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On the Equivalence Problem of Control System and Control Systems With Time-Lags in the Theory of Stabilization*

Liu Yongqing

(South China Institute of Technology, Guangzhou)

Abstract

By the symmetric positive definite solutions of the Riccati matrix equation and the Riccati differential matrix equation we construct positive definite, quadratic form functions, and give the equivalence of linear constant control systems and linear constant control systems with time-lags, and the equivalence of linear time-varying control systems and linear time-varying control systems with time-lags. At the same time we obtain the estimate formulae for the bound of the time-lags.

1. The Sourceof the Problem[1][2]

By Chin Yuanshun's idea,[2] We take the simplest open-loop control system as shown in Fig 1-1, its control equation is

$$I\frac{dX(t)}{dt}+CX(t)=U(t). \tag{1.1}$$

In addition, we have the simplest closed-loop control system as shown in Fig 1-2, the equation of which is usually written as

$$I\frac{dX(t)}{dt}+(C+K)X(t)=U(t). \tag{1.2}$$

However, strictly speaking, this diagram should be represented by

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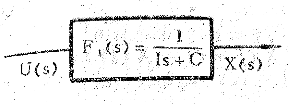

Fig1—1

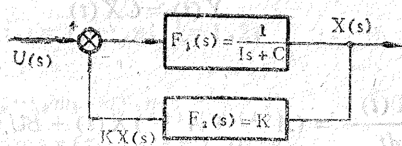

Fig 1—2

the following difference-differential equation

$$I\frac{dX(t)}{dt}+CX(t)+KX(t-\tau)=U(t), \qquad (1.3)$$

where $\tau>0$ may be a constant or a function of time t, since even electromagnetic waves need time to propagate. In the general problems of control engineering, (1.3) is replaced by (1.2) on the ground that τ is very small. For control systems this replacement requires mathematical justification, since, for example, when $U=0$, the equation

$$I\frac{dX(t)}{dt}+X(t)=0$$

possesses a stable trivial solution whereas the trivial solution of the equation

$$I\frac{dX(t)}{dt}+X(t+\tau)=0 \qquad (\tau>0)$$

is unstable, no matter how small the positive number τ may be. Therefore, regarding the problems of stabilization created by replacing (1.3) by (1.2), we should give a theoretically systematic mathematical proof.

This paper studies the general equivalence problems of the linear, constant control system and the linear, constant control system with time-lags, and the eruivalence of the linear, time-varying control system and the linear, time-varying control system with time-lags, in the theory of stabilization.

2. The Equivalence of the Linear, Constant Control System and the Linear, Constant Control System with Time-lags in the Theory of Stabilization

The general, linear, constant control system with time-lags is

$$\frac{dX(t)}{dt}=A^{(1)}X(t)+A^{(2)}X(t-\tau)+BU(t), \qquad (2.1)$$

$$Y(t)=CX(t) \tag{2.1$_1$}$$

or

$$\frac{dX(t)}{dt}=(A^{(1)}+A^{(2)})X(t)+BU(t)+A^{(2)}(X(t-\tau)-X(t))$$

$$=AX(t)+BU(t)+A^{(2)}(X(t-\tau)-X(t)), \tag{2.2}$$

where $A=A^{(1)}+A^{(2)}$, $\tau=\tau_{ij}(t)>0$ $(i,j=1,\cdots,n)$ is a constant or a function of time, when $\tau=0$, (2.1) and (2.2) become constant control systems

$$\frac{dX(t)}{dt}=AX(t)+BU(t) \tag{2.3}$$

$$Y(t)=CX(t) \tag{2.3$_1$}$$

where constant matrice $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times m}$, $C(c_{ij})_{p\times n}$, $A^{(l)}=(a_{ij}^{(l)})_{n\times n}$ $l=1,2$; vectors $X(t)=(x_1(t),\cdots,x_n(t))^T$, $U(t)=(u_1(t),\cdots u_m(t))^T$, $Y(t)=(y_1(t),\cdots,y_p(t))^T$.

Suppose that (2.3) is controllable and observable. For the linear, constant control system, we can choose an optimal negative vector function

$$U(t)=-KX(t), \tag{2.4}$$

which minimizes the quadratic performance index

$$J=\int_{t_0}^{\infty}(X^T(t)\,Q\,X(t)+U^T(t)\,RU(t))\,dt, \tag{2.5}$$

such that all the roots of the charateristic equation

$$D(\lambda)=|(A-BK)-\lambda I|=0 \tag{2.6}$$

of the closed-loop system of (2.3)

$$\frac{dX(t)}{dt}=(A-BK)X(t) \tag{2.7}$$

have negative real parts, that is, the trivial solution of (2.7) is asymptotically stable, where

$$K=R^{-1}B^TP, \tag{2.8}$$

P is the unique symmetric positive definite solution of the Riccati matrix equation

$$A^TP+PA-PBR^{-1}B^TP+C^TC=0, \tag{2.9}$$

R is a $m\times m$ symmetric, positive definite constant matrix, $Q=C^TC$ is an $n\times n$ symmetric, nonnegative definite matrix. Let

$$|a_{ij}^{(l)}| \le \frac{1}{2} a_1, \quad |a_{ij}| \le a_1, \quad i,j = 1,\cdots,n, \quad l = 1,2;$$

$$|b_{ij}| \le b_1, \quad i = 1,\cdots,n, \quad j = 1,\cdots,m; \quad m \le n. \tag{2.10}$$

$$|p_{ij}| \le p_1, \quad i,j = 1,\cdots,n; \quad \tau = \max(\tau_{ij}, \ i,j = 1,\cdots,n),$$

$$|k_{ij}| \le k_1, \quad i = 1,\cdots,m, \quad j = 1,\cdots,n.$$

We choose symmetric positive definite, quadratic form

$$V(X(t)) = X^T(t) P X(t), \tag{2.11}$$

from[3], there exist numbers $\beta_1 > 0$ and $\beta_2 > 0$ such that

$$\beta_1 X^T(t) X(t) \le V(X(t)) \le \beta_2 X^T(t) X(t) \tag{2.12}$$

Lemma 1 Suppose that (2.3) is controllable and observable, then there exists a function in(2.11) such that

$$\left.\frac{dV}{dt}\right|_{(2.7)} < 0 \quad ,$$

that is, the trivial solution of (2.7) is asymptotically stable.

proof The derivative of V along the trajectories of (2.7) is computed as [7]

$$\left.\frac{dV}{dt}\right|_{(2.7)} = -X(t)^T (C^T C + PBR^{-1}B^T P) X(t). \tag{2.13}$$

since $X^T(t)$ $(C^T C + PBR^{-1}B^T P)$ $X(t)$ is a symmetric, positive, definite quadratic form, from [3][8] there exist numbers $\beta_3 > 0$ and $\beta_4 > 0$ such that

$$\beta_3 X^T(t) X(t) \le X^T(t) (C^T C + PBR^{-1}B^T P) X(t) \le \beta_4 X^T(t) X(t); \tag{2.14}$$

therefore,

$$\left.\frac{dV}{dt}\right|_{(2.7)} \le -\beta_3 X^T(t) X(t) < 0. \tag{2.15}$$

The proof of lemma 1 is complete.

Lemma 2 $|x_k(t-\tau_{ik}) - x_k(t)| = \left| \int_{t-\tau_{ik}}^{t} \frac{d}{dt} x_k(t)\, dt \right| \le |\tau_{ik}| \, |\dot{x}_k(t_k')|$

$$\le \tau \left[a_1 \sum_{j=1}^{n} (|x_j(t_k')| + |x_j(t_k' - \tau_{kj})|) + b_1 \sum_{j=1}^{m} |u_j(t_k')| \right]. \tag{2.16}$$

Lemma 3 If $(x_1(t_k' - \tau_{k1}), \cdots, x_n(t_k' - \tau_{kn}))$ is in $4V(x_1(t), \cdots, x_n(t))$, that is, $V(x_1(t_k' - \tau_{k1}), \cdots, x_n(t_k' - \tau_{kn})) \le 4V(x_1(t), \cdots, x_n(t))$, from (2.12), then

$$\sum_{j=1}^{n} x_j^2(t_k' - \tau_{kj}) \le \frac{V(x_1(t_k'-\tau_{k1}),\cdots,x_n(t_k'-\tau_{kn}))}{\beta_1}$$

$$\le \frac{4V(x_1(t)),\cdots,x_n(t))}{\beta_1} \le \frac{4\beta_2}{\beta_1}\sum_{j=1}^{n} x_j^2(t), \qquad (2.17)$$

Similarly,

$$\sum_{j=1}^{n} x_j^2(t_k') \le \frac{4\beta_2}{\beta_1}\sum_{j=1}^{n} x_j(t) \qquad (2.18)$$

Now, suppose that (2.5) is the negative feedback function of (2.2), and that (2.11) is the positive definite function of (2.2), therefore, we have

Theorem 1 Suppose that (2.3) is cotrollable and observable, then there exists $\Delta_1 > 0$, the trivial solution of the closed-loop system of (2.2) is asymptotically stable, provided that

$$0 \le \tau \le \Delta_1, \qquad (2.19)$$

where

$$\Delta_1 = \frac{\beta_3}{a_1 P_1 n^3(2a_1 + b_1 k_1 m)\left(1 + \frac{4\beta_2}{\beta_1}\right)}. \qquad (2.20)$$

proof Substituting (2.4) in (2.2) and rewriting (2.2) and (2.11) as scalar forms respectively, we take the derivative of V along the trajectories of (2.2)

$$\left.\frac{dV}{dt}\right|_{(2.2)} = \sum_{i=1}^{n}\frac{\partial V}{\partial x_i}\left.\frac{dx_i(t)}{dt}\right|_{(2.2)} = \sum_{i=1}^{n}\frac{\partial V}{\partial x_i}\left(\sum_{j=1}^{n} a_{ij}x_j(t)\right.$$

$$\left. + \sum_{j=1}^{m} b_{ij}u_j(t) + \sum_{j=1}^{n} a_{ij}^{(2)}(x_j(t-\tau_{ij}) - x_j(t))\right),$$

from lemma 1 and (2.15) and (2.16),

$$\left.\frac{dV}{dt}\right|_{(2.2)} \le \left.\frac{dV}{dt}\right|_{(2.7)} + 2\sum_{i=1}^{n}\sum_{l=1}^{n}|P_{il}|\,|x_l(t)|\left[\sum_{j=1}^{n}|a_{ij}^{(2)}|\,|x_j(t\right.$$

$$-\tau_{ij})-x_j(t)|\Big]\le\frac{dV}{dt}\Big|_{(2.7)}+\tau a_1P_1n\sum_{l=1}^{n}|x_l(t)|\Big[a_1\sum_{j=1}^{n}\sum_{i=1}^{n}$$

$$(|x_i(t_j')|+|x_1(t_j'-\tau_{ij})|+b_1\sum_{j=1}^{n}\sum_{i=1}^{m}|u_i(t_j')|\Big]. \qquad (2.21)$$

Rewriting (2.4) as the scalar form

$$|u_i(t)|\le\sum_{l=1}^{n}|k_{il}||x_l(t)|\le K_1\sum_{l=1}^{n}|x_l(t)|,\ i=1,\cdots,m, \qquad (2.22)$$

and substituting (2.22) in (2.21), we get

$$\frac{dV}{dt}\Big|_{(2.2)}\le\frac{dV}{dt}\Big|_{(2.7)}+\tau a_1P_1n\sum_{l=1}^{n}|x_l(t)|\Big[(a_1+b_1k_1m)\sum_{j=1}^{n}\sum_{i=1}^{n}$$

$$|x_i(t_j')|+a_1\sum_{j=1}^{n}\sum_{i=1}^{n}|x_i(t_j'-\tau_{ij})|\Big]\le\frac{dV}{dt}\Big|_{(2.7)}+\tau a_1P_1n(a_1+b_1k_1m).$$

$$\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{i=1}^{n}(x_l^2(t)+x_i^2(t_j'))+\tau a_1P_1n\sum_{j=1}^{n}\sum_{l=1}^{n}\sum_{i=1}^{n}(x_l^2(t)+$$

$$x_i^2(t_j'-\tau_{ij})),$$

from lemma 3, (2.17) and (2.18), then

$$\frac{dV}{dt}\Big|_{(2.2)}\le\frac{dV}{dt}\Big|_{(2.7)}+\tau a_1P_1n^3(a_1+b_1k_1m)\Big[\sum_{l=1}^{n}x_l^2(t)+$$

$$+\frac{4\beta_2}{\beta_1}\sum_{l=1}^{n}x_l^2(t)\Big]+\tau a_1P_1n^3\Big[\sum_{l=1}^{n}x_l^2(t)+\frac{4\beta_2}{\beta_1}\sum_{l=1}^{n}x_l^2(t)\Big]$$

$$\le-\beta_3\sum_{j=1}^{n}x_j^2(t)+\tau a_1P_1n^3(2a_1+b_1k_1m)\left(1+\frac{4\beta_2}{\beta_1}\right)\sum_{j=1}^{n}x_j^2(t) \qquad (2.23)$$

When $0\le\tau\le\Delta_1$, we have

$$\left.\frac{dV}{dt}\right|_{(2.2)}<0,$$

therefore, the trivial solution of the closed-loop system of (2.2) is asymptotically stable.

Theorem 2 If the trivial solution of the closed-loop system (2.7) is asymptotically stable, then there exists a number $\Delta_1>0$, the trivial solution of the closed-loop system of (2.2) is also asymptotically stable, provided that

$$0\le\tau<\Delta_1 \tag{2.20}$$

When the matrices $B=0$ and $C=0$, (2.1) and (2.2) become

$$\frac{dX(t)}{dt}=A^{(1)}X(t)+A^{(2)}X(t-\tau) \tag{2.24}$$

and

$$\frac{dX(t)}{dt}=(A^{(1)}+A^{(2)})X(t)=AX(t) \tag{2.25}$$

respectively. From theorem 2, we have the equivalence of differential equations and difference-differential equations in the theory of stability [5] [6].

3. The Equivalence of Linear, Time-Varying Control Systems and Linear, Time-Varying Control Systems with Time-Lags in the Theory of Stabilization

Consider the linear, time-varying control system with time-lags

$$\dot X(t)=A^{(1)}(t)X(t)+A^{(2)}(t)X(t-\tau)+B(t)U(t) \tag{3.1}$$

$$Y(t)=C(t)X(t) \tag{3.1$_1$}$$

or rewrite (3.1) as

$$\begin{aligned}\dot X(t)&=(A^{(1)}(t)+A^{(2)}(t))\,X(t)+B(t)U(t)+A^{(2)}(t)(X(t-\tau)-X(t))\\&=A(t)X(t)+B(t)U(t)+A^{(2)}(t)(X(t-\tau)-X(t))\end{aligned} \tag{3.2}$$

where $\tau=\tau_{ij}(t)\ge0$ ($i,j=1,\cdots,n$) is a constant or a function of time t, and $A(t)=A^{(1)}(t)+A^{(2)}(t)$.

When $\tau=0$, (3.1) becomes the linear, time-varying control system

$$\dot X(t)=A(t)X(t)+B(t)U(t) \tag{3.3}$$

$$Y(t)=C(t)X(t) \tag{3.3$_1$}$$

Time-varying coefficient matrices $A(t)=(a_{ij}(t))_{n\times n}$, $B(t)=(b_{ij}(t))_{n\times m}$, $C(t)=(c_{ij}(t))_{p\times n}$, $A^{(l)}(t)=(a_{ij}^{(l)}(t))_{n\times n}$, $l=1,2$. Vectors $X(t)=(x_1(t),\cdots,x_n(t))^T$, $U(t)=(u_1(t),\cdots,u_m(t))^T$, $Y(t)=(y_1(t),\cdots,y_p(t))^T$. For $t\ge t_0$, the elements of $A(t)$, $B(t)$ and $C(t)$ are sectionally continuous and uniformly bounded, that is,

$$|a_{ij}^{(l)}(t)| \leq \frac{a_2}{2}, |a_{ij}(t)| \leq a_2, i,j=1,\cdots,n; l=1,2, \tag{3.4}$$
$$|b_{ij}(t)| \leq b_2, i=1,\cdots,n, j=1\cdots,m,$$

where a_2 and b_2 are positive constants independent of t and t_0.

Suppose that (3.3) is completely uniformly controllable and completely uniformly observable for $t(\geq t_0)$, then there exists the optimal negative feedback vector function

$$U(t) = -K(t)X(t), \tag{3.5}$$

which minimizes the quadratic performance index

$$J = \int_{t_0}^{\infty} [X_{(t)}^T(t)Q(t)X(t) + U(t)^T R(t)U(t)]dt \tag{3.6}$$

such that all the roots of the characteristic equation

$$D(\lambda(t)) = |(A(t) - B(t)K(t)) - \lambda I| = 0 \tag{3.7}$$

of the closed-loop system

$$\dot{X}(t) = (A(t) - B(t)K(t))X(t) \tag{3.8}$$

of (3.3) have negative real parts

$$\mathrm{Re}(\lambda(t)) < -\delta < 0 \ (\delta \text{ is a positive constant}), \tag{3.9}$$

that is, the trivial solution of the closed-loop system (3.8) of (3.3) is uniformly asymptotically stable, where

$$K(t) = R^{-1}(t)\, B^T(t)P(t) \tag{3.10}$$

and $P(t)$ is the symmetric, positive definite solution of the Riccati differential matrix equation

$$\dot{P}(t) + P(t)A(t) + A^T(t)P(t) - P(t)B(t)R^{-1}(t)B^T(t)P(t) + C^T(t)C(t) = 0. \tag{3.11}$$

For $t \geq t_0$, the elements of matrix $P(t)$ are uniformly bounded, therefore,

$$\begin{aligned} &|P_{ij}(t)| \leq P_2, \ i, j=1,\cdots, n, \\ &|K_{ij}(t)| \leq K_2, \ i=1,\cdots, m, \ j=1,\cdots, n, \end{aligned} \tag{3.12}$$

P_2 and K_2 are positive constants independent of t and t_0.

$R(t)$ is a time-varying, symmetric, positve definite $m\times m$ matrix, $Q(t) = C(t)^T C(t)$ is an $n\times n$ symmetric, nonnegative definite matrix, for $t \geq t_0$, its elements are sectionally continuous and uniformly bounded.

We construct the positive definite, quadratic form

$$V(t, X(t)) = X^T(t)P(t)X(t), \tag{3.13}$$

then, from[3], there exist numbers $\beta_5>0$ and $\beta_6>0$ such that

$$\beta_5 X^T(t)X(t)\leq X^T(t)P(t)X(t)\leq\beta_6 X^T(t)X(t) \tag{3.14}$$

therefore, we have

Lemma 4 Suppose that for $t\geq t_0$, the elements of matrices $A(t)$, $B(t)$, $C(t)$ and $R(t)$ are sectionally continuous and uniformly bounded, and that (3.3) is uniformly completely controllable and uniformly completely observable, then there exists the positive definite function (3.13) such that

$$\left.\frac{dV}{dt}\right|_{(3.8)}<0,$$

that is, the trivial solution of (3.8) is uniformly asymptotically stable.

Proof We take the derivative of V along the trajectories of (3.8)[7]

$$\left.\frac{dV}{dt}\right|_{(3.8)}=-X^T(t)[C^T(t)C(t)+P(t)B(t)R^{-1}(t)B^T(t)P(t)]X(t). \tag{3.15}$$

Since $C^T(t)C(t)+R(t)B(t)P^{-1}(t)B^T(t)P(t)$ is the symmetric, positive definite matrix, from [3][8] there exist numbers $\beta_7>0$ and $\beta_8>0$ such that

$$\begin{aligned}\beta_7 X^T(t)X(t)&\leq X^T(t)(C^T(t)C(t)+P(t)B(t)R^{-1}(t)B^T(t)P(t))X(t)\\&\leq\beta_8 X^T(t)X(t),\end{aligned} \tag{3.16}$$

therefore, we obtain

$$\left.\frac{dV}{dt}\right|_{(3.8)}\leq-\beta_7 X^T(t)X(t)<0. \tag{3.17}$$

Now, suppose that the optimal negative feedback function (3.5) of (3.3) is the negative feedback function of (3.2) and that the positive definite, quadratic form (3.13) is that of (3.2), hence we have

Theorem 3 Suppose that for $t\geq t_0$, the elements of time-varying matrices $A(t)$, $B(t)$, $C(t)$ and $R(t)$ are sectionally continuous and uniformly bounded and that $(A(t), B(t))$ of (3.2) is uniformly completely controllable and that $(A(t), C(t))$ of (3.2) is uniformly completely observable, then there exists a number $\Delta_2>0$; The trivial solution of the closed-loop system of (3.2) is uniformly asymptotically stable, provided that

$$0\leq\tau<\Delta_2, \tag{3.18}$$

where

$$\Delta_2 = \frac{\beta_7}{a_2 P_2 n^3 (2a_2 + b_2 K_2 m)\left(1 + \dfrac{4\beta_6}{\beta_5}\right)}. \qquad (3.19)$$

proof We rewrite (3.2) as the scalar form of difference-differential equations. The derivative of $V(t, X(t))$ in (3.13) along the trajectories of (3.2) is computed and (3.5) is substituted in (3.2). We get

$$\frac{dV}{dt}\bigg|_{(3.2)} = \left(\frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \frac{dx_i(t)}{dt}\right)_{(3.2)} = \frac{\partial V}{\partial t} + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \cdot$$

$$\cdot \left[\sum_{j=1}^{n} a_{ij}(t) x_j(t) + \sum_{j=1}^{m} b_{ij}(t) u_j(t) + \sum_{j=1}^{n} a_{ij}^{(2)}(t)\left(x_j(t-\tau_{ij}) - x_j(t)\right)\right]$$

$$\leq \frac{dV}{dt}\bigg|_{(3.8)} + \sum_{i=1}^{n}\left|\frac{\partial V}{\partial x_i}\right|\left(\sum_{j=1}^{n}|a_{ij}^{(2)}(t)|\,|x_j(t-\tau_{ij}) - x_j(t)|\right)$$

$$\leq \frac{dV}{dt}\bigg|_{(3.8)} + 2a_2 \sum_{i=1}^{n}\left\{\sum_{l=1}^{n}|P_{il}(t)|\,|x_l(t)|\left|\sum_{j=1}^{n}|x_j(t-\tau_{ij}) - x_j(t)|\right|\right\}$$

$$\leq \frac{dV}{dt}\bigg|_{(3.8)} + 2a_2 P_2 n \sum_{i=1}^{n}|x_i(t)|\left|\sum_{j=1}^{n}|x_j(t-\tau_{ij}) - x_j(t)|\right|, \qquad (3.20)$$

Then, using lemma 2, lemma 3, (2.16), (2.17), (2.18) and (3.5), we obtain

$$\frac{dV}{dt}\bigg|_{(3.2)} \leq \frac{dV}{dt}\bigg|_{(3.8)} + \tau a_2 P_2 n^3 (2a_2 + b_2 k_2 m)\left(1 + \frac{4\beta_6}{\beta_5}\right)\sum_{j=1}^{n} x_j^2(t)$$

$$\leq -\beta_7 \sum_{j=1}^{n} x_j^2(t) + \tau a_2 P_2 n^3 (2a_2 + b_2 k_2 m)\left(1 + \frac{4\beta_6}{\beta_5}\right)\sum_{j=1}^{n} x_j^2(t).$$

(3.21)

When $0 \leq \tau < \Delta_2$,

$$\frac{dV}{dt}\bigg|_{(3.2)} < 0, \qquad (3.22)$$

therefore, the trivial solution of the closed system of (3.2) is uniformly asymptotically stable.

Theorem 4 Suppose that for $t \geq t_0$, the elements of $A(t), B(t), C(t)$ and $R(t)$ are sectionally continuous and uniformly bounded and that all the characteristic roots of the closed-loop system (3.8) of (3.3) satisfy $Re(\lambda(t)) < -\delta < 0$, then there exists a number $\Delta_2 > 0$, The trivial solution of the closed-loop system of (3.2) is uniformly asymptotically stable, provided that

$$0 \leq \tau < \Delta_2. \tag{3.18}$$

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在镇定理论中控制系统与具有滞后的控制系统的等价性

刘 永 清

（华南工学院）

摘 要

本文由勒卡提矩阵方程与勒卡提矩阵微分方程的正定对称解构造了正定二次型函数，给出了在镇定理论中定常及时变线性控制系统与具有滞后的定常及时变线性控制系统的等价性．同时给出了滞后界限的估计公式．

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其中$v_1(t)$和$v_2(t)$分别表示随机的测量误差．书中讲述了如何利用数据$z_1(t)$和$z_2(t)$辨识上述参数的方法，此外还研究了葡萄糖注入速度应如何变化方能使参数辨识的精度提高．作者把这两个问题归结为微分方程的两点边值问题．为此，作者在本书第一部份的引言后面，在第二部份中先行介绍了线性与非线性两点边值问题之求解方法，然后才在第三部份和第四部份中讨论上述的两个问题，并在第五部份中附上计算机程序．这样就使本书内容自成一体．此外，在第四部份中还顺便讲了桁条理论及特征值问题，前者可供力学家及土木工程师参考，后者可能会引起数值分析家的兴趣．全书共五部份，包括十一章．目录如下：第一部份内有第一章，它是引言．第二部份内有第二章，为最优控制；第三章为线性两点边值问题的数值解法；第四章为非线性两点边值问题的数值解法．第三部份内有第五章，为系统辨识的高斯-牛顿法；第六章为系统辨识的准线性化方法；第七章为系统辨识之应用．第四部份内有第八章，为最优输入；第九章为最优输入的其他问题；第十章为最优输入之应用．第五部份内有第十一章，它是求解边值问题和辨识问题之计算机程序．

作者中的R. Kalaba是美国著名应用数学家，与R. Bellman曾合写过不少自控理论方面的文章．思路清晰，观点明确，文笔流畅，是他的一贯文风．