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A New Conclusion on the Parameter Stability Domain of Linear Time-Invariant Large-Scale Systems

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Abstract: In this paper we discuss the parameter stability domain of linear time-invariant large-scale systems by using the scalar Lyapunov function method and combining with Bailey inequality. And a new form of the parameter stability domain is acquired. Finally, we take one nth-order system with two subsystems as an example and point out that the new parameter stability domain and the parameter stability domain resulted from the vector Lyapunov function method are not included each other.

Key words: large-scale system; stability; Lyapunov method; matrix theory

1 Introduction

It was generally considered that the parameter stability domain of linear time-invariant systems resulted from the vector Lyapunov function method is larger than that resulted from the scalar Lyapunov function method[1][2]. Recently, Tang Gongyou pointed out that this view is not correct and has proved that the parameter stability domain of a 2nd-order linear time-invariant system resulted from the scalar Lyapunov function method is not smaller than that resulted from the vector Lyapunov function method. In this paper, the nth-order system with m subsystems is restudied by using scalar Lyapunov fuction decomposition method and the conclusion mentioned above is extended to the nth-order systems. The basic idea is that we treat the derivates of Lyapunov function of the large-scale system by putting the incidence terms of the pairwise incidence subsystems together and making good use of Bailey inequality, thus, one new form of parameter stability domain is acquired. By comparing the two parameter stability domains of the nth-order system with two subsystems resulted respectively from both the vector method and the scalar method, the conclusion that the two domains are not included each other is also obtained.

2 The New Result of the Parameter Stability Domain of Linear Time-Invariant Large-Scale Systems

In this section, we study the parameter stability domain of linear time-invariant large-scale systems by using the scalar Lyapunov function decomposition method combined with Bailey

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inequality. For this reason, we introduce the following lemma:

Lemma If the real number $a>0$, then $\forall z\in[0,\infty)$

$$-az^2+bz\leqslant-\frac{a}{2}z^2+\frac{b^2}{2a}.$$

Now, consider a nth-order time-invariant large-scale system described in the following matrix form

$$\dot{X}_i=A_{ii}X_i+\sum_{\substack{j=1\\j\neq i}}^{m}A_{ij}X_j,\quad i=1,2,3,\cdots,m,\tag{1}$$

and its m isolated subsystems

$$\dot{X}_i=A_{ii}X_i,\quad i=1,2,3,\cdots,m.\tag{2}$$

Where

$$X_i=(X_1^{(i)},X_2^{(i)},\cdots,X_{n_i}^{(i)})^T\in R^{n_i},\quad \sum_{i=1}^{m}n_i=n,$$

$$A_{ij}\in R^{n_i\times n_j},\quad i,j=1,2,\cdots,m.$$

Assume that the eigenvalues of the ith isolated subsystem are satisfied with

$$\mathrm{Re}(\lambda_r(A_{ii}))<0,\quad r=1,2,\cdots,n_i,\tag{3}$$

and then, according to the Lyapunov stability theory, there exists unique symmetric matrix B_i satisfied with the Lyapunov equation

$$A_{ii}^TB_i+B_iA_{ii}=-I_i,\tag{4}$$

and B_i is positive definite. Take the Lyapunov function of the isolated subsystem (2) as

$$V_i(X_i)=X_i^TB_iX_i,\tag{5}$$

and then, the total derivative of V_i with respect to t, obtained along the solution of (2) is

$$\dot{V}_i|_{(2)}=-X_i^TX_i.$$

Take the Lyapunov function of the system (1) as

$$V=\sum_{i=1}^{m}V_i,$$

where V_i is defined with (5), then the total derivative of V with respect to t, obtained along the system (1) is

$$\begin{aligned}\dot{V}_{(1)}&=\sum_{i=1}^{m}\Big[-X_i^TX_i+\sum_{\substack{j=1\\j\neq i}}^{m}(X_i^TB_iA_{ij}X_j+X_j^TA_{ij}^TB_iX_i)\Big]\\&=\sum_{i=1}^{m}(-X_i^TX_i)+\sum_{i=1}^{m}\sum_{\substack{j=1\\j\neq i}}^{m}X_i^TB_iA_{ij}X_j+\sum_{j=1}^{m}\sum_{\substack{i=1\\i\neq j}}^{m}X_j^TA_{ij}^TB_iX_i\\&=\sum_{i=1}^{m}(-X_i^TX_i)+\sum_{i=1}^{m}\sum_{\substack{j=1\\j\neq i}}^{m}X_i^TB_iA_{ij}X_j+\sum_{i=1}^{m}\sum_{\substack{j=1\\j\neq i}}^{m}X_i^TA_{ji}^TB_jX_j\\&=\sum_{i=1}^{m}\Big\{-X_i^TX_i+\sum_{\substack{j=1\\j\neq i}}^{m}[X_i^T(B_iA_{ij}+A_{ji}^TB_j)X_j]\Big\}.\end{aligned}$$

For convenience, let $B_iA_{ij}=[B_{rs}^{(ij)}]n_i\times n_j$,

where

$$B_{rs}^{(ij)} = \sum_{k=1}^{n_i} b_{rk}^{(i)} a_{ks}^{(ij)}, \quad i,j = 1,2,\cdots,m,$$

hence

$$\begin{aligned}
\dot V_{(1)} &= \sum_{i=1}^{m}\Big[\sum_{r=1}^{n_i}\big(-X_r^{(i)^2}\big) + \sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{r=1}^{n_i}\sum_{s=1}^{n_j}\big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)X_r^{(i)}X_s^{(j)}\Big] \\
&= \sum_{i=1}^{m}\sum_{r=1}^{n_j}\Big[-X_r^{(i)^2} + \sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{s=1}^{n_i}\big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)X_r^{(i)}X_s^{(j)}\Big] \\
&= \sum_{i=1}^{m}\sum_{r=1}^{n_j}\Big\{\sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{s=1}^{n_i}\Big[-\frac{1}{n-n_i}X_r^{(i)^2} + \big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)X_r^{(i)}X_s^{(j)}\Big]\Big\} \\
&\leqslant \sum_{i=1}^{m}\sum_{r=1}^{n_j}\Big\{\sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{s=1}^{n_i}\Big[-\frac{1}{2(n-n_i)}X_r^{(i)^2} + \frac{n-n_i}{2}\big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)^2X_s^{(j)^2}\Big]\Big\} \\
&= \sum_{i=1}^{m}\sum_{r=1}^{n_i}\Big[-\frac{1}{2}X_r^{(i)^2} + \sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{s=1}^{n_i}\frac{n-n_i}{2}\big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)^2X_s^{(j)^2}\Big] \\
&= \sum_{i=1}^{m}\Big[-\frac{1}{2}X_i^TX_i + \sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{r=1}^{n_i}\sum_{s=1}^{n_j}\frac{n-n_i}{2}\big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)^2X_s^{(j)^2}\Big].
\end{aligned} \tag{6}$$

Let

$$L_{ij} = \max\Big\{(n-n_i)\sum_{r=1}^{n_i}\big(B_{rs}^{(ij)} + B_{sr}^{(ji)}\big)^2, \quad s = 1,2,\cdots,n_j\Big\}, \tag{7}$$

then

$$\begin{aligned}
\dot V_{(1)} &\leqslant \sum_{i=1}^{m}\Big(-\frac{1}{2}X_i^TX_i + \frac{1}{2}\sum_{\substack{j=1\\ j\neq i}}^{m}\sum_{s=1}^{n_j}L_{ij}X_s^{(j)^2}\Big) \\
&= \sum_{i=1}^{m}\Big(-\frac{1}{2}X_i^TX_i + \sum_{\substack{j=1\\ j\neq i}}^{m}\frac{1}{2}L_{ij}X_j^TX_j\Big) \\
&= \sum_{i=1}^{m}\Big(-\frac{1}{2}X_i^TX_i\Big) + \sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\frac{1}{2}L_{ij}X_j^TX_j \\
&= \sum_{i=1}^{m}\Big(-\frac{1}{2}X_i^TX_i\Big) + \sum_{i=1}^{m}\sum_{\substack{j=1\\ j\neq i}}^{m}\frac{1}{2}L_{ji}X_i^TX_i \\
&= \sum_{i=1}^{m}\Big(-1 + \sum_{\substack{j=1\\ j\neq i}}^{m}L_{ji}\Big)\frac{1}{2}X_i^TX_i.
\end{aligned}$$

it can be written as

$$\dot V_{(1)} \leqslant \frac{1}{2}\sum_{i=1}^{m}\Big(-1 + \sum_{\substack{j=1\\ j\neq i}}^{m}L_{ji}\Big)X_i^TX_i. \tag{8}$$

Obviously, if $\sum_{j=1, j\neq i}^{m} L_{ji} < 1$ $(i = 1,2,\cdots m)$, then $\dot V_{(1)} < 0$. Hence, the balanced state of the large-scale system (1) is of asymptotic stability. Therefore, we can obtain the following theorem:

Theorem For the nth-order linear time-invariant large-scale system (1), assume that the eigenvalues of its m isolated subsystems (2) are satisfied with the condition (3), if $\sum_{j=1, j\neq i}^{m} L_{ji} < 1$ $(i=1,2,\cdots m)$, then the balanced state of (1) is of asymptotic stability, where L_{ji} is defined with (7).

3 Comparison of the Parameter Stability Domains

It was considered that the parameter stability domain resulted from the vector Lyapunov function decomposition method is larger than that from the scalar one. In fact, this view is not correct. In this section, taking one nth-order large-scale system with two subsystems as an example, we show that the parameter stability domains resulted respectively from these two methods are not included each other.

Now, consider the parameter stability domain resulted from the vector Lyapunov method.

$$\because \quad \dot{V}_1 = -X_1^T X_1 + 2X_1^T B_1 A_{12} X_2$$

$$= -X_1^T X_1 + 2\sum_{r=1}^{n_1}\sum_{s=1}^{n_2} X_r^{(1)} X_s^{(2)} B_{rs}^{(12)}$$

$$= \sum_{r=1}^{n_1}\left(-X_r^{(1)^2} + 2\sum_{s=1}^{n_2} B_{rs}^{(12)} X_r^{(1)} X_s^{(2)}\right)$$

$$\leqslant \sum_{r=1}^{n_1}\left(-\frac{1}{2}X_r^{(1)^2} + 2n_2\sum_{s=1}^{n_2} B_{rs}^{(12)^2} X_s^{(2)^2}\right)$$

$$= -\frac{1}{2}X_1^T X_1 + 2n_2\sum_{s=1}^{n_2}\sum_{r=1}^{n_1} B_{rs}^{(12)^2} X_s^{(2)^2},$$

$$\therefore \quad \text{Let} \quad C_{ij} = \max\{B_{rs}^{(ij)}, \quad r = 1,2,\cdots,n_i;\ s = 1,2,\cdots,n_j\}, \tag{9}$$

$$i,j = 1,2,\cdots; \quad i \neq j,$$

then

$$\dot{V}_1 \leqslant -\frac{1}{2}X_1^T X_1 + 2n_2 n_1 C_{21}^2 X_2^T X_2.$$

Similarly,

$$\dot{V}_2 \leqslant -\frac{1}{2}X_2^T X_2 + 2n_1 n_2 C_{21}^2 X_1^T X_1.$$

According to the theorem concerned of the vector Lyapunov function decomposition method, when

$$4n_1^2 n_2^2 C_{12}^2 C_{21}^2 \leqslant \frac{\lambda_m(B_1)\lambda_m(B_2)}{4\lambda_M(B_1)\lambda_M(B_2)},$$

i. e.

$$|C_{12}C_{21}| \leqslant \frac{1}{4n_1 n_2}\sqrt{\frac{\lambda_m(B_1)\lambda_m(B_2)}{\lambda_M(B_1)\lambda_M(B_2)}}, \tag{10}$$

the balanced state of the large-scale system (1) is of asymptotic stability.

Let
$$\Delta_1 = \frac{1}{4n_1n_2}\sqrt{\frac{\lambda_m(B_1)\lambda_m(B_2)}{\lambda_M(B_1)\lambda_M(B_2)}},$$
where $\lambda_M(B_i), \lambda_m(B_i)$ $(i=1,2)$ is respectively the largest and smallest eigenvalue of the matrix B_i.

In terms of $|C_{12}C_{21}| \leqslant \Delta_1$, the incidence parameter stability domain can be drawn, see Fig. 1.

Secondly, consider the parameter stability domain resulted from the scalar Lyapunov method. In terms of (6)

$$\dot{V}_{(1)} \leqslant \sum_{i=1}^{2}\Big[-\frac{1}{2}X_i^TX_i + \frac{1}{2}\sum_{\substack{j=1\\ j\neq i}}^{2}\sum_{s=1}^{n_j}\sum_{r=1}^{n_i} n_j(B_{rs}^{(ij)} + B_{sr}^{(ji)})^2X_s^{(j)^2}\Big]$$
$$\leqslant \sum_{i=1}^{2}\Big[-\frac{1}{2}X_i^TX_i + \frac{1}{2}\sum_{\substack{j=1\\ j\neq i}}^{2} n_jn_i(C_{ij} + C_{ji})^2X_j^TX_j\Big]$$
$$= \frac{1}{2}[-1 + n_1n_2(C_{21} + C_{12})^2]X_1^TX_1 + \frac{1}{2}[-1 + n_2n_1(C_{12} + C_{21})^2]X_2^TX_2,$$

hence, when $n_1n_2(C_{12}+C_{21})^2 \leqslant 1$, i. e.
$$|C_{12} + C_{21}| \leqslant \frac{1}{\sqrt{n_1n_2}}, \tag{11}$$
the balanced state of the large-scale system (1) is of asymptotic stability.

Let
$$\Delta_2 = \frac{1}{\sqrt{n_1n_2}},$$
in terms of $|C_{12}+C_{21}| \leqslant \Delta_2$, its incidence parameter stability domatin can also be drawn, see Fig. 2.

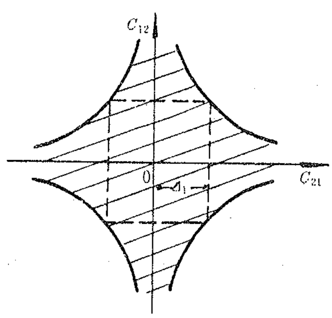

Fig. 1 Domain from vector method

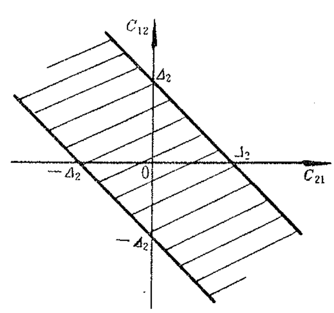

Fig. 2 Domain from scalar method

In terms of both Fig. 1 and Fig. 2, it can be seen that the incidence parameter stability domains resulted respectively from the scalar and the vector Lyapunov function decomposition method are not included each other.

Renferences

[1] Liu Yongqing and Song Zhongkun. Theory and Application of Large-Scale Dynamic Systems. The South China Institute of Technology Press, 1988 (in Chinese)

[2] Wang Muqiu. The Expansion of the Parameter Stability Domain. Journal of Mathematics, 1975, 18(2):107—122 (in Chinese)

[3] Bailey, F. N.. The Application of Lyapunov's Second Method to Interconnected Systems. J. SIAM. Control Ser. A, 1966, 3(3):443—462

线性定常大系统参数稳定域的一个新结论

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摘要:本文应用标量李亚普诺夫函数分解法结合 Bailey 不等式对线性定常大系统的参数稳定域进行了讨论,得到参数稳定域的一种新形式,并以具有两个子系统的 n 阶大系统为例,指出新的参数稳定域与用向量李亚普诺夫函数分解法得到的参数稳定域互不包含这一新的结论.

关键词:大系统;稳定性;李亚普诺夫方法;矩阵理论

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