A Multivariable Adapitive Controller with Integral Action in the Presence of Plant-Model Order Mismatch

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Abstract: A robust adaptive controller is presented for a multivariable plant described by a controlled auto-regressive integrated moving average (CARIMA) model in the presence of bounded disturbances and plant-model order mismatches. The bounded-input bounded-output stability is proven. Finally, a simulation example illustrates the main feature of the controller.

Key words: adaptive control; multivariable system; stability; robustness

1 Introduction

Robust adaptive controllers dealing with plant-model order mismatches and bounded disturbances are proposed recently[1-3], there, however, exists a steady state error in these adaptive control systems. In this paper a robust adaptive controller is presented for a multivariable plant described by a CARIMA model and an integral action is provided automatically in this way, therefore steady state errors due to unmodeled dynamics and step disturbances can be eliminated.

2 A MIMO adaptive controller

Consider a MIMO plant described by the following CARIMA model

$$A(q^{-1})y(t) = B(q^{-1})u(t) + \xi(t)/\Delta, \tag{2.1}$$

where $y(t)\in R^m, u(t)\in R^m$ and $\xi(t)\in R^m$ are the plant output, input and unmeasured bounded disturbance vectors respectively. $\Delta = 1-q^{-1}$. $A(q^{-1}) = \mathrm{diag}(A_i(q^{-1}))$ and $B(q^{-1}) = (q^{-d_{ij}}B_{ij}(q^{-1}))$ where $A_i(q^{-1})$ and $B_{ij}(q^{-1})$ $(1\leqslant i\leqslant m, 1\leqslant j\leqslant m)$ are polynomials in the backward shift operator $q^{-1}$with unknown constant coefficients. $A_i(0)=1$ and $B_{ij}(0)\neq 0$. d_{ij} is a time delay between the i th input and the j th outputs. In the following we denote $x_i(t)$ as the ith element of a vector $x(t)$.

Using the following equations

$$1 = A_i(q^{-1})F_i(q^{-1})\Delta + q^{-d_i}(1+\Delta G_i(q^{-1})), \quad i=1,\cdots,m,$$

where $F_i(q^{-1})$ and $G_i(q^{-1})$ are polynomials in q^{-1} with orders d_i-1 and n_i-1 respectively, n_i is the order of $A_i(q^{-1})$, $d_i=\min\limits_{1\leqslant j\leqslant m}\{d_{ij}\}$, (2.1) can be written as

$$\begin{aligned} y_i(t+d_i) &= y_i(t) + \alpha_i(q^{-1})\Delta y_i(t) + \sum_{j=1}^{m}\beta_{ij}(q^{-1})\Delta u_i(t) + \delta_i(t+d_i) \\ &= y_i(t) + X_i(t)^T\Theta_i + \delta_i(t+d_i), \quad i=1,\cdots,m, \end{aligned} \tag{2.2}$$

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where $\alpha_i(q^{-1}) = G_i(q^{-1})$, $\beta_{ij}(q^{-1}) = F_i(q^{-1})B_{ij}(q^{-1})q^{d_i-d_{ij}}$, $\delta_i(t) = F_i(q^{-1})\xi_i(t)$

and
$$\Theta_i^T = [\alpha_0^i, \cdots, \alpha_{n_i-1}^i, \beta_0^{i1}, \cdots, \beta_0^{im}, \cdots, \beta_{b_i+d_i-1}^{i1}, \cdots, \beta_{b_i+d_i-1}^{im}],$$
$$X_i(t)^T = [\Delta y_i(t), \cdots, \Delta y_i(t - n_i + 1), \Delta u(t)^T, \cdots, \Delta u(t - b_i - d_i + 1)^T],$$
where $b_i = \max\limits_{1\leqslant j\leqslant m}$ {the order of polynomial $q^{d_i-d_{ij}}B_{ij}(q^{-1})$}.

The model order used in the design of a controller is , in general, chosen such that it is less than the actual plant order for simplifying the design of a controller and implementing the algorithm easily, even in non-adaptive cases. Suppose we choose $p_i\leqslant n_i$ and $q_i\leqslant b_i+d_i$ for model orders, then (2. 2) can be rewritten as

$$y_i(t + d_i) = y_i(t) + x_i(t)^T\theta_i + \gamma_i(t + d_i), \tag{2. 3}$$

where
$$x_i(t)^T = [\Delta y_i(t), \cdots, \Delta y_i(t - p_i + 1), \Delta u(t)^T, \cdots, \Delta u(t - q_i + 1)^T],$$
$$\theta_i^T = [\alpha_0^i, \cdots, \alpha_{p_i-1}^i, \beta_0^{i1}, \cdots, \beta_0^{im}, \cdots, \beta_{q_i-1}^{i1}, \cdots, \beta_{q_i-1}^{im}],$$
$$\gamma_i(t + d_i) = z_i(t)^T\rho_i + \delta(t + d_i),$$
where
$$z_i(t)^T = [\Delta y_i(t - p_i), \cdots, \Delta y_i(t - n_i + 1), \Delta u(t - q_i)^T, \cdots, \Delta u(t - b_i - d_i + 1)^T],$$
$$\rho_i^T = [\alpha_{p_i}^i, \cdots, \alpha_{n_i-1}^i, \beta_{q_i}^{i1}, \cdots, \beta_{q_i}^{im}, \cdots, \beta_{b_i+d_i-1}^{i1}, \cdots, \beta_{b_i+d_i-1}^{im}].$$
In order to guarantee the boundedness of $z_i(t)^T\rho_i$, the normalization technique is used here. Define
$$\bar{y}_i(t) - \bar{y}_i(t - d_i) = (y_i(t) - y_i(t - d_i))/\mu_i(t),$$
$$\bar{x}_i(t - d_i) = x_i(t - d_i)/\mu_i(t),$$
$$\mu_i(t) = \max\{2 \max_{1\leqslant k\leqslant g_i} |\varphi_i(t - d_i)_k|, C\},$$
where
$$\varphi_i(t)^T = [y_i(t), \cdots, y_i(t - r_i), u(t)^T, \cdots, u(t - s_i)^T],$$
with $n_i\leqslant r_i$ and $b_i+d_i\leqslant s_i$. The dimension of $\varphi_i(t)$ is $g_i = r_i + ms_i$. It is evident that $\varphi_i(t)$ contains all of the inputs and the outputs included in $X_i(t)$. $\varphi_i(t-d_i)_k$ denotes the kth element of $\varphi_i(t-d_i)$. C is a positive constant and is used to prevent division by zero.

Using these normalized variables, (2. 3) can be rewritten as
$$\bar{y}_i(t) = \bar{y}_i(t - d_i) + \bar{x}_i(t - d_i)^T\theta_i + \bar{\gamma}_i(t), \tag{2. 4}$$
where $\bar{\gamma}_i(t) = \bar{z}_i(t-d_i)^T\rho_i + \delta_i(t)/\mu_i(t)$. It is clear that $\bar{\gamma}_i(t)$ is bounded. Suppose M_i to be an upper bound of $|\bar{\gamma}_i(t)|$, i. e. $|\bar{\gamma}_i(t)| \leqslant M_i (i = 1, \cdots, m)$.

The recursive parameter estimation is as follows. For $i = 1, \cdots, m$.
$$\bar{\varepsilon}_i(t) = \bar{y}_i(t) - \bar{y}_i(t - d_i) - \bar{x}_i(t - d_i)^T\hat{\theta}_i(t - 1), \tag{2. 5}$$
$$\hat{\theta}_i(t) = \hat{\theta}_i(t - 1) + \frac{\lambda_i(t)P_i(t - 2)\bar{x}_i(t - d_i)\bar{\varepsilon}_i(t)}{1 + \bar{x}_i(t - d_i)^TP_i(t - 2)\bar{x}_i(t - d_i)}, \tag{2. 6}$$
$$P_i(t - 1) = P_i(t - 2) - \frac{\lambda_i(t)P_i(t - 2)\bar{x}_i(t - d_i)\bar{x}_i(t - d_i)^TP_i(t - 2)}{1 + \bar{x}_i(t - d_i)^TP_i(t - 2)\bar{x}_i(t - d_i)}, \tag{2. 7}$$
$$\lambda_i(t) = \begin{cases} 0 & \text{if } |\bar{\varepsilon}_i(t)| < M_i, \\ \sigma & \text{otherwise}, \end{cases} \tag{2. 8}$$
where $\tau < \sigma < 3(1-\tau)/4, 0 < \tau < 3/7$.

The control u(t) is chosen such that

$$y_i^0(t+d_i) = y_i(t) + x_i(t)^T\hat{\theta}_i(t), \quad i=1,\cdots,m, \tag{2.9}$$

where $\{y_i^0(t)\}$ $(i=1,\cdots,m)$ are known reference sequences.

3 Stability results

The following assumptions are necessary for the stability analysis.

A1: $d_i(i=1,\cdots,m)$ are known.

A2: The upper bounds of n_i and $b_i(i=1,\cdots,m)$ are known.

A3: The upper bounds of $|\bar{\gamma}_i(t)|(i=1,\cdots,m)$ are known.

A4: The sequence $\{\|\Phi(t)\|\}$ is unbounded only if there is a subsequence $\{t_n\}$ such that

$$\lim_{t_n\to\infty}\|\Phi(t_n)\| = \infty, \tag{3.1}$$

$$\|Y(t_n)\| > 2K_1\|\Phi(t_n)\| - K_2, \quad t_n > 0, \tag{3.2}$$

where $0<K_1<\infty$, $0\leqslant K_2<\infty$ and

$$\Phi(t)^T = [\varphi_1(t-d_1)^T,\cdots,\varphi_m(t-d_m)^T].$$

Note the assumption A2 does not require knowledge of the plant orders, but only upper bounds of their orders. The assumption A2 is used here to decide the dimension of $\varphi_i(t)$ in the normalization variable $\mu_i(t)$.

Theorem 3.1 Under the assumptions A1-A4 if K_1 in (3.2) satisfies

$$K_1 > 2\sqrt{m}M, \quad \text{where} \quad M = \max_{1\leqslant i\leqslant m}\{M_i\}, \tag{3.3}$$

the adaptive control system formed by the plant (2.1), the parameter estimation (2.5)—(2.8) and the control law (2.9) have following properties:

1) $\{\|u(t)\|$ and $\{\|y(t)\|\}$ are bounded for all t.

2) There exists a $T>0$ such that for $t\geqslant T$.

$$|y_i(t) - y_i^0(t)| < 2M_i\mu_i(t), \quad i=1,\cdots,m.$$

Proof: From lemma 2 in [3] and (3.3) we have

$$K_1 > 2\sqrt{m}M > \sqrt{m}\max_{1\leqslant i\leqslant m}\{|y_i(t) - y_i^0(t)|/\mu_i(t)\}$$

$$> \sqrt{m}\max_{1\leqslant i\leqslant m}\{|y_i(t) - y_i^0(t)|/\max\{2\max_{1\leqslant k\leqslant g_i}|\varphi_i(t-d_i)_k|, C\}\}$$

$$> \sqrt{m}\max_{1\leqslant i\leqslant m}\{|y_i(t) - y_i^0(t)|/\max\{2\|\Phi(t)\|, C\}\}.$$

Following the same lines as in [3] the results are obtained immediately.

The theoretical results presented here, particularly condition (3.3) in the theorem 3.1, show that the inherent nature of the plant may limit the allowable model reduction. A discussion of the effect of the model reduction on the performance of the control system can be found in [1].

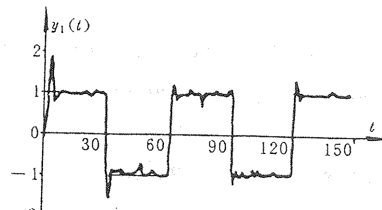
4 Simulation

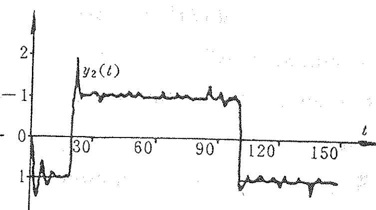
Consider the following MIMO plant

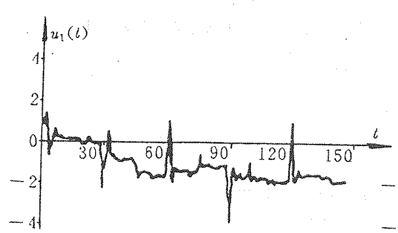
$$\begin{bmatrix} 1-0.9q^{-1}+0.02q^{-2} & 0 \\ 0 & 1-0.7q^{-1}-0.03q^{-2} \end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

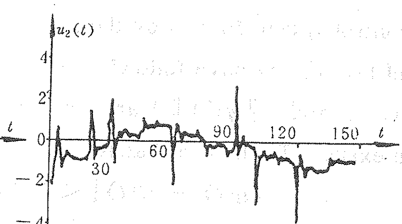
$$= \begin{bmatrix} 1+0.04q^{-1} & 0.2 \\ 1-0.05q^{-1} & 1 \end{bmatrix} \begin{bmatrix} u_1(t-1) \\ u_2(t-1) \end{bmatrix} + \begin{bmatrix} \xi_1(t)/\Delta \\ \xi_2(t)/\Delta \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}, \quad (4.1)$$

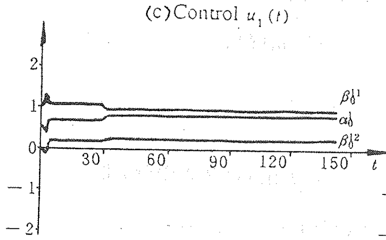
where $\xi_1(t)$ and $\xi_2(t)$ are random noises with zero mean and $|\xi_i(t)| \leqslant 0.1$. $\eta_1 = 0.3$ $(45 < t \leqslant 75)$ and $\eta_2 = 0.3$ $(85 < t \leqslant 135)$ are step load disturbances. Although (4.1) is a second order plant $(n_i = 2, d_i = 1, b_i = 1)$, a first order model is used in our algorithm $(p_i = 1, q_i = 1)$. There, therefore, are 6 parameters to be estimated, i. e. $\theta_1^T = [\alpha_0^1, \beta_0^{11}, \beta_0^{12}]$ and $\theta_2^T = [\alpha_0^2, \beta_0^{21}, \beta_0^{22}]$. The controller parameters are chosen as $\sigma = 0.7$, $M_1 = 0.12$, $M_2 = 0.14$ and $C = 2$. Figs. (a) and (b) show the behaviour of $y_1(t)$ and $y_2(t)$ tracking reference sequences $y_1^0(t)$ and $y_2^0(t)$. Figs. (c) and (d) show the corresponding control $u_1(t)$ and $u_2(t)$. It can be seen that the controller drives $y_1(t)$ and $y_2(t)$ to track $y_1^0(t)$ and $y_2^0(t)$ quite well, even though there are step disturbances and plant-model order mismatches. Figs. (e) and (f) show the parameter estimates of $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$, which demonstrate the good convergence properties of the parameter estimates.

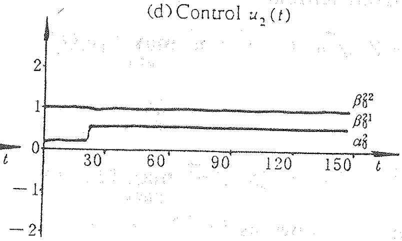

(a) Output $y_1(t)$ and reference $y^\circ_1(t)$


(b) Output $y_2(t)$ and reference $y^\circ_2(t)$


(c) Control $u_1(t)$


(d) Control $u_2(t)$


(e) Estimated parameters $\theta_1^T = [\alpha_0^1, \beta_0^{11}, \beta_0^{12}]$


(f) Estimated parameters $\theta_2^T = [\alpha_0^2, \beta_0^{21}, \beta_0^{22}]$

References

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存在模型阶失配时多变量积分型自适应控制器

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摘要：本文对用受控自回归积分滑动平均模型描述并具有有界扰动和模型阶失配的多变量系统提出一种自适应控制方法. 同时给出了有界输入有界输出的稳定性证明. 最后，一个仿真例子表明了该方法的特点.

关键词：自适应控制；多变量系统；稳定性；鲁棒性

1991 全国"控制理论与应用"年会在威海举行

中国自动化学会"控制理论与应用"年会于 1991 年 10 月 10 日至 15 日在山东威海举行. 来自全国十几个省（市）、自治区的 140 多名代表参加了会议. 代表中除来自全国各科研部门和高等学校的理论和应用科研工作者外，还有相当数量的来自全国各地的工、农业生产第一线和有关管理部门的科技工作者. 从代表的年龄结构上看，45 岁以下的青年人约占 2/3. 从论文分类看，具有实际应用效果和具有应用前景的论文数量和历届年会相比有所增多. 上述两个特点是系统控制理论及其应用事业在我国蓬勃发展、兴旺发达的标志. 本届年会由中国自动化学会常务理事、控制理论专业委员会副主任秦化淑研究员主持.

本届年会的学术活动有如下三方面.

1. 大会综述性学术报告. 会上，北京控制工程研究所吴宏鑫教授、北京航空航天大学陈宗基教授和中国科学院系统科学研究所秦化淑研究员分别就"自适应控制技术的应用"、"控制在海湾战争中的应用"（本报告原由航空航天部二十五所夏国洪教授准备，因夏教授会前突然生病，改由陈宗基教授报告）和"复杂控制系统的理论——基于数学和计算机的研究"作了综述性报告. 三个报告从不同角度评述了控制理论中几个方面的理论和应用研究现状及今后值得深入研究的课题，使与会者受到启发.

2. 分组学术交流. 本届年会共分 30 个专题小组，每天有 8 个专题小组进行学术交流. 会上不仅有问有答，且有讨论，有建议，气氛热烈、生动活泼.

3. 专题学术讨论会. 本届年会组织了两次专题性学术讨论会：一是鲁棒控制，二是自适应控制中的鲁棒性. 讨论会上发言非常踊跃. 有的概述该专题国内外研究情况（包括已获得的结果和尚待解决的问题）；有的发表了关于该专题研究重点的见解；有的提出了在理论研究和实际应用中所遇到的困难. 事实证明，这种专题性讨论会已成为"控制理论与应用"年会的一个有机组成部分. 这两个专题讨论会也从一个侧面反映了 1990 年年会综述报告的效果.

会议期间召开了"控制理论专业委员会"会议. 对本届年会和会后的工作进行了讨论. 一致认为：组织综述性报告和专题性讨论会是提高年会质量的有力措施. 它不但向与会者介绍系统控制理论前沿课题的研究概况，而且具有研究导向的作用. 今后应继续加强. 会议就明年年会综述报告的题目和人选进行了研究. 控制理论委员会决定：1992 年"控制理论与应用"年会于 1992 年 10 月 10 日至 15 日在南京召开.

据悉，1992 年"控制理论与应用"年会的征文通知，不日即可发出. 希望广大系统控制理论与应用科技工作者踊跃投稿.

（王朝珠）