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The Decomposition of An Operator on Product B-Spaces And Its Applications

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Abstract

In this paper, two fundamental theorems are given to discuss several types of decomposition of an operator and conditions for the operator to generate a C_0 semigroup under such decompositions. An example is also given to indicate the application of the above theorems.

I. Introduction

The conditions that a linear operator generates a C_0 semigroup on product B-spaces or decomposition of a linear operator and its application to initial value problems with non-local boundary conditions and boundary value control are discussed in a few papers [1], [2], [3], [4]. Now, we want to develop these works specializing for multiple coupling systems.

II. Fundamental Theorems

Consider the following system;

$$dx/dt = Ax, \quad x(0) = x_0. \tag{1}$$

We assume that

$$x = (x_1, x_2, x_3)^T \in X = X_1 \oplus X_2 \oplus X_3$$

where X_1, X_2 and X_3 are B-spaces, A is a linear operator from X to X with domain $D(A) = ((x_1, x_2, x_3)^T \in X: x_i \in S_{X_i} \subset X_i \; i = 1, 2, 3)$ We also assume that A_i is a linear operator on X_i respectively.

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We have the following theorems on decomposition of A and condition to generate a C_0 semigroup.

Theorem 1 We assume that

i) $D(A_i)=S_{X_i}$, $i=2, 3$. $D(A_1)=(\tilde{x}_1\in X_1: \tilde{x}_1=x_1-F_{21}x_2-F_{31}x_3$, $x_i\in S_{X_i}$, $i=1, 2, 3)$. F_{21} is a bounded operator from x_2 to x_1, F_{31} is a bounded operator from x_3 to x_1).

ii) A_i $(i=1, 2, 3)$ generates C_0 semigroup T_t^i on X respectively.

iii) A can be decomposed into the following form

$$A=\begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix}\begin{pmatrix} I & -F_{21} & -F_{31} \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}+\begin{pmatrix} 0 & F_{21}A_2 & F_{31}A_3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (2)$$

where $x_i\in S_{X_i}$. Then, A can generate a C_0 semigroup T_t on X, and T_t is expreseed as follows

$$T_t=\begin{pmatrix} T_t^1 & F_{21}T_t^2-T_t^1F_{21} & F_{31}T_t^3-T_t^1F_{31} \\ 0 & T_t^2 & 0 \\ 0 & 0 & T_t^3 \end{pmatrix}. \quad (3)$$

Proof According to assumptions i) and ii) as well as (3), we can verify that

1, T_t is an operator from R^+ to $\mathbf{L}(X)$;

2, $T_0=I$;

3, $T_tT_s=T_{t+s}$, $t>0$, $s>0$;

4, for any $x_0\in X$, $\|T_tx_0-x_0\|\longrightarrow 0$ as $t\to 0^+$;

5,

$$\lim_{t\to 0^+}(T_tx-x)/t=\lim_{t\to 0^+}\begin{pmatrix} (T_t^1(x_1-F_{21}X_2-F_{31}x_3)/t-x_1/t+ \\ +F_{21}T_t^2x_2/t+F_{31}T_t^3x_3/t \\ (T_t^2x_2-x_2)/t \\ (T_t^3x_3-x_3)/t \end{pmatrix}$$

$$=\lim_{t\to 0^+}\begin{pmatrix} (T_t^1(x_1-F_{21}x_2-F_{31}x_3)-(x_1-F_{21}x_2-F_{31}x_3)/t+ \\ +F_{21}(T_t^2x_2-x_2)/t+F_{31}(T_t^3x_3-x_3)/t \\ (T_t^2x_2-x_2)/t \\ (T_t^3x_3-x_3)/t \end{pmatrix}$$

$$= \begin{bmatrix} A_1(x_1 - F_{21}x_2 - F_{31}x_3) + F_{21}A_2x_2 + F_{31}A_3x_3 \\ A_2x_2 \\ A_3x_3 \end{bmatrix} = Ax$$

by (2) for $x_i \in S_{X_i} = D(A_i)$ $(i=2,3)$ and $x_1 \in S_{X_1}$, since $x_1 - F_{21}x_2 - F_{31}x_3 \in D(A_1)$ for $x_i \in S_{X_i}$ $(i=1,2,3)$.

The proof of theorem is completed by conclusions 1~5.

Remark 1 We can generalize the theorem to more general cases;

$$X = X_1 \oplus X_2 \oplus \quad \oplus X_N,\ N \geq 2,$$

$$D(A_i) = S_{X_i},\ i = 2, 3, \cdots N,$$

$$D(A_1) = (\widetilde{x}_1 \in X_1 : \widetilde{x}_1 = x_1 - \sum_{i=2}^{N} F_{i1}x_i,\ x_i \in S_{X_i},\ i=1,\cdots,N)$$

$$A = \begin{bmatrix} A_1 & & & 0 \\ & \ddots & & \\ & & \ddots & \\ 0 & & & A_1 \end{bmatrix} \begin{bmatrix} I & -F_{21} & \cdots\cdots & F_{N1} \\ & \ddots & & \\ & & \ddots & 0 \\ 0 & & & I \end{bmatrix} + \begin{bmatrix} O & F_{21}A_2 & \cdots\cdots & F_{N1}A_N \\ & \ddots & & \\ & & \ddots & 0 \\ 0 & & & O \end{bmatrix}, \tag{4}$$

where A_i ($i=1,\cdots,N$) is densely defined as an closed operator in X_i and generates a C_0 semigroup T_t^i on X_i respectively, F_i ($i=2,\cdots N$) is a bounded operator from X_i to X_1 repectively.

Then, A generates a C_0 semigroup T_t on X and

$$T_t x = \begin{pmatrix} T_t^1\left(x_1 - \sum_{i=2}^{N} F_{i1}x_i\right) + \sum_{i=2}^{N} F_{i1}T_t^i x_i \\ T_t^2 x_2 \\ \vdots \\ T_t^N x_N \end{pmatrix} \tag{5}$$

Moreover, we can assume that

$$\|R(\lambda, A_i)^r\| \leq m_i/(\lambda - a_i)^r,\ i = 1,\cdots,N,$$

by Hille-Yosida theorem and we can conclude that

$$\|R(\lambda, A)^r\| \le m(N+2(N-1)c)/(\lambda-a)^r, \tag{6}$$

$$\|T_t\| \le m(N+2(N-1)c)\exp(at), \quad t>0, \tag{7}$$

where $m=\max(m_1,m_2,\cdots,m_N)$, $a=\max(a_1,a_2,\cdots,a_N)$, $c=\max(c_1,c_2\cdots,c_N)$, $\|F_{i1}\|\le c$ ($i=1,2,\cdots,N$), and taking

$$\|\cdot\|_X=\sqrt{\|\cdot\|^2_{X_1}+\|\cdot\|^2_{X_2}+\cdots\cdots+\|\cdot\|_{X_N}}\ .$$

Remark 2 In some cases, operator A usually is decomposed into the following form

$$A=\tilde{A}+\phi$$

where $\tilde{A}$ satisfies the assumptions of the theorem or remark 1 and ϕ is a bounded operator from X to X. As it is well known, A generates a C_0 semigroup on X.

Especially, if A is as the first term in (4) and ϕ is as the minus second term in (4), the solution for the system

$$\frac{dx}{dt}=Ax, \quad x(0)=x_0=(x_{10},\cdots, x_{N0})$$

is as follows

$$x_i(t)=T_t^i x_i, \quad i=2,\cdots, N,$$

$$x_1(t)=T_t^1\left(x_{10}-\sum_{i=2}^{N}F_{i1}x_{i0}\right)+\sum_{i=2}^{N}F_{i1}T_t^i x_{i0}+$$

$$+\int_0^t T_{t-s}^1\left(-\sum_{i=2}^{N}F_{i1}A_iT_s^i x_{i0}\right)ds. \tag{8}$$

In what follows, we give more general form of decomposition of A.

Theorem 2 We assume that

i) $D(A_3)=S_{X_3}$,

$$D(A_2)=(\tilde{x}_2\in X_2:\tilde{x}_2=x_2-F_{32}x_3,\quad x_i\in S_{X_i},\quad i=2,3), \tag{9}$$

$$D(A_1)=(\tilde{x}_1\in X_1:\tilde{x}_1=x_1-F_{21}\tilde{x}_2-F_{31}x_3,\quad x_1\in S_{X_1},$$

$$\tilde{x}_2\in D(A_2),\quad x_3\in S_{X_3})$$

where F_{32} is a bounded operator from X_3 to X_2, F_{21} is a bounded operator from X_2 to X_1 and F_{31} is a bounded operator from X_3 to X_1.

ii) As the same as assumption ii) of theorem 1.

iii) A can be decomposed into the following form;

$$A=\begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix}\begin{pmatrix} I & -F_{21} & -F_{31}+F_{21}F_{32} \\ 0 & I & -F_{32} \\ 0 & 0 & I \end{pmatrix}+$$

$$+\begin{pmatrix} 0 & F_{21}A_2 & F_{31}A_3-F_{21}A_2F_{32} \\ 0 & 0 & F_{32}A_3 \\ 0 & 0 & 0 \end{pmatrix}. \tag{10}$$

Then, A can generate a C_0 semigroup T_t on X and T_t is expressed as follows

$$T_t=\begin{pmatrix} T_t^1 & -T_t^1F_{21}+F_{21}T_t^2 & T_t^1F_{21}F_{32}-T_t^1F_{31}-F_{21}T_t^2F_{32}+F_{31}T_t^3 \\ 0 & T_t^2 & F_{32}T_t^3-T_t^2F_{32} \\ 0 & 0 & T_t^3 \end{pmatrix}. \tag{11}$$

Proof We can rewrite (10) as follows

$$A=\begin{pmatrix} A_1 & -A_1F_{21}+F_{21}A_2 & A_1F_{21}F_{32}-A_1F_{31}-F_{21}A_2F_{32}+F_{31}A_3 \\ 0 & & \\ 0 & & \tilde{A} \end{pmatrix}, \tag{12}$$

where

$$\tilde{A}=\begin{pmatrix} A_2 & 0 \\ 0 & A_3 \end{pmatrix}\begin{pmatrix} I & -F_{32} \\ 0 & I \end{pmatrix}+\begin{pmatrix} 0 & F_{32}A_3 \\ 0 & 0 \end{pmatrix}. \tag{13}$$

According to the assumption of the theorem and results of theorem 1 and remark 1, we can see $\tilde{A}$ generates a C_0 semigroup $\tilde{T}_t$ on $\tilde{X}=X_2\oplus X_3$ and we have

$$\tilde{T}_t\tilde{x}=\begin{pmatrix} T_t^2(x_2-F_{32}x_3)+F_{32}T_t^2x_3 \\ T_t^3x_3 \end{pmatrix}, \quad \tilde{x}=\begin{pmatrix} x_2 \\ x_3 \end{pmatrix}. \tag{14}$$

On the one hand, it follows from (9) that;

$$D(A_1)=(\tilde{x}_1=x_1-\tilde{F}\,\tilde{\tilde{x}},\ x_1\in S_{X_1},\ \tilde{\tilde{x}}\in D(\tilde{A}),$$

$$\tilde{\tilde{x}}=(\tilde{x}_2,\ x_3)^T,\ \tilde{x}_2\in D(A_2),\ x_3\in S_{X_3}), \tag{15}$$

where $\widetilde{F}=(F_{21}, F_{31})$ and we have

$$\widetilde{F}\widetilde{\widetilde{x}} = F_{21}x_2 + F_{31}x_3 = F_{21}x_2 - F_{21}F_{32}x_2 + F_{31}x_3$$

$$= (F_{21}, F_{31} - F_{21}F_{32})\widetilde{x} \triangleq F\widetilde{x}, \quad \widetilde{x} = (x_2, x_3)^T. \tag{16}$$

On the other hand, we can also express (12) by the following form

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & \widetilde{A} \end{pmatrix} \begin{pmatrix} I & -F \\ 0 & I \end{pmatrix} + \begin{pmatrix} 0 & F\widetilde{A} \\ 0 & 0 \end{pmatrix}, \tag{17}$$

and $F=(F_{21}, F_{31}-F_{21}F_{32})$ is a bounded operator from $\widetilde{X}$ to X_1 since F_{21}, F_{31} and F_{32} all are bounded operators.

So, we can use the results of theorem 1 and remark 1 again by (15), (16) and (17) and we obtain that A generates a C_0 semigroup T_t on $X=X_1\oplus\widetilde{X}=X_1\oplus X_2\oplus X_3$,

and

$$T_t x = \begin{pmatrix} T_t^1 x_1 - T_t^1 F\widetilde{x} + F\widetilde{T}_t\widetilde{x} \\ \widetilde{T}_t\widetilde{x} \end{pmatrix} = (11)x .$$

The proof of the theorem is completed.

Remark 3 It is easy to verify that the assumptions and results of theorem 2 are reduced to the assumptions and results of theorem 1 when $F_{32}=0$.

Remark 4 When $F_{31}=0$ we can obtain another form of decomposition of A distinct from the form stated in theorm 1:

$$D(A_3) = S_{X_3}$$

$$D(A_2) = (\widetilde{x}_2 \in X_2 : \widetilde{x}_2 = x_2 - F_{32}x_3, \quad x_i \in S_{X_i}, \quad i=2, 3)$$

$$D(A_1) = (\widetilde{x}_1 \in X_1 : \widetilde{x}_1 = x_1 - F_{21}\widetilde{x}_2, \quad x_1 \in S_{X_1}, \quad \widetilde{x}_2 \in D(A_2))$$

and

$$A = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix} \begin{pmatrix} I & -F_{21} & F_{21}F_{32} \\ 0 & I & -F_{32} \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & F_{21}A_2 & -F_{21}A_2F_{32} \\ 0 & 0 & F_{32}A_3 \\ 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

$$T_t x = \begin{pmatrix} T_t^1(x_1 - F_{21}(x_2 - F_{31}x_3)) + F_{21}T_t^2(x_2 - F_{32}x_3) \\ T_t^2(x_2 - F_{32}x_3) + F_{32}T_t^3 x_3 \\ T_t^3 x_3 \end{pmatrix}. \tag{19}$$

Remark 5 When $D(A_1)=(\widetilde{x}_1\in X_1:\widetilde{x}_1=x_1-F_{21}x_2-F_{31}x_3, x_i\in S_{X_i}, i=1,2,3)$ instead of (9), we can also acquire another form of decomposition of A distinct from (2), (3) or (18), (19):

$$A=\begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & A_3 \end{pmatrix}\begin{pmatrix} I & -F_{21} & -F_{31} \\ 0 & I & -F_{32} \\ 0 & 0 & 0 \end{pmatrix}+$$

$$+\begin{pmatrix} 0 & F_{21}A_2 & F_{21}F_{32}A_3+F_{31}A_3-F_{21}A_2F_{32} \\ 0 & 0 & F_{32}A_3 \\ 0 & 0 & 0 \end{pmatrix}. \tag{20}$$

$$T_tx=\begin{bmatrix} T_t^1(x_1-F_{21}x_2-F_{31}x_3)+F_{21}T_t^2(x-F_{32}x_3)+ \\ \qquad\qquad +(F_{21}F_{32}+F_{31})T_t^3x_3 \\ T_t^2(x_2-F_{32}x_3)+F_{32}T_t^3x_3 \\ -T_t^3x_3 \end{bmatrix}, \tag{21}$$

since $F_{21}x_2=F_{21}(\widetilde{x}_2+F_{32}x_3)=F_{21}\widetilde{x}_2+F_{21}F_{32}x_3$.

The result implies that the domains of operators A_1, A_2 and A_3 are connected with each other like a chain.

Remark 6 It follows from the proof of theorem 2 that the results of theorem 2 can be extended to the case of $X=X_1\oplus X_2\oplus\cdots\oplus X_N$, $N\geq 2$ by using the method as was stated in the proof repeatedly.

Remark 7 The key point to apply theorem 1 and 2 is to find out F_{ij} so that the assumptions of theorems are satisfied.

III. An example of more than two systems couple with each other on the boundaries.

consider the system as follows

$$\partial x_1/\partial t=A_1x_1=\partial^2x_1/\partial\xi^2,\quad \xi\in(0,1),\quad t\geq 0,$$
$$x_1(\xi,t)|_{t=0}=x_{10}\in L^2(0,1),$$
$$x_1(0,t)=\sum_{i=1}^{m}x_3^ig_i,\qquad f(\cdot)\in C_0^1(0,1), \tag{22}$$
$$x_1(1,t)=\int_0^1 f(\xi)x_2(\xi,t)d\xi,$$

$\partial x_2/\partial t=A_2x_2=-v\partial x_2/\partial\xi$, A_2——convection operator

v——positive constant (convection speed),

$x_2|_{\xi=0}=\langle b,x_3\rangle_{R^m}$, $b=(b^1,\cdots,b^m)^T$; $x_2(t,\xi)|_{t=0}=x_{20}\in L^2(0,1)$,

$dx_3/dt = A_3x_3 + Du$, A_3——$m\times m$ matrix,

$x_3 = (x_3^1, \cdots, x_3^m) \in R^m$, $x_3(0) = x_{30} \in R$,

$u(\cdot) \in L^2_{\text{loc}}(0, \infty)$, $u(t) \in R^n (n \leq m)$, D——$m\times m$ matrix

Now, we have

$S_{X3} = (x_3 \in R)$,

$S_{X2} = (x_2 \in H^1(0, 1)$: $x_2|_{\xi=0} = \langle b, x_3\rangle_{R^m})$,

$$S_{X1} = \left(x_1 \in H^2(0, 1): \ x_1|_{\xi=0} = \sum_{i=1}^{m} x_3^i g_i, \right.$$

$$\left. x_1|_{\xi=1} = \int_0^1 f(\xi)x_2(\xi, t)d\xi \right).$$

If we take

$D(A_3) = S_{X3}$,

$D(A_2) = (\tilde{x}_2 \in H^1(0,1)$: $\tilde{x}_2 = x_2 - F_{32}x_3$, $x_i \in S_{X_i}$, $i = 2, 3)$,

$D(A_1) = (\tilde{x}_1 \in H^2(0, 1)$: $\tilde{x}_1 = x_1 - F_{21}x_2 - F_{31}x_3$, $x_i \in S_{X_i})$,

where

$F_{32}x_3 = \langle b, x_3\rangle_{R^m}$,

$F_{31}x_3 = \langle G(\xi), x_3\rangle_{R^m}$, $G(\xi) = (G^1(\xi), \cdots, G^m(\xi))^T$

satisfying $d^2G^i(\xi)/d\xi^2 = 0$, $G^i|_{\xi=1} = 0$,

$F_{21}x_2 = \overline{G}\int_0^1 f(\xi)x_2(\xi, t)d\xi$, $\overline{G} = \xi$,

then, A_1 generates a C_0 semigroup T_t^1 on $X_1 = L^2(0, 1)$ and A_i ($i = 2$, 3) generates a C_0 group T_t^i on X ($i = 2, 3$) respectively, $X_2 = L^2(0, 1)$, $X_3 = R$.

Now, the systems (22) can be described by

$$dx/dt = Ax + \phi x + \overline{D}u$$

wheae A is expressed by (20) and

$$\phi = -\begin{pmatrix} 0 & F_{21}A_2 & F_{21}F_{32}A_3 + F_{31}A_3 - F_{21}A_2F_{32} \\ 0 & 0 & F_{32}A_3 \\ 0 & & 0 \end{pmatrix},$$

$\overline{D}u = (0, 0, Du)^T$, and A generates a C_0 semigroup T_t given by (21) on $X = X_1 \oplus X_2 \oplus X_3 = L^2(0, 1) \oplus L^2(0, 1) \oplus R$ by remark 5.

we can verify that $F_{21}A_2$ is a bounded operator from $L^2(0, 1)$ to itself by the definition of distribution derivative and $f \in C_0^1(0, 1)$. Hence, ϕ is a bounded operator from X to X and $(A+\phi)$ generates a C_0 semigroup $\tilde{T}_t$ on X:

$$\tilde{T}_t x_0 = T_t x_0 + \int_0^t T_{t-s} \phi T_s x_0 \, ds$$

where T_t is given by (21) and

$$(T_t^1 x_{10})(\xi) = \sum_1^{\infty} 2e^{-n^2\pi^2 t} \sin n\pi\xi \int_0^1 \sin n\pi y \; x_{10}(y) dy,$$

$$(T_t^2 x_{20})(\xi) = x_2(\xi - vt),$$

$$T_t^3 x_{30} = e^{A_{2}t} x_{30}.$$

So,

$$x(t) = T_t x_0 + \int_0^t T_{t-s} \phi T_s x_0 \, ds + \int_0^t T_{t-\tau} D u(\tau) d\tau + $$
$$+ \int_0^t \int_0^{t-\tau} T_{t-\tau-s} \phi T_s D u(\tau) \, ds\tau.$$

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在乘积B空间上算子的分解及应用

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文中，讨论了一个算子在巴拿赫乘积空间上的几种分解形式，以及该算子在这些分解下产生一个 C_0 半群的条件，从而给出了两个基本定理。此外，还给出了一个例子说明这些定理的应用。

全国控制系统分析、综合与计算机辅助设计学术会议在西安召开

由全国自动化学会控制理论专业委员会筹办的全国控制系统分析、综合与计算机辅助设计学术会议于1984年11月16日至11月21日在西安召开。会议由全国自动化学会秘书长秦化淑主持，自动化学会副理事长疏松桂代表自动化学会致词。会议还特邀了参加1984年4月在布达佩斯举行的IFAC辨识与估计、1983年7月在华沙举行的IFAC大系统会议的部分同志作了专题报告，并由有关方面负责同志介绍了近期内将进行的国内外学术活动。接着，会议分两大组进行：一组是学术交流组，交流论文共33篇。另一组是中国控制系统计算机辅助软件包(CCSCAD)联合设计组，进行工作讨论，初步确定CCSCAD软件包在1985年12月问世。会议期间，与会代表还举行了多次研讨会，探讨了自动控制在中国发展的特点和方向，理科高等院校自动控制专业的教学等问题。与会代表一致认为，近年我国自动控制事业有了很大发展，在某些方面已经赶上了世界水平，如线性系统理论等。但就整体来看，尤其是应用，尚有很大差距。代表们高度赞扬了党的经济体制改革政策，坚信随着改革事业的不断发展，我国的自动控制事业必将有较大发展。代表们还呼吁全国同行们把精力更多地投入到控制理论的应用上去，为祖国的四个现代化事业作出更大的贡献。

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