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Robust Stabilization for a Class of Uncertain Bilinear Systems with Time-Delay

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Abstract: This paper presents a sufficient condition of robust stabilization of bilinear systems in which both structural perturbations and the states with time-delay are contained. According to the knowledges of the matrix measure, the problem of asymptotically robust stability of the systems with higher dimensions is transformed into that with one dimension. Thus the main result is easily deduced by using comparision principle. Finally the numerical example is given to illustrate the robust controller design procedures.

Key words: uncertain bilinear systems; time-delay; robust stability; matrix measure; comparision principle

1 Introduction

It is well known that any control design based on the mathematical model exhibiting the robustness with respect to the modelling uncertainties or imprecisions is desirable. The studies of robust stabilization of the controlled systems subjected to a class of structural perturbations have been developing in depth by many researchers in recent years. As the important approach of dealing with some classes of uncertain systems, many techniques with the aid of time domain designs are announced from the different applied aspects in the literatures. The robust stabilization of the systems with time-delay is also actively discussed in the related areas because these researches are very significant for the practical engineering applications[1-3]. In this paper the robust stabilization of the multivariable bilinear systems with time-delay using a possible linear state feedback controller, avoiding the nonlinear design[4], is proposed. It should be noted that the proof of the main theorem is based on the structural methods of the differential equations and the basic theory of the matrix measure[2,6]. Then we can proceed to change the stabilization problem of the systems with higher dimensions into that with one and the simplifing is obtained. To prove the so-called comparision principle[5,7]the discretization technique and the inductive method are used.

This paper is organized as follows: in section 2, the useful result of the matrix measure is briefly introduced and the main throrem is expounded. The proof of the theorem is arranged in section 3. Finally, in section 4, the numerical example is placed to illustrate the design.

2 Basic concepts and main result

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Let us consider the following bilinear system with both structural perturbations and time-delay:

$$\begin{cases} \dot{X}(t) = (A + \Delta A)X(t) + \sum_{s=1}^{l}(P_s + \Delta P_s)X(t - h_s) \\ \qquad + \sum_{s=1}^{l}(N_s + \Delta N_s)X(t - h_s)u_s(t) + (B + \Delta B)U(t), \\ X(t) = \Phi(t), \quad -h \leqslant t \leqslant 0, \end{cases} \tag{2.1}$$

where $h = \max_{1\leqslant s\leqslant l}\{h_s\}$, A, P_s, $N_s \in R^{n\times n}$. $B \in B^{n\times l}$, the state $X \in R^n$ and the controls $u_s \in R$ for every fixed t. $U = (u_1, u_2, \cdots, u_l)^T$. Δ represents the corresponding matrix perturbation. h_s is time delay in the system and $\Phi(t)$ is a given continuous vector function over the domain under consideration. According to the basic concepts about the matrix measure we can obtain

Lemma[6] Let A and B be the square matrices and $\mu_k(A)$ represent the matrix measures, then we have

1) $\|\exp(At)\|_k \leqslant \exp\{\mu_k(A)t\}$, (2.2)

2) $\mu_k(A+B) \leqslant \mu_k(A) + \mu_k(B)$. (2.3)

Remark For the sake of simplicity of the computations k is generally taken $1, 2$, or ∞ in the practical calculations in the sequel.

Now we analyze the stabilization of the system (2.1) via the state feedback control. If the possible control law is taken as

$$U = K_0X(t) + \sum_{i=1}^{l}K_iX(t - h_i), \tag{2.4}$$

where k_i is given by

$$K_i = \begin{bmatrix} k_{11}^{(i)} & k_{12}^{(i)} & \cdots & k_{1n}^{(i)} \\ k_{21}^{(i)} & k_{22}^{(i)} & \cdots & k_{2n}^{(i)} \\ \vdots & & & \vdots \\ k_{l1}^{(i)} & k_{l2}^{(i)} & \cdots & k_{ln}^{(i)} \end{bmatrix} = \begin{bmatrix} K_1^{(i)} \\ K_2^{(i)} \\ \vdots \\ K_l^{(i)} \end{bmatrix}, \tag{2.5}$$

$i = 1, 2, \cdots, l$. Substituting (2.4) into (2.1) yields

$$\begin{cases} \dot{X} = (A + BK_0)X + \sum_{s=1}^{l}[(P_s + BK_s) + (\Delta P_s + \Delta BK_s)]X(t - h_s) \\ \qquad + \sum_{s=1}^{l}(N_s + \Delta N_s)X(t - h_s)[K_s^{(0)}X + \sum_{j=1}^{l}K_s^{(j)}X(t - h_j)] + (\Delta A + \Delta BK_0)X, \\ X = \Phi(t), \quad -h \leqslant t \leqslant 0. \end{cases} \tag{2.6}$$

The stabilization of the system (2.1) can be stated as follows:

Definition The bilinear system (2.1) is robustly stabilizable against the structural perturbations by means of the state feedback control (2.4), if every solution $X(t)$ of the closed loop system (2.6) starting from an arbitrary initial curve converges asymptotically to $X(t) = 0$ as $t \to \infty$.

The sufficient condition of the robust stabilization of the bilinear system shown above is addressed as the main result in the paper.

Theorem Let the pair $(A,B)\in M$, where $M=\{(A,B)\mid$ there exists a K such that $\mu_k(A+BK)<0\}$. If we can choose the appropriate K_i in the control (2. 4), $i=1,2,\cdots,l$, such that

1) $\mu_k(A+BK_0)<-\|\Delta A+\Delta BK_0\|_k$, (2. 7)

2) $\sum_{s=1}^{l}(\|P_s+BK_s\|_k+\|\Delta P_s+\Delta BK_s\|_k)<-\mu_k(A+BK_0)-\|\Delta A+\Delta BK_0\|_k.$ (2. 8)

then the closed loop system (2. 6) is robust stable against the structural perturbations, Δ.

Remark The feedback gain matrix K_0 is generally assumed to be real. If this requirment is properly relaxed, that is, allowed to be complex which means that the gain possess both amplitudes and phases, the design of controller may be more convenient.

3 The proof of the theorem

For $t>0$, the differential equation (2. 6) can be equivalently expressed the integral equation and hence we have

$$\begin{aligned}\|X(t)\| \leqslant &\exp\{\mu_k(A+BK_0)t\}\|\Phi(0)\| + \int_0^t \exp\{\mu_k(A+BK_0)\}(t-\tau)\{\sum_{s=1}^{l}[\|P_s+BK_s\|_k \\ &+ \|\Delta P_s+\Delta BK_s\|_k]\|X(\tau-h_s)\| + \sum_{s=1}^{l}\|N_s+\Delta N_s\|_k\|X(\tau-h_s)\|[\|K_s^{(0)}\|\|X(\tau)\| \\ &+ \sum_{j=1}^{l}\|K_s^{(j)}\|\|X(\tau-h_j)\|] + \|(\Delta A+\Delta BK_0)\|_k\|X(\tau)\|\}d\tau, \end{aligned} \tag{3. 1}$$

according to the lemma (1). The next step of our proceeding is to investigate the following differential equation in one dimension.

$$\begin{cases} \dot{x}(t)=\mu_k(A+BK_0)x(t)+\sum_{s=1}^{l}[\|P_s+BK_s\|_k+\|\Delta P_s+\Delta BK_s\|_k]x(t-h_s)+\sum_{s=1}^{l}\|N_s \\ \quad +\Delta N_s\|_k\|x(t-h_s)\|[\|K_s^{(0)}\|x(t)+\sum_{j=1}^{l}\|K_s^{(j)}\|x(t-h_j)]+\|\Delta A+\Delta BK_0\|_k x(t), \\ x(t)=\Psi(t), \quad -h\leqslant t\leqslant 0, \end{cases} \tag{3. 2}$$

where $\|\Phi(t)\|<\Psi(t)$. It is obvious that the solution of above equation has the similar form to the right hand side of the inequality (3. 1) when $t>0$. Thus the so-called comparision principle has to be proved, that is,

$$\|X(t)\|<x(t), \quad t>0, \tag{3. 3}$$

in order to change the problem with higher dimensions into that with one. To do so, discreting the inequality (3. 1) for any time period T we have

$$\begin{aligned}\|X[(k+1)T]\| \leqslant &\exp\{\mu_k(A+BK_0)T\}\|X(kT)\| \\ &+\int_{kT}^{(k+1)T}\exp\{\mu_k(A+BK_0)\}[(k+1)T-\tau] \\ &\times\{\sum_{s=1}^{l}[\|P_s+BK_s\|_k+\|\Delta P_s+\Delta BK_s\|_k]\|X(\tau-h_s) \\ &+\sum_{s=1}^{l}\|N_s+\Delta N_s\|_k\|X(\tau-h_s)\|\end{aligned}$$

$$\times \left[\|K_s^{(0)}\|\|X(\tau)\| + \sum_{j=1}^{l}\|K_s^{(j)}\|\|X(\tau - h_j)\|\right] + \|(\Delta A + \Delta BK_0)\|_k\|X(\tau)\|\}d\tau, \quad (3.4)$$

where the constant k is the number of sample times. we also have

$$x[(k+1)T] = \exp\{\mu_k(A + BK_0)T\}x(kT) + \int_{kT}^{(k+1)T}\exp\{\mu_k(A + BK_0)\}[(k+1)T - \tau]$$

$$\times \{\sum_{s=1}^{l}[\|P_s + BK_s\|_k + \|\Delta P_s + \Delta BK_s\|_k]x(\tau - h_s)$$

$$+ \sum_{s=1}^{l}\|N_s + \Delta N_s\|_k x(\tau - h_s)$$

$$\times \left[\|K_s^{(0)}\|x(\tau) + \sum_{j=1}^{l}\|K_s^{(j)}\|x(\tau - h_j)\right] + \|(\Delta A + \Delta BK_0)\|_k x(\tau)\}d\tau \quad (3.5)$$

Because $\|X(t)\|$ and $x(t)$ are continuous for $t>0$, if $t_0>0$ is given such that $\|X(t_0)\|<x(t_0)$ then $\exists$ a δ neighborhood $\delta(t_0:T(t_0))$ such that $\|X(t)\|<x(t)$ for all $t\in\delta(t_0)$. It is assumed that the time period T satisfies·

$$T < \min\{T(0), h_1, h_2, \cdots, h_l\}. \quad (3.6)$$

Let $k=0$, we have

$$\|X(T)\| < x(T), \quad (3.7)$$

according to (3.6) and the fact that $\|X(0)\|=\|\Phi(0)\|<\Psi(0)=x(0)$. It is assumed that for any positive integer k we have

$$\|X(kT)\| < x(kT), \quad (3.8)$$

where $kT>h$ is supposed without loss of generality. Since T is arbitrarily given, the inequality (3.8) implies

$$\|X(t)\| < x(t), \quad \text{for all } t \leqslant kT. \quad (3.9)$$

To prove the fact that $\|X[(k+1)T]\|<x[(k+1)T]$ the following technique of reduction to absurdity is introduced. If we assume that the function

$$y(t) = \|X(t)\| - x(t), \quad (3.10)$$

has at least one zero point in the interval $[kT,(k+1)T]$, and the first is denoted by ξ, say, that is

$$y(\xi) = \|X(\xi)\| - x(\xi) = 0, \quad (3.11)$$

which implies

$$\|X(t)\| \leqslant x(t), \quad (3.12)$$

as $t\in[kT,\xi]$. On the other hand,

$$\|X(\xi)\| \leqslant \exp\{\mu_k(A + BK_0)\xi\}\|\Phi(0)\| + \int_0^{\xi}\exp\{\mu_k(A + BK_0)\}(\xi - \tau)$$

$$\times \{\sum_{s=1}^{l}[\|P_s + BK_s\|_k + \|\Delta P_s + \Delta BK_s\|_k]\|X(\tau - h_s)\| + \sum_{s=1}^{l}\|N_s + \Delta N_s\|_k\|X(\tau - h_s)\|$$

$$\times \left[\|K_s^{(0)}\|\|X(\tau)\| + \sum_{j=1}^{l}\|K_s^{(j)}\|\|X(\tau - h_j)\|\right] + \|(\Delta A + \Delta BK_0)\|_k\|X(\tau)\|\}d\tau. \quad (3.13)$$

It is quite evident that $\xi-h_s<kT$, $s=1,2,\cdots,l$, so according to the inductive hypothesis (3.8)

and (3.12)、(3.13) we have

$$\|X(\xi)\| < \exp\{\mu_k(A+BK_0)\xi\}\Psi(0) + \int_0^\xi \exp\{\mu_k(A+BK_0)\}(\xi-\tau)$$
$$\times \{\sum_{s=1}^{l}[\|P_s+BK_s\|_k + \|\Delta P_s+\Delta BK_s\|_k]x(\tau-h_s) + \sum_{s=1}^{l}\|N_s$$
$$+\Delta N_s\|_k x(\tau-h_s)[\|K_s^{(0)}\|x(\tau) + \sum_{j=1}^{l}\|K_s^{(j)}\|x(\tau-h_j)] + \|(\Delta A+\Delta BK_0)\|_k x(\tau)\}d\tau$$
$$= x(\xi), \tag{3.14}$$

which acts contrary to (3.11). This contradictory fact shows that

$$\|X[(k+1)T]\| < x[(k+1)T]. \tag{3.15}$$

In terms of the inductive principle, we have the conclusion cited in (3.3).

Let us consider the Liapunov functional of the form

$$V(\psi) = \frac{1}{2}\psi^2(0) + \sum_{s=1}^{l}\int_{-h_s}^{0} d_s\psi^2(\theta)d\theta, \tag{3.16}$$

where d_s is a constant to be determined. Hence the derivative of V along the solution of the system (3.2) is obtained

$$\dot{V}(x_t) = V_1 + V_2, \tag{3.17}$$

where $\quad a = \mu_k(A+BK_0) + \|\Delta A+\Delta BK_0\|_k, \quad b_s = \|P_s+BK_s\|_k + \|\Delta P_s+\Delta BK_s\|_k \quad (3.18)$

and

$$V_1 = (a+\sum_{s=1}^{l}d_s)x^2(t) + \sum_{s=1}^{l}b_s x(t)x(t-h_s) - \sum_{s=1}^{l}d_s x^2(h-h_s), \tag{3.19}$$

$$V_2 = \sum_{s=1}^{l}\|N_s+\Delta N_s\|_k\|K_s^{(0)}\|x(t-h_s)x^2(t) + \sum_{s=1}^{l}\sum_{j=1}^{l}\|N_s$$
$$+\Delta N_s\|_k\|K_s^{(j)}\|x(t-h_s)x(t-h_j)x(t). \tag{3.20}$$

The quadratic form V_1 can be expressed the following compact form

$$V_1 = -zQz^T, \quad z = [x(t), x(t-h_1), \cdots, x(t-h_l)], \quad \text{and}$$

$$Q = \begin{bmatrix} -a-\sum_{s=1}^{l}d_s & -b_1/2 & \cdots & -b_l/2 \\ -b_1/2 & d_1 & \cdots & 0 \\ \vdots & & & \vdots \\ -b_l/2 & 0 & \cdots & d_l \end{bmatrix} \tag{3.21}$$

If the conditions (2.7)-(2.8) hold it is easily verified that

$$\sum_{s=1}^{l}d_s + \sum_{s=1}^{l}b_s^2/4d_s < -a, \tag{3.22}$$

as long as d_s is taken to be $b_s/2$. This fact implies that V_1 is negative definite. On the other hand we can find the fact (1) if $x(t)\to 0$ and not all $x(t-h_i)\to 0$, $i=1,2,\cdots,l$, we have $V_1/\|z\|^2\to 0$; (2) if $\|z\|\to 0$, as $t\to\infty$, we also have $V_1/\|z\|^2\to 0$. Therefore if $\forall\ \varepsilon>0$, $\exists\ \delta>0$ such that $V_1<\varepsilon\|z\|^2$ for any $x\in\rho(0,\delta)$, a δ-neighborhood of zero point, where $x\neq 0$ and hence $\|z\|\neq 0$.

It is noted that $-V_1$ is positive definite and so we can assume, without loss of generality, that there exists a constant $E>0$, such that

$$\min_{\|z\|=E} -V_1(z) = e > 0, \tag{3.23}$$

hence
$$-V_1(z) = -\|z\|^2 V_1(Ez/\|z\|)/E^2 \geq e\|z\|^2/E^2. \tag{3.24}$$

Taking $\varepsilon = e/(2E^2)$ given in above we conclude

$$-\dot{V} = -V_1(z) - V_2 > e\|z\|^2/(2E^2). \tag{3.25}$$

It follows form the work of Hale and the definition stated above that the system under consideration is robustly stabilizable with respect to the structural perturbations via the linear feedback control. The theorem is thus completely proved.

Remark It can be seen from the processes of the proof that N_i, $i=1,2,\cdots,l$, have no effect upon the robustness of the system. If (A,B) is a controllable pair, the matrix K_0 of determination is guaranteed. Moreover, the experienced method may be used for reference.

Step 1: The K_i, $i=1,2,\cdots,l$, are at first designed such that the induced matrix norm $\|P+BK_i\|_k$ is as small as possible. Taking Forbenius norm this implies

$$\min J(K_i) = \min \operatorname{tr}\{(P+BK_i)^T(P+BK_i)\}. \tag{3.26}$$

Thus the quantity on the left hand side of (2.8) is determined.

Step 2: According to (2) in the lemma, designing a appropriate K_0 such that $\mu_k(A)+\mu_k(BK_0)$ is so small that (1) and (2) in the theorem are satisified is generally feasible.

4 Numerical example

Consider a process governed by the bilinear time-delay controlled system with structural perturbations.

$$\begin{aligned} \dot{X}(t) = &\begin{bmatrix} (-6.5,-6) & (1.9,2.1) \\ (0.5,1.2) & (-5,-4.5) \end{bmatrix} X(t) + \begin{bmatrix} (0.9,1) & (-0.5,0.5) \\ (-0.5,0) & (0.8,1) \end{bmatrix} X(t-\tau_1) \\ &+ \begin{bmatrix} (-2.2,-1.9) & (1,1.1) \\ (-1,-0.5) & (-0.5,0.5) \end{bmatrix} X(t-\tau_2) + \begin{bmatrix} (-1,1) & (0.5,1) \\ (-2,-1) & (0,0.5) \end{bmatrix} X(t-\tau_1)u_1(t) \\ &+ \begin{bmatrix} (0,1) & (-1,1) \\ (0.5,1) & (-0.5,0) \end{bmatrix} X(t-\tau_2)u_2(t) + \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \end{aligned} \tag{4.1}$$

In this example the euclid norm and the corresponding matrix measure are used. The related nominal system matrices are given by

$$A = \begin{bmatrix} -6 & 2 \\ 1 & -5 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}, \quad \Delta B = \begin{bmatrix} 0.2\sin t & 0.5\exp(-t) \\ 0.5\exp(-t) & 0.1\cos t \end{bmatrix}. \tag{4.2}$$

The perturbations are denoted by $\Delta A = (a_{ij})$, $\Delta P_1 = (P_{ij}^{(1)})$ and $\Delta P_2 = (P_{ij}^{(2)})$ where all elements a_{ij}, $P_{ij}^{(1)}$, $P_{ij}^{(2)}$ are able to be directly obtained.

First we find the feedback gain such that $J(K_1)\to\min$. It is not hard to verify by using the optimization technique that

$$K_1 = \begin{bmatrix} -1 & 0 \\ -1.5 & 1 \end{bmatrix} \tag{4.3}$$

and in this case $J(K_1)=0$. Due to the same reason as above we have

$$K_2 = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} \tag{4.4}$$

and $J(K_2)=0$. The simple computations lead to

$$\|\Delta P_1 + \Delta BK_1\|_2 \leqslant 1.7918, \quad \|\Delta P_2 + \Delta BK_2\|_2 \leqslant 2.3705. \tag{4.5}$$

It is clear that we can obtain $\mu_2(A) = -3.9189$. Thus K_0 is figured out

$$K_0 = \begin{bmatrix} -5 & 0 \\ -5 & 5 \end{bmatrix}, \tag{4.6}$$

by using the eigenvalue placement and hence $\mu_2(BK_0) = -5$. So we have

$$\mu_2(A + BK_0) \leqslant -8.9189. \tag{4.7}$$

Because the fact that

$$\|\Delta A + \Delta BK_0\|_2 \leqslant 4.5338, \tag{4.8}$$

we have

$$\mu_2(A + BK_0) \leqslant -8.9189 < -4.5338 \leqslant -\|\Delta A + \Delta BK_0\|_2 \tag{4.9}$$

and

$$\sum_{s=1}^{2} \|\Delta P_s + \Delta BK_s\|_2 \leqslant 4.1623 < 4.3851 < -\mu_2(A + BK_0) - \|\Delta A + \Delta BK_0\|_2. \tag{4.10}$$

In accordance with the theorem the system can therefore be robustly stabilizable via the linear state feedback control

$$U = K_0X(t) + K_1X(t - \tau_1) + K_2X(t - \tau_2), \tag{4.11}$$

where K_i, $i=0,1,2$ is determined by (4.3), (4.4) and (4.6).

5 Conclusions

The robust stabilization for a class of MIMO bilinear systems with structural perturbations and time-delay is discussed and the linear state feedback is available. As shown in the paper, the derivation of the proposed sufficient condition is succinct since the comparision principle is used. The design of the controller is also relatively concise provided that the norm is chosen appropriately. The result presented in this paper generalizes some recent criteria of robust stability of the linear system. It is noted that the similar method can be applied to a kind of nonlinear time-delay systems.

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一类具有时滞的不确定双线性系统的Robust镇定

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摘要：本文给出了一类既有结构摄动又有状态时滞的双线性系统的Robust镇定的充分条件. 基于矩阵测度的概念，本文将高维系统的渐近Robust稳定性问题转化为一维情形，以使本文的主要问题易于处理. 在证明主要定理时，本文采用了比较原理及离散化、归纳法等技巧. 最后我们还给出了数值算例以示控制器的设计.

关键词：不确定双线性系统；时滞；Robust稳定性；矩阵测度；比较原理