $\sigma\in\Sigma(q_0)$, $\delta(\sigma,q_0)\in P$; and

(3) $\forall$ $q\in P, a_1, a_2\in\Sigma, \delta(a_1,q)\in P, \delta(a_2,q)\in P$, there is no $\sigma\in\Sigma_c$ such that $C_a(\delta(a_1,q))=C_a(\delta(a_2,q))=1$ with $\delta(\sigma,\delta(a_1,q))\in P\wedge\delta(\sigma,\delta(a_2,q))\notin P$.

The Predictive State Feedback Controller is constructed as following. Let $S(f/G)\subset 2^Q$ be the given set of closed loop state trajectories. For $q\in S(f/G)$, $C_{a_1}(q)=1$, $\delta(a_1,q)\in S(f/G)$, $a\in\Sigma$, $C_a(\delta(a_1,q))=1$, define the feedback function f as:

$$f_a(q)=\begin{cases}0, \text{if } \delta(\alpha_1,\delta(\alpha_1,q)) \notin S(f/G \wedge \alpha\in\Sigma_c, \\ 1, \text{if } \delta(\alpha,\delta(\alpha_1,q)) \in S(f/G \wedge \alpha\in\Sigma_c \text{ or } \sigma\in\Sigma_u\cap\Sigma(q').\end{cases} \quad (3.1)$$

Definition 3. 2 PSFC f is called conflict-free if (1). $\forall\ \alpha\in\Sigma(q_0), \delta(\alpha,q_0)\in S(f/G)$; (2). there is no $\alpha\in\Sigma_c, q\in Q$ such that $f_a(q)=\{1,0\}$.

Definition 3. 3 PSFC f is called complete if (1). $q\in S(f/G)$; (2). $\delta(\alpha_1,q))\in S(f/G)$, $\alpha\in\Sigma, C_a(\delta(\alpha_1,q))=1$; (3) $f_a(q)=1$ together imply that (4). $\delta(\alpha,\delta(\alpha_1,q))\in S(f/G)$.

The following proposition establishes the relation between controllable predictable predicates and complete conflict-free predictive PSFC.

Proposition 3. 1 There is a complete conflict-free predictive PSFC f such that $S(f/G)=P$ if and only if P is controllable and predictable.

We omit the detailed proof here for the sake of clarity. The interested reader can ask the authors for the complete version of the paper.

Now we turn to the computation of classes of predicates which are controllable and predictable.

Denote

$C'(P)=\{Y\subset 2^Q \mid Y\subset P, Y \text{ is controllable and predictable}\}$

$C(P)=\{Y\subset 2^Q \mid Y\supset P, Y \text{ is controllable and predictable}\}$

The following example shows that $C'(P)$ is not closed under arbitrary union.

EXAMPLE 3. 1 For system depicted in Fig. 2, let $P_1=\{q_0,q_1,q_2,q\}$, $P_2=\{q_0,q_1,q_2'\}$.

Since $\Sigma(q_0)=\beta$, we have (1). P_1 is predictable; and (2). P_2 is predictable. However, the union of P_1 and P_2, denoted $P_1\vee P_2$, is not predictable, as depicted in Fig. 3.

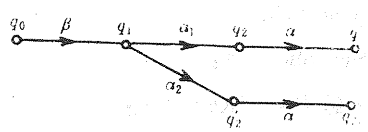

Fig. 2 Transition diagram of the system of example 3. 1

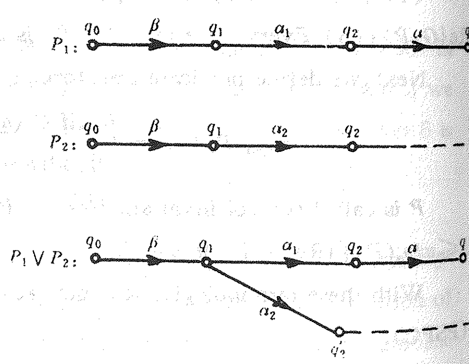

Fig. 3 $P_1\vee P_2$

For the class of predicates $C(P)$, we have

Proposition 3. 2 $C(P)$ is closed under arbitrary intersection.

We know from Proposition 3. 2 that for the task of maintaining a predicate invariant in a predictive fashion, no optimal solution exists in general. However, for a special class of DES, an optimal solution does exist, provided a mild condition is satisfied, as discussed in the following section.

4 Predictive State Feedback Control of a Special Class of DES

DES modeled by state-variable formalism are called Vector DES(VDES)[7]. Composition and control of such a class of DES are well studied. We describe briefly the modeling approach of

VEDS and give some comments on such models here.

Suppose the state set of a DES is descrided by state variables $q_1, q_2, \cdots, q_m$, ranging over integer domain Z. Then the state space of G is

$$Q = \{q = (q_1, q_2, q_3, \cdots, q_m) | q_i \in Z.\}.$$

Let the event set of G be $\Sigma = \{\sigma_1, \sigma_2, \cdots, \sigma_n\}$, the occurence condition $C: \Sigma^* Q \rightarrow \{0,1\}$ is a predicate on Q for each $\sigma \in \Sigma$ specified by

$$C_\sigma(q) = 1 \Leftrightarrow (q \geqslant J_\sigma),$$

where $J_\sigma \in Q$. We write $q \in C_\sigma$ in place of $C_\sigma(q) = 1$. The state transition function of G is $\delta: \Sigma^* Q \rightarrow Q$ specified by

$$\delta_\sigma(q) = q + I_\sigma \quad \text{for each} \quad \sigma \in \Sigma \tag{4.1}$$

with I_σ being constants in Q.

A mild condition

$$(\forall \ q \in Q) \ q \geqslant J_\sigma \Rightarrow q + I_\sigma \geqslant 0,$$

is imposed to guarantee that the system has nonnegative state components when initialized with nonnegative state components.

In order to characterize systems satisfying Equ. (4.1), we give

Definition 4.1 (1) G is called The First Type Strictly Simply System (S_1^s) if $\forall \ q_1, q_2 \in Q$, $q_1 \neq q_2 \Rightarrow (\Sigma(q_1) \cap \Sigma(q_2)) = \varphi \wedge ((\forall \ i = 1,2) |\Sigma(q_i)| \leqslant 1)$. (2) G is called The Second Type Strictly Simple System (S_2^s) if $\forall \ q_1, q_2 \in Q, q_1 \neq q_2 \Rightarrow (\Sigma(q_1) \cap \Sigma(q_2)) = \varphi$.

Obviously, if $G \subset S_1^s$, then $G \subset S_2^s$.

It is remarked here that a large portion of practically encountered DES, e. g., cyclic discrete event processes[17], database management systems[11], are either S_2^s or more strictly, S_1^s.

Denote $\|$ the symbol of shuffle product, now we focus on the languages generated by $G = G_1 \| G_2$, with each G_i being $S_1^s (\forall \ i = 1,2)$, we have

Lemma 4.1 For system $G = G_1 \| G_2, \forall \ q \in Q, \sigma_1, \sigma_2 \in \Sigma(q)$, there is no $\sigma \in \Sigma$ s. t. $\delta(\sigma, \delta(\sigma_1, q))! \wedge \delta(\sigma, \delta(\sigma_2, q))!$

The next lemma follows directly from (2) of Definition 4.1.

Lemma 4.2 If G is S_2^s, then for arbitrary $q \in Q, \sigma_1, \sigma_2 \in \Sigma, \delta(\sigma_1, q))! \wedge \delta(\sigma_2, q))!$, there is no $\sigma \in \Sigma$ s. t. $\delta(\sigma, \delta(\sigma_1, q))! \wedge \delta(\sigma, \delta(\sigma_2, q))!$

In light of lemmas 4.1 and 4.2, we have

Theorem 4.1 If G is the synchronous product of two S_1^s or G is S_2^s, then $\forall \ P \subset 2^Q$, P is controllable $\wedge q_0 \in P \wedge \forall \ \sigma \in \Sigma(q_0), \delta(\sigma, q_0) \in P \Rightarrow P$ is controllable and predictable.

5 Example

Let $G = G_1 \| G_2, G_i (i = 1,2)$ is the model of one user of a shared resource[1][18]. The transition deagrams of G_1 and G_2 are shown in Fig. 4.

Our control objective is the mutual exclusion of the use of the shared resource. For this let $P = Q/(u_1, u_2)$. The trimified transition diagram of the supremal controllable predicate of P is depicted in Fig 5.2. Since G_1, G_2 are all S_1^s, and (1). $q_0 \in P$; (2). $\forall \ \sigma \in \Sigma(q_0), \delta(a, q_0) \in P$, it follows from theorem 4. 1 that P is controllable and predictable.

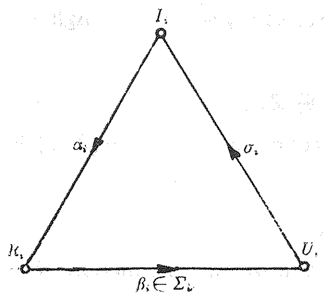

Fig. 4 Transition diagram of $G_i(i=1,2)$

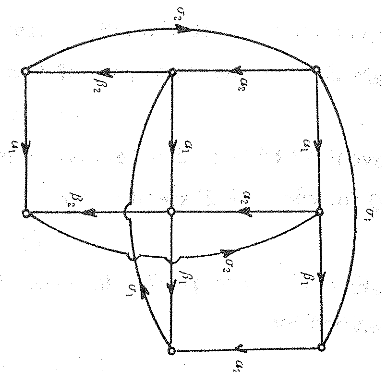

Fig. 5 Trim recognizer of the optimal solution of SCP[1][18]

6 Conclusion

In this paper, results on predictive supervisory control have been extended to the case when a (static) state feedback policy is employed. It is shown that there is a complete conflict-free PSFC f s. t. some given predicate is guaranteed to be invariant in the closed loop system if and only if the predicate is controllable and predictable. For a special class of DES, PSFC has an optimal solution, provided a mild condition is satisfied.

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实时离散事件系统的预测状态反馈控制

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摘要：本文我们将预测监控的一般结果推广到状态反馈控制的情况. 给出了预测状态反馈控制器存在的充要条件及设计方法. 对一类特殊的系统，我们证明了任一传统的状态反馈控制下的解也是预测状态反馈控制问题的解，从而预测状态反馈控制对这类系统有最优解存在. 最后，我们给出了一个例子来说明这一结果.

关键词：离散事件系统；反馈控制；不变性