THE MIXED PENALTY FUNCTION METHODS FOR SOLVING THE CONSTRAINED OPTIMAL CONTROL PROBLEMS

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Abstract

The articles [7, 8] have taken a unified approach to define the concepts of exterior and interior functions, and have taken the generalized exterior and interior penalty functions to solve the constrained optimal control problems. In this paper, our work is founded on [7, 8]. We combine the exterior and interior penalty functions to produce a mixed method for solving the constrained optimal control problems in the following form:

$$\begin{cases} \dfrac{dx}{dt} = g(t, x) + B(t, x)u, \\ x(a_u) = x_0, \quad x(b_u) = x_1, \\ x(t) \in B = B' \cap B'', \quad u(t) \in U, \quad a_u \leq t \leq b_u, \\ J[u] = \int_{a_u}^{b_u} \{ g_0(t, x) + \langle h_0(t, x), u \rangle \} dt = \min , \end{cases}$$

where $\langle \cdot , \cdot \rangle$ denotes inner product; $g(t, x)$, $g_0(t, x)$ and $h_0(t, x)$ are n-vectors, 1-vector and r-vectors functions, respectively; $B(t, x)$ is an $n \times r$ matrix function; all of the above functions are continuous for $(t, x) \in [a, b] \times R^n$ and continuously differentiable for $x \in R^n$; u is a vector and its range is in U; U is a convex compact set in R^r with nonempty interior; B' and B'' are clased sets in R^n with nonempty interior.

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Identification of Multivariable Continuous-Time Systems From Samples of Input-Output Data

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Abstract

Several methods for the identification of linear multivariable continuous-time systems from the samples of input-output data are discussed. These include three new methods proposed by the authors. The suitability of these methods for estimating the parameters of the system using a recursive least-squares algorithm is compared using a simulated example. The results indicate that the best results are obtained using the block pulse function method as proposed by the authors.

1. Introduction

In most practical situations, one must identify the process parameters, using a digital computer, from the samples of the input and output observations. On the other hand, the dynamical model of the process is usually described in terms of continuous-time state equations. The problem may, therefore, be stated as the estimation of the parameters of a continuous-time model from the samples of the input-output data for a multivariable system.

In this paper we consider five different methods for the identification of linear multivariable continuous-time systems from the input-output data.

(i) the state transition method

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(ii) the bilinear z-transformation method

(iii) the modified state transition method

(iv) the trapezoidal integration method

and (v) the block pulse function method.

The first three methods are based on the indirect approach, i. e. a discrete-time model is first identified, and then an equivalent continuous-time model is obtained. Among these, only the first two are well-known. The last two are direct methods, and have not been studied earlier in connection with the identification of multivariable continuous-time systems from the samples of input and output data.

Finally, a comparison of the five methods is made for a simulated two-input two-output system. Since a recursive least-squares algorithm has been utilized for parameter estimation in each case, only a very small amount of noise was added to the output. For higher noise level, we must use more sophisticated methods, like maximum likelihood or generalized least-squares for better estimates.

2. Statement of the Problem

Consider an nth-order linear time-invariant system with m inputs and p outputs. The outputs of the system are assumed to be contaminated with additive noise. The system can be described by the following equations.

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \\ z(t) &= y(t) + w(t) \end{aligned}\right\} \qquad (1)$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$. The noise vector $w(t)$ is assumed to be a zero-mean random noise vector of dimension p.

The problem of system identification may be stated as the determination of the matrices A, B and C from records of samples of $u(kT)$ and $z(kT)$, where k is an integer, and T is the sampling interval. For convenience, these sampled observations will be denoted as $u(k)$ and $z(k)$, respectively. It will be assumed that the sampling interval, T, has been selected carefully, and in particular, $|\lambda_* T| < 0.5$, where λ_* is the eigenvalue of A farthest from the origin of the complex frequency plane [11]. It will also be assumed that the order, n, of the model is known a priori.

As is well known, the matrices A, B and C are not unique and for any given input-output description, many such matrices can be obtained through a similar transformation of the state. Advantage of this fact can be taken to define the matrices in a canonical form. Several canonical forms have been proposed for the identification of multivariable systems [1, 2, 12—14]. Alternatively, one may utilize the transfer function matrix description of equation (1), which is unique.

$$Y(s) = G(s)U(s) + W(s) \tag{2}$$

where

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1m}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2m}(s) \\ \vdots & \vdots & & \vdots \\ g_{p1}(s) & g_{p2}(s) & \cdots & g_{pm}(s) \end{bmatrix} \tag{3}$$

is the transfer function matrix of the system. If necessary, it is always possible to obtain the state equations from the transfer function matrix. Another advantage of using the transfer functionmatrix is that one can decompose the multivariable system into p subsystems, each with one output and m inputs, corresponding to each row of $G(s)$. Hence, each output may be calculated in the following form

$$Y_i(s) = \sum_{j=1}^{m} g_{ij}(s)u_j(s) + w_i(s) \tag{4}$$

$$i = 1,\ 2, \cdots,\ p$$

It may be pointed out that, in general, we may have to estimate more parameters when we use equation (4) than when we use a canonical form of the state equations.

3. The State Transition Method

This is the most well-known method. If we assume that the input is held constant during each sampling interval, and allowed to vary only at the sampling instants, we get the following state transition equation

$$\begin{aligned} x(k+1) &= Fx(k) + Gu(k) \\ y(k) &= Cx(k) + w(k) \end{aligned} \tag{5}$$

where for notational convenience, $x(kT)$ has been represented by $x(k)$ and

$$F = e^{AT} \tag{6}$$

$$G=\int_0^T e^{At}\cdot B\,dt \tag{7}$$

The problem of estimating the parameters of the discrete-time model descrided by equations (5) has been discussed by several authors [1—5], who have proposed using canonical forms of these matrices to minimize the number of unknown parameters. The next problem, then, is to determine A and B from the estimates of F and G.

An efficient approach, which does not require the computation of the eigenvalues and eigenvectors of F has been proposed earlier [6] and will be described briefly. We first make an initial guess of AT as

$$(AT)^{(0)}=\frac{1}{2}(F-F^{-1}) \tag{8}$$

Further improvements are then made by using the algorithm

$$(AT)^{(k+1)}=(AT)^{(k)}+F^{-1}(F-F^{(k)}) \tag{9}$$

where

$(AT)^{(k)}$ is the guess for AT at the kth iteration, and

$$F^{(k)}=e^{(AT)^{(k)}}=I+AT^{(k)}+\frac{((AT)^{(k)})^2}{2!}+\frac{((AT)^{(k)})^3}{3!}+\cdots \tag{10}$$

It should be emphasized that the convergence of this algorithm is guaranteed only if the initial guess $(AT)^{(0)}$, as given by equation (8), is sufficiently close to the actual value of AT. This will be possible in all cases where A repesents a stable system, and the spectral norm of AT is less than 0.5.

After determining A from F, it is now quite straightforward to obtain B through the relationship

$$B=R^{-1}G \tag{11}$$

where

$$R=IT+\frac{1}{2!}AT^2+\frac{1}{3!}A^2T^3+\cdots \tag{12}$$

and can be easily calculated on a computer if A is known. Furthermore, the nonsingularity of R is guaranteed if the condition on the spectral norm of AT is satisfied.

4. The bilinear Z-Transformation

Instead of using state-space techniques, one may like to use the transfer function matrix representation of the multivariable system, and decompose it into p subsystems with one output and m inputs, as in equation (4). A procedure for identifying the parameters for this representation has been discussed in an earlier paper [15]. From the transfer function matrix of the discrete-time system obtained in this manner, we can obtain the transfer function matrix of the corresponding continuous-time system by using the bilinear z-transformation, given by

$$s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} \tag{13}$$

or

$$z^{-1} = \frac{2-sT}{2+sT} \tag{14}$$

Thus, one only has to replace z^{-1} in the transfer function matrix of the discrete-time system by the right-hand side of equation (14) to obtain the corresponding continuous-time transfer function matrix. This is a very good approximation as long as the sampling interval has been selected suitably, so that $|\lambda_* T| < 0.5$, where λ_* is the pole of the continuous-time transfer function matrix farthest from the origin of the s-plane [5]. This corresponds exactly to the spectral norm condition discussed in the previous section.

5. The Modified State Transition Method

The method described in section 3 is based on the assumption that the input $u(t)$ is held constant during each sampling interval. Unless a sampler and zero-order hold are included with the continuous-time system, this assumption is seldom valid, with the possible exception of the case in which $u(t)$ is piece-wise constant. Furthermore, our objective is to estimate the model of a continuous-time system, with the knowledge of only the samples of the input and the output. In the absence of any further information about the variation of $u(t)$ between the sampling instants, it appears that a more reasonable approach is to utilize the mean value of $u(k)$ and $u(k+1)$ to obtain $x(k+1)$. Hence, equation (5) is modified as follows

$$\left.\begin{aligned} x(k+1) &= F\,x(k) + G \cdot \frac{1}{2}\{u(k)+u(k+1)\} \\ y(k) &= C\,x(k) + w(k) \end{aligned}\right\} \tag{15}$$

where F and G are as defined earlier in equations (6) and (7).

The procedure for identifying the discrete-time model is the same as before except for the fact that instead of using $u(k)$, we now use $\frac{1}{2}\{u(k)+u(k+1)\}$. With this minor modification, the earlier algorithms can be utilized [1—3] to estimate F and G, in a canonical form. The matrices A and B can be determined from F and G in the same manner, as described in section 3.

6. The Trapezoidal Integration Method

As pointed out by Hung et al. [10], one may use the trapezoidal rule of integration to directly estimate the parameters of the continuous-time model from the samples of the input-output data. The method proposed by them required the measurement of the states to obtain the model, and was applicable only to single-input single-output systems. We shall present a method which will remove these limitations.

If we integrate the state equation over the interval $kT<t\leq(k+1)T$, and use the trapezoidal rule, we get the following approximation.

$$x(k+1)-x(k)=\int_{kT}^{(k+1)T} A\,x\,dt+\int_{kT}^{(k+1)T} B\,u\,dt$$

$$\cong\frac{AT}{2}[x(k+1)+x(k)]+\frac{BT}{2}[u(k+1)+u(k)] \qquad (16)$$

Equation (16) can be rearranged in the form

$$x(k+1)=\overline{F}\,x(k)+\overline{G}\,\frac{1}{2}[u(k+1)+u(k)] \qquad (17)$$

$$y(k)=C\,x(k)+w(k)$$

where

$$\overline{F}=\left(I-\frac{AT}{2}\right)^{-1}\left(I+\frac{AT}{2}\right) \qquad (18)$$

and

$$\overline{G}=\left(I-\frac{AT}{2}\right)^{-1}BT \qquad (19)$$

The matrices $\overline{F}$ and $\overline{G}$ can be estimated if we know the order, n, of the model. Let us consider that the matrix A can be diagonalized by the linear transformation to obtain

$$A = \text{diag.}\ [\lambda_1,\ \lambda_2, \cdots,\ \lambda_n] \tag{20}$$

Then, from equations (18) and (19).

$F = \text{diag}\,[f_1,\ f_2, \cdots, f_n]$

where

$$f_i \triangleq \frac{1+\frac{1}{2}\lambda_i T}{1-\frac{1}{2}\lambda_i T} \tag{21}$$

The transfer function of each subsystem is first estimated from $y_i(k)$ and $0.5\,\{u(k)+u(k-1)\}$ by expressing the relationships in the form of a difference equation. A partion fraction expansion then leads to f_i and the other related terms. The details of the derivation are given in an ealier paper [18].

The case when A cannot be diagonalized is also easily handled by using the corresponding Jordan form. In this case, the partial fraction expansion leads to some repeated eigenvalues, from which the corresponding A, B and C are obtained in a straightfoward manner.

7. The Block Pulse Function Method

The block pulse functions were first used in the analysis and synthesis of dynamic systems by Sannuti [8], who utilized them for integrating state equations. Since this approach requires matrix inversion, it is not convenient for on-line identification. We shall present an algorithm which does not require matrix inversion.

Although both block pulse and Walsh functions constitute complete sets of orthogonal piecewise-constant functions for approximation, the former have several advantages [7, 8].

Following Shieh et al. [9] and Palanisamy [17], we shall define an N-element block pulse function over the interval $0<t<T_0$, where $T_0 = NT$, as

$$\phi_j(t) \triangleq \begin{cases} 1 & \text{for} \quad (j-1) \quad T \le t < jT \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

and let

$$\underline{\phi}(t) = [\phi_1(t) \quad \phi_2(t) \cdots \phi_N(t)]' \tag{23}$$

Given any function $y(t)$ which is integrable over the interval $0<t<T_0$, we can approximate it as

$$\hat{y}(t) = \underline{\phi}'(t) \cdot \underline{y} \tag{24}$$

where

$$\underline{y} = [y_1 \ y_2 \cdots y_N]' \tag{25}$$

with the superscript' representing transposition and

$$y_i \triangleq \text{average value of } y(t) \text{ over the interval } (i-1)T \leq t \leq iT \tag{26}$$

It is easily shown that

$$\int_0^t \underline{\phi}(\tau)\, d\tau = TH\, \underline{\phi}(t) \tag{27}$$

where

$$H = \begin{pmatrix} \frac{1}{2} & 1 & 1 \cdots & 1 \\ 0 & \frac{1}{2} & 1 \cdots & 1 \\ 0 & 0 & \frac{1}{2} \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \tag{28}$$

It may be noted that H is an $N \times N$ upper triangular matrix. It also follows that we may approximate the integral

$$I_1(t) = \int_0^t y(\tau) d\tau \cong \int_0^t \underline{\phi}'(\tau)\, \underline{y}\, d\tau = T\underline{\phi}' H' \underline{y} \tag{29}$$

We may also express $I_1(t)$ in the block pulse form

$$I_1(t) = \underline{I_1}'\, \underline{\phi}(t) = [I_{1,1}, I_{1,2}, \cdots, I_{1,N}]\underline{\phi}(t) \tag{30}$$

where

$$\left.\begin{aligned} I_{1,1} &= \tfrac{1}{2} T y_1 \\ I_{1,2} &= T y_1 + \tfrac{1}{2} T y_2 \\ I_{1,3} &= T y_1 + T y_2 + \tfrac{1}{2} T y_3 \\ &\vdots \end{aligned}\right\} \tag{31}$$

If $y_0 = 0$, then we get the following recursive relationship

$$I_{1,i} = I_{1,i-1} + \frac{T}{2}(y_i + y_{i-1}) \text{ for } i = 1, 2, \cdots \tag{32}$$

Similarly,

$$I_2(t) = \int_0^t \int_0^t y(\tau) d\tau\, d\tau \cong \underline{I_2}'\, \underline{\phi}(t) \tag{33}$$

where

$$\overline{I_2}' = [I_{2,1}, I_{2,2}, \cdots, I_{2,N}] \tag{34}$$

Again, if $I_{1,0} = 0$, we get the recursive relationship

$$I_{2,i} = I_{2,i-1} + \frac{T}{2}(I_{1,i} + I_{1,i-1}) \text{ for } i = 1, 2, \cdots \tag{35}$$

Substituting for $I_{1,i}$ from epuation (32) we get

$$I_{2,i}=I_{2,i-1}+TI_{1,i-1}+\frac{T^2}{4}(y_i+y_{i-1}) \tag{36}$$

Proceeding in this manner, we can show that the k^{th} integral

$$I_k(t)\cong \underline{I_k}'\underline{\phi}(t) \tag{37}$$

and

$$I_{k,i}=I_{k,i-1}+TI_{k-1,i-1}+\frac{T^2}{2}I_{k-2,i-1}+\frac{T^3}{4}I_{k-3,i-1}+\cdots$$

$$+\frac{T^{k-1}}{2^{k-2}}I_{1,i-1}+\frac{T^k}{2^k}(y_i+y_{i-1}) \tag{38}$$

We shall now show how these relationships can be utilized for identifying the parameters of a continuous-time multivariable system from the samples of the input-output data. First we decompose it into p subsystems, each with one output and m inputs. Then, the output at the kth sampling instant can be expressed in terms of the unknown parameters as well as the past samples of the output, the inputs, and terms of the form $I_{i,j}$. For example, consider a second-order system with two inputs and two outputs. The differential equation for one of the outputs can then be written as

$$\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_0y=b_1\frac{du}{dt}+b_0u+c_1\frac{dv}{dt}+c_0v \tag{39}$$

where $u(t)$ and $v(t)$ are the two inputs.

Integrating the above twice with respect to time, and expressing in terms of block pulse functions, we get, for zero initial conditions

$$\underline{y}'\underline{\phi}(t)+a_1\underline{I_1}'\underline{\phi}(t)+a_0\underline{I_2}'\underline{\phi}(t)=b_1\underline{J_1}'\underline{\phi}(t)+b_0\underline{J_2}'\underline{\phi}(t)$$
$$+c_1\underline{k_1}'\underline{\phi}(t)+c_0\underline{k_2}'\underline{\phi}(t) \tag{40}$$

where

$$I_1(t)=\int_0^t y(\tau)d\tau\cong\underline{I_1}'\underline{\phi}(t) \tag{41}$$

$$I_2(t)=\int_0^t\int_0^t y(\tau)\,d\tau\,d\tau\cong\underline{I_2}'\underline{\phi}(t) \tag{42}$$

$$J_1(t)=\int_0^t u(\tau)\,d\tau\cong\underline{J_1}'\underline{\phi}(t) \tag{43}$$

$$J_2(t)=\int_0^t\int_0^t u(\tau)\,d\tau\,d\tau\cong\underline{J_2}'\underline{\phi}(t) \tag{44}$$

$$K_1(t)=\int_0^t v(\tau)\,d\tau=\underline{K_1}'\underline{\phi}(t) \tag{45}$$

and

$$K_2(t) = \int_0^t \int_0^t v(\tau)\, d\tau\, d\tau = \underline{K}_2{}' \, \underline{\phi}(t) \qquad (46)$$

Equating the k^{th} coefficient of $\phi(t)$ we have

$$y_k = -a_1 I_{1,k} - a_0 I_{2,k} + b_1 J_{1,k} + b_0 J_{2,k} + c_1 k_{1,k} + c_0 k_{2,k} \qquad (47)$$

The values of the terms $I_{i,k}$, $J_{i,k}$ and $K_{i,k}$ are obtained from the samples of the output and input according to equations (32) and (36). Hence, the unknown parameters of the subsystem can be estimated by using the recursive least – squares (or the maximum likelihood) algorithm from the concatenation of equations obtained for different values of k. It should be noted that while calculating $I_{i,k}$, $J_{i,k}$ and $K_{i,k}$, we have

$$\left.\begin{aligned} y_i &= \frac{y(iT) + y((i-1)T)}{2} \\ u_i &= \frac{u(iT) + u((i-1)T)}{2} \\ \text{and} \quad v_i &= \frac{v(iT) + v((i-1)T)}{2} \end{aligned}\right\} \qquad (48)$$

8. Results of Simulation

To compare the methods, a two – input two – output second – order system with the following transfer function matrix was simulated

$$G(s) = \begin{pmatrix} \dfrac{1}{s^2+3s+2} & \dfrac{2}{s+1} \\ \dfrac{3}{s+2} & \dfrac{2s+1}{s^2+3s+2} \end{pmatrix} \qquad (49)$$

The output of the system was calculated for the following input

$$u_1(t) = 1.5 \cos 0.9871t + 2.5 \cos 0.2137t - 4 \cos 5.8763t \qquad (50)$$

$$u_2(t) = 2 \cos 0.4769t + 2 \cos 3.83t - 4 \text{ oos } 2.317t \qquad (51)$$

Assuming zero initial conditions, the exact output is given by

$$\begin{aligned} y_1(t) = {} & 1.63123\, e^{-2t} - 5.295588\, e^{-t} + 0.478639 \cos(0.9871t - 1.237807) \\ & + 1.215481 \cos(0.2137t - 0.316979) - 0.108106 \cos(5.8736t \\ & - 2.64498) + 3.601447 \cos \\ & (0.4769t - 0.444997) + 1.01051 \cos(3.83t - 1.315401) \\ & - 3.170091 \cos(2.317t - 1.163355) \end{aligned} \qquad (52)$$

$$\begin{aligned} y_2(t) = {} & 1.1286\, e^{-t} - 5.8133\, e^{-2t} + 2.01764 \cos(0.9871t \\ & - 0.458474) + 3.728775 \cos(0.2137t - 0.106446) \end{aligned}$$

$$-1.9332\cos(5.8763t-1.242744)+1.21333\cos(0.4769t+0.08268)+0.903337\cos(3.83t-0.963971)-2.454973\cos(2.317t-0.663793) \quad (53)$$

A small amount of noise was added to the outputs, and the samples of the data were utilized for estimating the models with noise to signal ratio of 0.15%. For parameter estimation, the system was decomposed into two subsystems, each of the form

$$G(s)=\left[\frac{b_1s+b_0}{s^2+a_1s+a_0} \quad \frac{c_1s+c_0}{s^2+a_1s+a_0}\right] \quad (54)$$

The recursive least-squares method (at sampling intervals of 0.05 second and 0.1 second), was used to estimate the model parameters from 200 samples of input-output data with and without noise.

For the smaller sampling interval, T=0.05 second, all the methods except the block pulse function method gave poor estimates in the presence of noise. The reason is that with smaller sampling interval, the differencing used in these methods causes the noise to be accentuated. On the other hand, the block pulse function method does not appear to be affected as it uses integration which smooths out the effect of the noise.

9. Conclusions

The results of simulation indicate that the state transition method gives very poor estimates of the parmeters when the input is not held constant between the sampling instants. If the bilinear z-transformation is used with the transfer function obtained as above, the results are not much better. It was also found that in this case the bilinear z-transformation gave an extra term (b_2 and c_2) in each of the numerators.

The modified state transition method gave much better estimates, as expected. The corresponding models obtained with bilinear z-transformation were also good, and nearly identical with those obtained with the trapezoidal rule.

The best results were obtained with the block pulse function method. It is probably due to the fact that the implied integration used in this method has the effect of smoothing out the noise. This was noticed even further when the sampling interval was reduced, In the presence of noise all the other methods gave poor estimates, since all the transformations require division by T, as well as some differencing. It is felt by the authors that this methodi s very promising and merits further investigation, especially regarding bias and consistency in the presence of noise while using recursive least - squares methods.

Another interesting feature is the choice of the sampling interval. It was found that for most systems, there is an optimum choice of the range from which the sampling interval should be selected. Larger as well as smaller sampling intervals give poorer estimates of the parameters, especially for the first four methods. This will be discussed in more detail in a subsequent paper.

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