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A Note on The Estimation of Cardiac Output

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Abstract

A nonlinear least squares fit technique with a moving window is used to estimate a time varying coefficient of as et of differential equations describing the cardiac output for a one body compartment lung model.

Computer simulations demonstrated the feasiblityof this method.

1.

Knowledge of cardiac output is important in patient monitoring, physiological experiments and heart function testings. Standard clinical methods used today of measurement of cardiac output are thermal dilution and dye solution, both of which are invasive methods.

Many noninvasive methods for measuring cardiac output have been developed since the work of Fick in 1870. One of them was proposed recently by O. Brovko and co-workers using an extended Kalman filter to estimate the pulmonary blood flow which is nearly equal to the cardiac output.

In our work we have tried to approach the same problem by using the method of nonlinear least squares fit with a moving window, and achieved satisfactory results in computer simulations.

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Following Zwart it can be shown by a minor transformation of variables that the cardiac output for a one body compartment lung model can be described mathematically to a proportional constant by the time-varying coefficient $b(t)$ of the following system of differential equations:

$$\begin{vmatrix} \dot{x}_1 \\ \dot{x}_2 \end{vmatrix} = \begin{vmatrix} -a & -b(t) \\ -a & -cb(t) \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \end{vmatrix} + \begin{vmatrix} a \\ a \end{vmatrix} \mathrm{P}(t) \qquad \begin{array}{l} x_1(0)=x_2(0)=0.0 \\ t \geq 0.0 \end{array} \tag{1}$$

The measurable output is:

$$z(t)=x_1(t)+r(t) \qquad t \geq 0.0 \tag{2}$$

In Eq. (1),

$P(t)$ is the inhaled gas partial pressures;

$x_1(t)$ is the end tidal gas partial pressures;

$x_2(t)=x_1(t)-p(t)$, where $p(t)$ is the unmeasurable partial pressures in the body compartment;

$a=(1-x)\dot{v}$, where x is the fraction of physiological dead space in the lung; $\dot{v}$ is the ventilation, volume per unit time. In our experiment $x=0.4$, $\dot{v}=7.0$ Lt/min, so that $a=4.2$.

$c=1+C_I/C_1$, where C_I is the equivalent gas volume of the gas exchanging part of the lung; C_1 is the equivalent gas volume of the body compartment. In our experiment $C_I=2.7$ Lt, $C_1=57.6$ Lt, so that $c=1.047$.

In Eq. (2), the measurement noise $r(t)$ is assumed to be the identically independently distributed random variable with zero mean and standard deviation σ. when $\sigma=0.0$ we refer to this case as the noiseless case, otherwise, the noisy case.

Our problem is to obtain an estimate of $b(t)$ at discrete time instant t given observations of $z(t)$ from time zero up to time t.

In approaching this time-varying problem we use a constant coefficient model:

$$\begin{vmatrix} \dot{y}_1 \\ \dot{y}_2 \end{vmatrix} = \begin{vmatrix} -a & -b \\ -a & -cb \end{vmatrix} \begin{vmatrix} y_1 \\ y_2 \end{vmatrix} + \begin{vmatrix} a \\ a \end{vmatrix} P(t) \qquad \begin{array}{l} y_1(0)=y_2(0)=0.0 \\ t \geq 0.0 \end{array} \tag{3}$$

where a, c, $P(t)$ are the same as in Eq. (1). Instead of making use of observations of $z(t)$ from time zero to time t, this model is fitted by observations of $z(t)$ only within a moving window on the time axis by minimizing the following penalty function with respect to b:

$$f(b) = \sum_{t=(L+1)T}^{(L+M)T} [z(t) - y_1(t)]^2 \tag{4}$$

This moving window is shown in Fig. 1, where T is the sampling period, MT is the window width and $(L+1)T$ is the starting point of the window. In our case $T=0.02$ min, $M=10$ and $L=0, 1, \ldots$. As all observations of $z(t)$ within the window are used in Eq. (4), the minimum point of $f(b)$ in Eq. (4) should reasonably be referred to as the estimate of $b(t)$ at the middle point of the window and will be denoted as $\hat{b}(L+M/2)$.

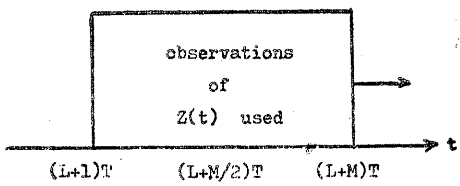

Fig. 1 Moving window with width MT.

2.

When $P(t)$ in Eq. (3) is a constant U, $y_1(t)$ can be obtained analytically as:

$$y_1(t) = U - \frac{U}{s_1 - s_2}[(s_1 - a)e^{-s_2 t} - (s_2 - a)e^{-s_2 t}] \tag{5}$$

where $-s_1$ and $-s_2$ are eigenvalues of Eq. (3):

$$-s_1 = 0.5[-(a+cb) + \sqrt{(a-cb)^2 + 4ab}] \tag{6}$$

$$-s_2 = 0.5[-(a+cb) - \sqrt{(a-cb)^2 + 4ab}] \tag{7}$$

both of them are real and negative when $a>0$ and $b>0$. Thus the penalty function to be minimized is:

$$f(b) = \sum_{t=(L+1)T}^{(L+M)T} \left\{ z(t) - U + \frac{U}{s_1 - s_2}[(s_1 - a)e^{-s_2 t} - (s_2 - a)e^{-s_1 t}] \right\}^2 \tag{8}$$

When $P(t)$ is a square wave train with constant amplitude and varying polarities through a given period of time as shown Fig. 2, the analytical solution $y_1(t)$ of Eq. (3) is:

$$y_1(t) = C_0 e^{-s_1 t} + C_0^* e^{-s_2 t} + \sum_{i=1}^{\infty} [C_{2i-1}\sin(2i-1)w_0 t + C_{2i-1}^* \cos(2i-1)w_0 t] \tag{9}$$

where

$$w_0 = 2\pi / T_N \tag{10}$$

$$D = [ab(c-1) - (2i-1)^2 w_0{}^2]^2 + (a+cb)^2 (2i-1)^2 w_0{}^2 \tag{11}$$

$$C_{2i-1} = \frac{1}{D}\left\{ \frac{4U}{(2i-1)\pi} ab(c-1)[ab(c-1) - (2i-1)^2 w_0{}^2] - \frac{8Ua}{T_N} \cdot (a+cb)(2i-1)w_0 \right\} \tag{12}$$

$$C_{2i-1}^* = \frac{1}{D}\left\{ \frac{8Ua}{T_N}[ab(c-1) - (2i-1)^2 w_0{}^2] - \frac{4U}{\pi} ab(c-1)(a+cb)w_0 \right\} \tag{13}$$

i = 1, 2, ⋯.

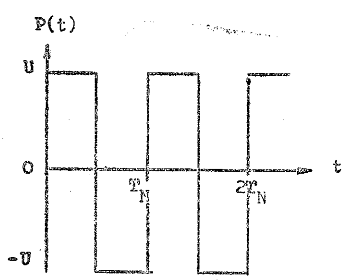

Fig. 2 P(t)

$$C_0 = \frac{U}{s_1 - s_2}(s_2 - a)\frac{1-\exp(0.5T_N s_1)}{1+\exp(0.5T_N s_1)} \tag{14}$$

$$C_0^* = \frac{U}{s_2 - s_1}(s_1 - a)\frac{1-\exp(0.5T_N s_2)}{1+\exp(0.5T_N s_2)} \tag{15}$$

here $-s_1$ and $-s_2$ are eigenvalues of Eq. (3), and are given by Eq. (6) and Eq. (7) respectively.

The penalty function to be minimized is

$$f(b) = \sum_{t=(L+1)T}^{(L+M)T} \left\{ z(t) - C_0 e^{-s_1 t} - C_0^* e^{-s_2 t} - \sum_{i=1}^{5} [C_{2i-1}\sin(2i-1)w_0 t + C_{2i-1}^* \cos(2i-1)w_0 t] \right\}^2 \tag{16}$$

where only the first five sine terms and the first five cosine terms in Eq. (9) are retained in order to facilitate practical computations.

3.

Computer simulations have been performed on the IBM 370 Model 3033, controlled by the Michigan Terminal System, at Rensselaer Polytechnic Institute.

In simulating Eq. (1) $P(t)$ is first assumed to be a constant $U=1.5$, and then is assumed to be a square wave train with amplitute $U=1.5$ and a period $T_N=0.2$ min.. For each case of $P(t)$, $b(t)$ is assumed respectively to be the following three functions,

a. $b(t)$ is a constant,

$$b(k)=23.00;$$

b. $b(t)$ is an exponentially decaying function:

$$b(k+1)=0.992b(k),\quad b(0)=23.00;$$

c. $b(t)$ is a linearly decaying function:

$$b(k+1)=b(k)-0.033,\ b(0)=23.00$$

The time between step k and step (k+1) is the sampling period T which is 0.02 min in our experiments as mentioned previously.

Data of $x_1(k)$ is obtaioned by solving numerically Eq. (1) using a 4th order RUNGE-KUTTA Subroutine. An IMSLLIB Subroutine is called to generate the normal noise $r(k)$. Each $r(k)$, multiplied by σ, is added to $x_1(k)$ to produce the noisy output $z(k)$.

In fitting these data into Eq. (3), a NAG Subroutine is called for obtaining the minimum points of $f(b)$ in Eq. (8) and Eq. (16) respectively. The results are shown in Fig. 3 to Fig. 14. In each figure both $b(k)$ and their estimate $\hat{b}(k)$ are plotted together in order for comparision. These results demonstrate the feasibility of applying the method of nonlinear least squares fit to the estimation of a time-varying coefficient (at present the cardiac output) in a set of differential equations obtained in modelling the cardiac output.

Practical experiments in estimating cardiac output are still going on in Rensselaer Polytechnic Institute.

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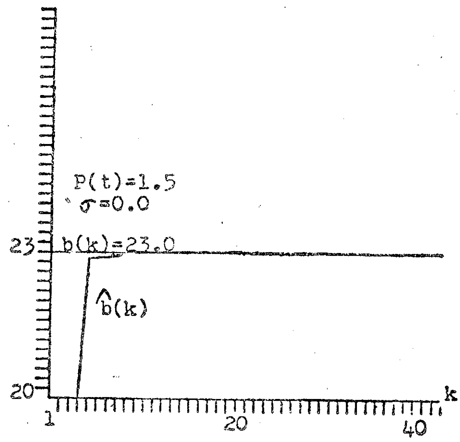

Fig.3

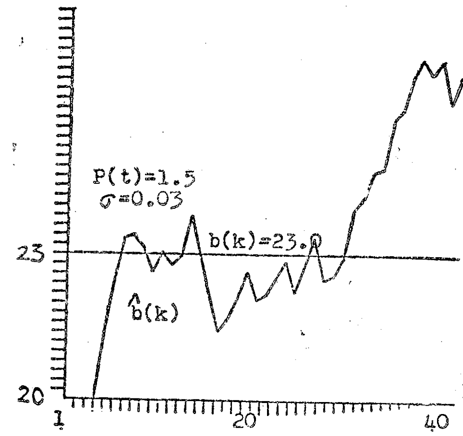

Fig.4

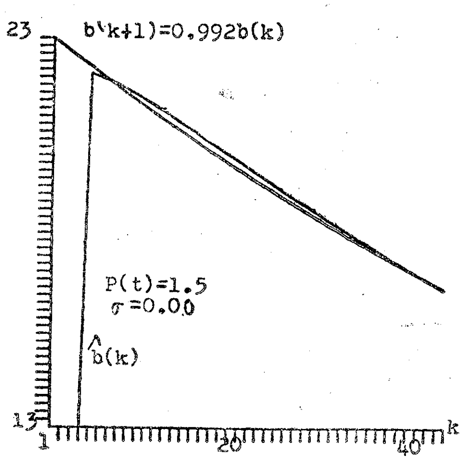

Fig.5

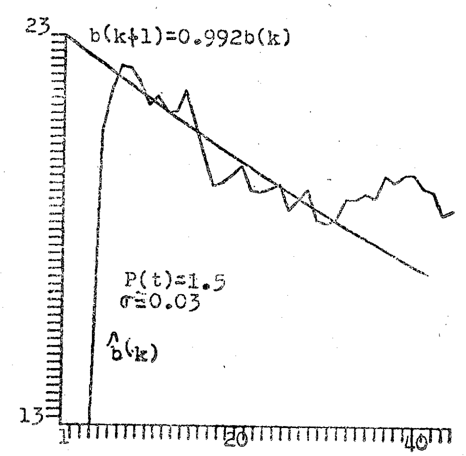

Fig.6

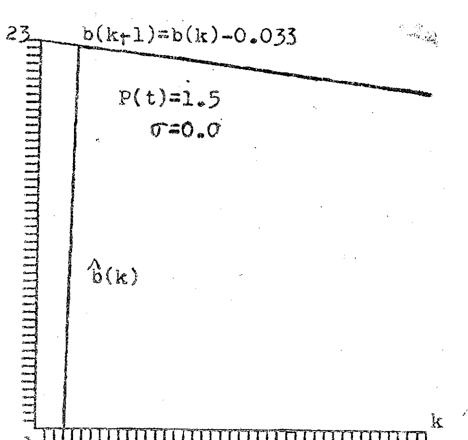

Fig.7

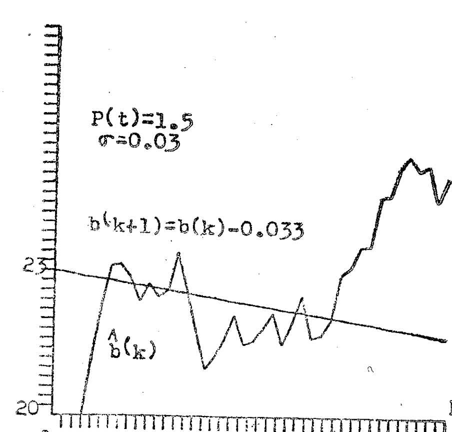

Fig.8

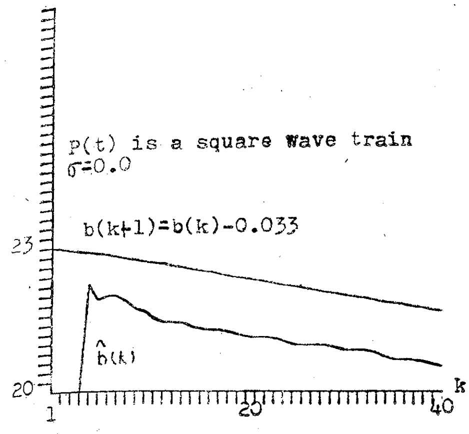

Fig.9

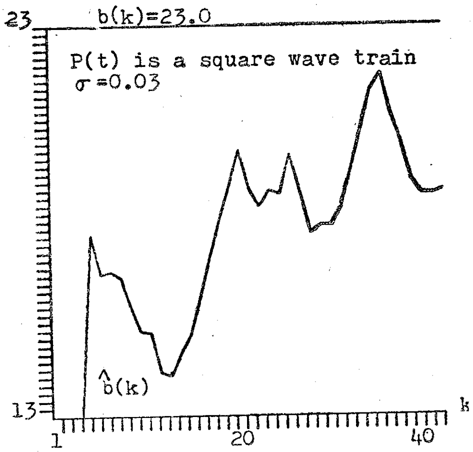

Fig.10

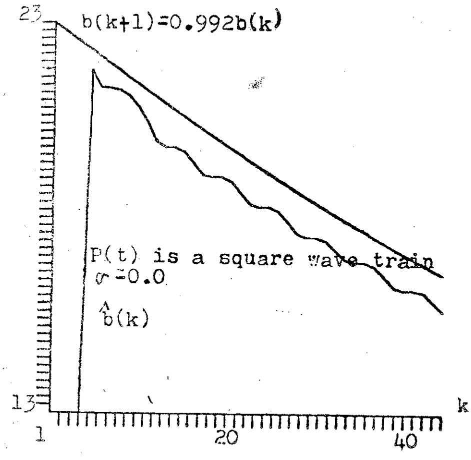

Fig.11

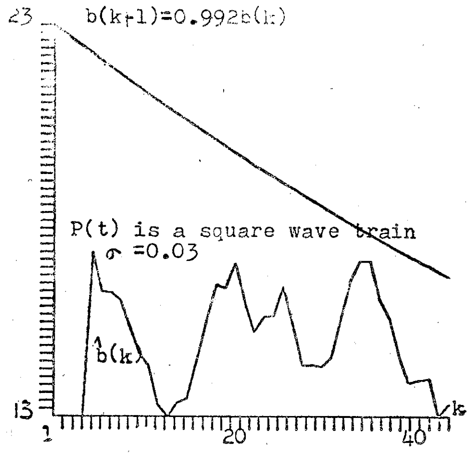

Fig.12

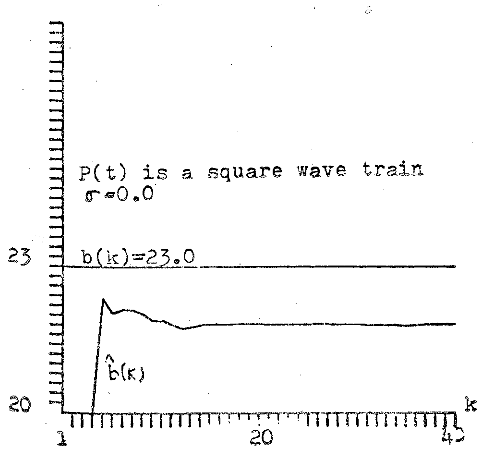

Fig.13

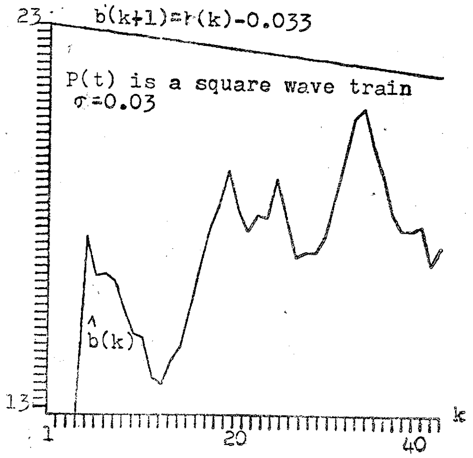

Fig.14

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