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The Structural Algorithm of Matrix Sequence and Linear Systems

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Abstract

A structural algorithm of matrix sequence is introduced in this paper. Some applications in linear system are presented to show that a considerable part of linear system theory can be treated by this approach.

Introduction

In control system theory, for the convienience of analysis and design, some relevant general processes are often summed up to a kind of algorithm for making program. For example, Rosenbrock[1] presented the algorithm of finding the system decoupling zeros. Wolovich[2] found some algorithms related to polynomial matrices. Silverman[3] got the input-output structural algorithm in state space. Wonham[4] obtained the algorithm of finding (A, B) - invariant and controllability subspaces from a geometric approach.

We proposed a couple of similar algorithm for realization of disturbance resistance by state feedback[5] and for finding the greatest common divisor of two polynomial matrices[6]. These algorithm are summed up to the structural algorithm of matrix sequence in this paper, some related problems are discussed as well.

Structural Algorithm of Matrix Sequence

Consider a matrix sequence

$$L \triangleq [L_0 L_1 L_2 \cdots L_\nu],$$

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where L_i is $m\times l$ matrix, $i=0,1,2,\cdots,\nu$. $(\nu+1)$ is called of the length of matrix sequence L. L_0 is called the leading matrix of L. The matrix sequence can be viewed as the coefficient matrix sequence of polynomial matrix (CMSPM).

We concentrate our attention to nonzero matrix sequence ($L\neq 0$).

Definition. If a CMSPM satisfies

i) $$L=\left[\begin{pmatrix}\overline{L}_0\\0\end{pmatrix}\begin{pmatrix}\overline{L}_1\\0\end{pmatrix}\begin{pmatrix}\overline{L}_2\\0\end{pmatrix}\cdots\begin{pmatrix}\overline{L}_\nu\\0\end{pmatrix}\right],$$

ii) $\overline{L}_0$ full row rank,

then the CMSPM L is called proper sequence.

Let

$$L_i=\begin{bmatrix}l_{1i}\\l_{2i}\\\vdots\\l_{mi}\end{bmatrix},\qquad i=0,1,2,\cdots,\nu.$$

We introduce the structural algorithm for the CMSPM.

Algorithm Step 0: Plumge a column vector with integer element α_j, before the 1-st column of L, we obtain

$$L'\triangleq\left[\begin{array}{c:c}\alpha_1 & \\ \vdots & L\\ \alpha_m & \end{array}\right].$$

Set

$$m_1\Longleftarrow m,$$
$$\alpha_j\Longleftarrow\nu,\qquad j\in\underline{m};$$

Step 1: For $j\in\underline{m_1}$, if $l_{j0}=0$, then

$$l_{ji}\Longleftarrow l_{ji+1},\qquad i=0,1,\cdots,\alpha_j-1,$$
$$l_{j\alpha_j}\Longleftarrow 0,$$
$$\alpha_j\Longleftarrow\alpha_j-1;$$

Step 2: Exchanging the rows of L', such that

$$\alpha_1\geq\alpha_2\geq\cdots\geq\alpha_{m_1}$$

And let

$$m_1\Longleftarrow\max\{k\,|\,\alpha_k\geq 0\};$$

Step 3: If $l_{10}, l_{20},\cdots l_{m_10}$ are linear dependent, that is

$$l_{j_10}=\beta_2 l_{j_20}+\beta_3 l_{j_30}+\cdots+\beta_t l_{j_t0},\quad j_i<j_{i+1},\quad i\in\underline{t-1},$$

then

$$l_{j_1 i} \Leftarrow l_{j_1 i} - \sum_{k=2}^{t} \beta_k l_{j_k i}, \quad i = 0,1,2,\cdots,\nu.$$

Obviously $l_{j_1 0} = 0$. Go to step 1;

Step 4: $l_{10}, l_{20}, \cdots, l_{m_1 0}$ are linear independent. STOP.

The algorithm can be summed up to a series of left shift, row exchanges and restricted eleminations of the leading matrix. The algorithm always stops by finite steps. At the end of the algorithm, the matrix sequence is proper. In that case, $\{m_1 : \alpha_1, \alpha_2, \cdots, \alpha_{m_1}\}$ is called the structural index of matrix sequence L.

Applying the above structural algorithm to the coefficient matrices of polynomial matrix

$$P(s) = L_0 s^{\nu} + L_1 s^{\nu-1} + \cdots + L_{\nu-1} s + L_{\nu}$$

is equivalent to carring out the row elementary transform of polynomial matrix for $P(s)$. And the step 2 is row exchange for $P(s)$, step 3 is left multipling $P(s)$ by the following unimodular matrix

$$\begin{array}{r} \\ \\ j_1\text{-th}- \\ \\ \\ j_2\text{-th}- \\ \\ \\ j_t\text{-th}- \\ \\ \\ \end{array}
\begin{bmatrix}
1 & & & & & & & \\
 & \ddots & & & & & & \\
 & & 1 & \cdots & -\beta_2 s^{\alpha_{j1}-\alpha_{j2}} & \cdots & -\beta_t s^{\alpha_{j1}-\alpha_{jt}} & \\
 & & & \ddots & \vdots & & \vdots & \\
 & & & & 1 & & \vdots & \\
 & & & & & \ddots & \vdots & \\
 & & & & & & 1 & \\
 & & & & & & & \ddots \\
 & & & & & & & & 1
\end{bmatrix} . \qquad (1)$$

Let the transformation matrix is $U(s)$, and

$$U(s)P(s) = \begin{bmatrix} \overline{P}(s) \\ 0 \end{bmatrix}, \qquad (2)$$

then m_1 is the rank of $P(s)$ and

$$\alpha_i = \partial_{r_i}[\overline{P}(s)], \quad i \in \underline{m_1},$$

$$\Gamma_r[\overline{P}(s)] = \overline{L_0},$$

where $\overline{L_0}$ is the part with full row rank of the leading matrix at the end of the algorithm.

We can obtain the $U(s)$ and/or $U^{-1}(s)$ through the structural algorithm in a similar way to the method proposed by Han & Chen [6].

Constructing of the (A, B) – Invariant Subspace

Consider the problem of disturbance resistance for linear time-invariant system

$$\begin{cases} \dot{x} = Ax + Bu + Ff, \\ y = Cx + Du. \end{cases} \tag{3}$$

In [5], we discussed this problem for arbitrary disturbance f, and presented the necessary and sufficient condition of the solvability for the problem and the required solution (i. e., the state feedback matrix).

Here we construct the matrix sequence

$$[D\ CB\ CAB \cdots CA^{\nu-1}B],$$

where

$$\nu = \min\{j \mid \operatorname{rank}[B\ AB \cdots A^{j-1}B] = \operatorname{rank}[B\ AB \cdots A^{j}B]\}.$$

Using the structural algorithm we get

$$\left(\begin{array}{c|cccc} \alpha_1 & & & & \\ \vdots & A_0 & A_1 & A_2 \cdots & A_\nu \\ \alpha_{m1} & & & & \\ -1 & & & & \\ \vdots & 0 & 0 & 0 & \cdots 0 \\ -1 & & & & \end{array}\right),$$

where A_0 has full row rank. Right shiftting the j-th row of matrix sequence

$$[A_0 \quad A_1 \quad A_2 \cdots A_\nu]$$

by $(\nu - \alpha_j)$ blocks, i.e.

$$a_{ji} \Longleftarrow a_{ji-(\nu-\alpha_j)}, \quad i = \nu,\ \nu-1, \cdots, \nu-\alpha_j,$$

$$a_{jk} \Longleftarrow 0, \quad k = 0, 1, \cdots, \nu - \alpha_j - 1,$$

where

$$A_i=\begin{pmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{m_1 i} \end{pmatrix}$$

Write the new matrix sequence as

$$[\overline{A}_0 \;\; \overline{A}_1 \;\; \overline{A}_2 \cdots \overline{A}_\nu].$$

Let

$$\widetilde{C}[B \;\; AB \cdots A^{\nu-1}B]=[\overline{A}_1 \;\; \overline{A}_2 \cdots \overline{A}_\nu], \tag{4}$$

and define

$$\widetilde{C}=\begin{bmatrix} \widetilde{C}_1 \\ \vdots \\ \widetilde{C}_{m_1} \end{bmatrix}.$$

Obviously,

$$\widetilde{C}_i A^k B=0, k=0,1,\cdots,(\nu-\alpha_i-2)$$

and

$$\begin{pmatrix} \widetilde{C}_1 \;\; A^{\nu-\alpha_1-1} \;\; B \\ \vdots \\ \widetilde{C}_{m_1} \;\; A^{\nu-\alpha_{m_1}-1} \;\; B \end{pmatrix}=A_0,$$

when $\alpha_i=\nu$, $\widetilde{C}_i A^{-1}B \triangleq \widetilde{d}_i$, in which

$$\widetilde{D} \triangleq \begin{pmatrix} \widetilde{d}_1 \\ \widetilde{d}_2 \\ \vdots \\ \widetilde{d}_m \end{pmatrix},$$

and $\widetilde{D}$ is determined by D. Introduce

$$\overline{C} \triangleq \begin{pmatrix} \widetilde{C}_1 \;\; A^{\nu-\alpha_1} \\ \vdots \\ \widetilde{C}_{m_1} \;\; A^{\nu-\alpha_{m_1}} \end{pmatrix}.$$

From [5], it is known thar

$$\prod_{j=1}^{m_1} \prod_{k=0}^{\nu-\alpha_j} K_{er}\, \widetilde{C}_j \overline{A}^{k-1} \tag{5}$$

is the supremal (A, B) - invariant subspace, where

$$\overline{A} \triangleq A - BK_0,$$

$$K_0 = A_0^- \overline{C},$$

and A_0^- is the right inverse of A_0, i.e.

$$A_0 A_0^- = I_{m_1}.$$

And define

$$\mathrm{Ker}\,\widetilde{C}_i \overline{A}^{-1} = x, \quad \forall i.$$

Applications To Linear Systems

1. Properizing of polynomial matrix

For a polynomial matrix

$$P(s) = P_0 s^\nu + P_1 s^{\nu-1} + \cdots + P_{\nu-1}s + P_\nu, \quad P_i : p \times m$$

which is of full rank, we can transform it into a row proper polynomial matrix by polynomial matrix transforms[2]. Now applying the structural algorithm to matrix sequence

$$[P_0 P_1 \cdots P_{\nu-1} P_\nu],$$

we have

$$\left(\begin{array}{c|cccc} \alpha_1 & & & & \\ \vdots & \overline{P}_0 & \overline{P}_1 \cdots \overline{P}_{\nu-1} & \overline{P}_\nu & \\ \alpha_{m_1} & & & & \\ -1 & & & & \\ \vdots & 0 & 0 \cdots 0 & 0 & \\ -1 & & & & \end{array}\right).$$

Obviously $m_1 = \min(p, m)$. Let $U(s)$ be the unimodular matrix determined by the structural algorithm, and

$$\overline{P}(s) = U(s)P(s).$$

Therefore $\overline{P}(s)$ is row proper.

2. Finding the greatest common divisor of two polynomial matrices.

Suppose two polynomial matrices $P(s)$ and $R(s)$ have same column number and $\begin{bmatrix} P(s) \\ R(s) \end{bmatrix}$ has full column rank,

$$\begin{bmatrix} P(s) \\ R(s) \end{bmatrix} = \begin{bmatrix} P_0 \\ R_0 \end{bmatrix} s^\nu + \begin{bmatrix} P_1 \\ R_1 \end{bmatrix} s^{\nu-1} + \cdots + \begin{bmatrix} P_{\nu-1} \\ R_{\nu-1} \end{bmatrix} s + \begin{bmatrix} P_\nu \\ R_\nu \end{bmatrix}.$$

Applying the structural algorithm to matrix sequence

$$\left[\begin{pmatrix} P_0 \\ R_0 \end{pmatrix}\begin{pmatrix} P_1 \\ R_1 \end{pmatrix}\cdots\begin{pmatrix} P_{\nu-1} \\ R_{\nu-1} \end{pmatrix}\begin{pmatrix} P_\nu \\ R_\nu \end{pmatrix}\right],$$

we have

$$\left[\begin{array}{c|cccc} \alpha_1 & & & & \\ \cdots & A_0 & A_1\cdots & A_{\nu-1} & A_\nu \\ \alpha m_1 & & & & \\ -1 & & & & \\ \vdots & 0 & 0\cdots & 0 & 0 \\ -1 & & & & \end{array}\right].$$

Obviously

$$m_1=\operatorname{rank}\begin{bmatrix} P(s) \\ R(s) \end{bmatrix},$$

i.e., A_0 is nonsingular matrix. Let

$$G(s)=\begin{bmatrix} s^{\alpha_1} & & 0 \\ & \ddots & \\ 0 & & s^{\alpha m_1} \end{bmatrix}A_0+\begin{bmatrix} s^{\alpha_1-1} & & 0 & \\ & \ddots & & \\ 0 & & s^{\alpha n_{1-1}} & 0 \\ & & 0 & \end{bmatrix}A_1+\cdots$$

$$+\begin{bmatrix} s^{\alpha_1-(\alpha_1-1)} & & & \\ & \ddots & 0 & 0 \\ 0 & & s^{\alpha_{n_{\alpha_1-1}}-(\alpha_1-1)} & \\ & & 0 & \end{bmatrix}A_{\alpha_1-1}+\begin{bmatrix} I_{n_{\alpha_1}} & 0 \\ & \\ 0 & 0 \end{bmatrix}A_{\alpha_1}.$$

Then $G(s)$ is a row proper nonsingular polynomial matrix, and

$$\begin{bmatrix} U_{11}(s) & U_{12}(s) \\ U_{21}(s) & U_{22}(s) \end{bmatrix}\begin{bmatrix} P(s) \\ R(s) \end{bmatrix}=\begin{bmatrix} G(s) \\ 0 \end{bmatrix}, \tag{6}$$

where

$$U(s)\triangleq\begin{bmatrix} U_{11}(s) & U_{12}(s) \\ U_{21}(s) & U_{22}(s) \end{bmatrix}$$

is the unimodular matrix determined by the structural algorithm. And $G(s)$ is the greatest common right divisor (r.c.r.d.) of $P(s)$ and $R(s)$. When $\alpha_1=0$, the two polynomial matrices $P(s)$ and $R(s)$ are right prime. E_q. (6) can also be expressed in the following form

$$\begin{bmatrix} P(s) \\ R(s) \end{bmatrix} = \begin{bmatrix} \hat{U}_{11}(s) & \hat{U}_{12}(s) \\ \hat{U}_{21}(s) & \hat{U}_{22}(s) \end{bmatrix} \begin{bmatrix} G(s) \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{U}_{11}(s) \\ \hat{U}_{21}(s) \end{bmatrix} \cdot G(s)$$

where $\hat{U}_{11}(s)$ and $\hat{U}_{21}(s)$ are right prime.

3. Solving of Diophantine equation

The polynomial matrix equation

$$X(s)P(s) + Y(s)R(s) = M(s) \tag{7}$$

is called unilateral Diophantine equation of polynomial matrix[7].

First, the equation (7) is solvable if and only if the r.c.r.d. $G(s)$ of two polynomial matrices $P(s)$ and $R(s)$ is a right divisor of $M(s)$.

Suppose $P(s)$, $R(s)$ and $G(s)$ are same as in "2".

i) Like the manner in "2", we obtain $G(s)$ by $P(s)$ and $R(s)$. If $\alpha_1 = 0$, then the equation (7) has solution.

ii) When $\alpha_1 \neq 0$, let

$$\begin{bmatrix} M(s) \\ G(s) \end{bmatrix} = \begin{bmatrix} M_0 \\ G_0 \end{bmatrix} s^{\mu} + \begin{bmatrix} M_1 \\ G_1 \end{bmatrix} s^{\mu-1} + \cdots + \begin{bmatrix} M_{\mu-1} \\ G_{\mu-1} \end{bmatrix} s + \begin{bmatrix} M_\mu \\ G_\mu \end{bmatrix}.$$

Obviously

$$\text{rank} \begin{bmatrix} M(s) \\ G(s) \end{bmatrix} = \text{rank } G(s) = \text{rank} \begin{bmatrix} P(s) \\ R(s) \end{bmatrix}.$$

If $\{ m_1 : \beta_1, \beta_2, \cdots, \beta_{m_1} \}$ is structural index of matrix sequence $\left[\begin{pmatrix} M_0 \\ G_0 \end{pmatrix} \begin{pmatrix} M_1 \\ G_1 \end{pmatrix} \cdots \begin{pmatrix} M_{\mu-1} \\ G_{\mu-1} \end{pmatrix} \begin{pmatrix} M_\mu \\ G_\mu \end{pmatrix} \right]$, then equation (7) is solvable if and only if

$$\alpha_1 + \alpha_2 + \cdots + \alpha_{m_1} = \beta_1 + \beta_2 + \cdots + \beta_{m_1}. \tag{8}$$

4. Decomposition of rational fraction matrix and minimal realization

Consider the $p \times m$ proper rational fraction matrix

$$W(s) = [w_{ij}(s)],$$

where

$$w_{ij}(s) = q_{ij}(s) / p_{ij}(s),$$

$$\partial(q_{ij}(s)) < \partial(p_{ij}(s)), \ \forall\ i,j.$$

Constructing $m \times m$ diagonal polynomial matrix

$$M_i(s) = \text{diag}\{ p_{i1}(s), p_{i2}(s) \cdots, p_{im}(s) \}$$

and $1 \times m$ polynomial matrix

$$N_i(s) = [q_{i1}(s)\ q_{i2}(s) \cdots q_{im}(s)].$$

Seeking the r.c.r.d. of $M_i(s)$ and $N_i(s)$ according to "2", we obtain

$$\begin{bmatrix} U_{11}^{(i)}(s) & U_{12}^{(i)}(s) \\ U_{21}^{(i)}(s) & U_{22}^{(i)}(s) \end{bmatrix} \begin{bmatrix} M_i(s) \\ N_i(s) \end{bmatrix} = \begin{bmatrix} G_i(s) \\ 0 \end{bmatrix}.$$

Write down

$$[\overline{q}_{i1}(s)\ \overline{q}_{i2}(s) \cdots \overline{q}_{im}(s)] = U_{21}^{(i)}(s), \quad p_i(s) = -U_{22}^{(i)}(s) .$$

Constructe

$$Q(s) = [\ \overline{q}_{ij}(s)\ ],$$

$$P(s) = \mathrm{diag}\{P_1(s),\ P_2(s),\ \cdots,\ P_p(s)\},$$

then

$$W(s) = P^{-1}(s) Q(s),$$

i.e., $P(s)$ and $Q(s)$ are a left decomposition of $W(s)$.

For $\begin{bmatrix} P^T(s) \\ Q^T(s) \end{bmatrix}$, according to "2", we obtain

$$\begin{bmatrix} P^T(s) \\ Q^T(s) \end{bmatrix} = \begin{bmatrix} \overline{P}^T(s) \\ \overline{Q}^T(s) \end{bmatrix} L(s),$$

here $\overline{P}(s)$ and $\overline{Q}(s)$ are a left irreducible decomposition of $W(s)$. Let

$$\overline{P}(s) = \overline{P}_0 s^{\nu} + \overline{P}_1 s^{\nu-1} + \cdots + \overline{P}_{\nu-1} s + \overline{P}_{\nu},$$

$$\overline{Q}(s) = \overline{Q}_0 s^{\nu} + \overline{Q}_1 s^{\nu-1} + \cdots + \overline{Q}_{\nu-1} s + \overline{Q}_{\nu}.$$

For the matrix sequence

$$[\overline{P}_0\ \overline{P}_1 \cdots \overline{P}_{\nu-1} \overline{P}_{\nu}],$$

carrying out the structural algorithm, we obtain

$$\left[\begin{array}{c|c} \alpha_1 & \\ \vdots & \overline{A}_0\ \ \overline{A}_1 \cdots \overline{A}_{\nu-1}\ \ \overline{A}_{\nu} \\ \alpha_p & \end{array}\right].$$

Doing the same thing to matrix sequence

$$[\overline{Q}_0\ \ \overline{Q}_1 \cdots \overline{Q}_{\nu-1}\ \ \overline{Q}_{\nu}],$$

we obtain

$$[\ \overline{B}_0\ \ \overline{B}_1 \cdots \overline{B}_{\nu-1} \overline{B}_{\nu}].$$

Since

$$\partial(q_{ij}(s))<\partial(p_{ij}(s)),\ \forall i,\ j,$$

therefore

$$\partial_{r_j}(U(s)\overline{Q}(s))<\partial_{r_j}(U(s)\overline{P}(s)),\ \forall j,$$

i.e., $\overline{B}_0=O$. Let

$$[\overline{\overline{P}}(s)\ \overline{\overline{Q}}(s)]=\begin{pmatrix} s^{\alpha_1} & & 0\\ & \ddots & \\ 0 & & s^{\alpha_p}\end{pmatrix}[\overline{A}_0\ \ 0]+\left(\begin{matrix} s^{\alpha_1-1} & & 0\\ & \ddots & \\ 0 & & s^{\alpha_{n_1}-1}\end{matrix}\ \ 0\right)[\overline{A}_1\ \ \overline{B}_1]$$

$$+\cdots+\left(\begin{matrix} s^{\alpha_1-(\alpha_1-1)} & & 0\\ & \ddots & \\ 0 & & s^{\alpha_{n_{\alpha_1-1}}-(\alpha_1-1)}\end{matrix}\ \ 0\right)[\overline{A}_{\alpha_1-1}\ \ \overline{B}_{\alpha_1-1}]$$

$$+\begin{pmatrix} I_{n_{\alpha_1}} & 0\\ 0 & 0\end{pmatrix}[\overline{A}_{\alpha_1}\ \ \overline{B}_{\alpha_1}]$$

Then

$$W(s)=\overline{\overline{P}}(s)^{-1}\overline{\overline{Q}}(s)=\overline{A}_0^{-1}[\overline{\overline{P}}(s)\overline{A}_0^{-1}]^{-1}\overline{\overline{Q}}(s).$$

Let

$$\begin{pmatrix} A_i\\ 0\end{pmatrix}=\overline{A}_i\overline{A}_0^{-1},\quad \begin{pmatrix} B_i\\ 0\end{pmatrix}=\overline{B}_i,$$

where A_1 is a $n_i\times p$ matrix and B_i is a $n_i\times m$ matrix.

Thus introduce

$$A=\begin{pmatrix} 0 & & & -A_{\alpha_1}\\ \begin{pmatrix} I_{\alpha_1}\\ 0\end{pmatrix} & 0 & 0 & -A_{\alpha_1-1}\\ & \begin{pmatrix} I_{\alpha_1-1}\\ 0\end{pmatrix} \ddots & & \vdots\\ & & 0 & -A_2\\ & \ddots & & \\ 0 & & \begin{pmatrix} I_2\\ 0\end{pmatrix} & -A_1\end{pmatrix},\quad B=\begin{pmatrix} B_{\alpha_1}\\ B_{\alpha_1-1}\\ \vdots\\ B_2\\ B_1\end{pmatrix},$$

$$C=[0\cdots 0\ \overline{A}_0^{-1}].$$

Then

$$\begin{cases} \dot{x}=Ax+Bu, \\ y=Cx \end{cases} \qquad (9)$$

is a minimal realization of $W(s)$ in the state space[8].

5. Determination of transformation matrix into Yokoyama canonical form

In [9], we express the complete controllable system

$$\begin{cases} \dot{x}=Ax+Bu, & \text{B full column rank,} \\ y=Cx \end{cases} \qquad (10)$$

as the following form

$$\begin{cases} (DI-A)x=Bu, \\ y=Cx. \end{cases}$$

Since system (10) is complete controllable system, therefore $(sI-A)$ and B are left prime. Thus applying "2" to matrix sequence

$$\left[\begin{pmatrix} I \\ 0 \end{pmatrix}\begin{pmatrix} -A^T \\ B^T \end{pmatrix}\right],$$

we get

$$\begin{bmatrix} U_1(s) & U_2(s) \\ U_3(s) & U_4(s) \end{bmatrix}\begin{bmatrix} (sI-A)^T \\ B^T \end{bmatrix}=\begin{bmatrix} G \\ 0 \end{bmatrix}.$$

After applying the structural algorithm to the coefficient matrix sequence of $U_3(s)$ and $U_3(s)$, we obtain the transformation matrix that transfer the system(10)to Yokoyama controllable canonical form and those submatrices.

When system (10) is not complete controllable, there is a similar result[9].

6. Eliminating zeros using the state feedback

Suppose the system (10) is complete controllable, $x\in \mathbf{R}^n$, $u\in \mathbf{R}^m$, $y\in \mathbf{R}^p$, B and C are full rank matrices. The transfer function of the system is

$$W(s)=R(s)P^{-1}(s),$$

where $P(s)$ is the column proper. Let

$$R(s)=R_l(s)R_r(s),$$

here $R_r(s)$ is $k\times n$ matrix

$$k=\begin{cases} p & p\leq m \\ m & p>m. \end{cases}$$

If $R_r(s)$ has full row rank and

$$\partial_{c_j}(R_r(s))<\partial_{c_j}(P(s)),\ \forall j,$$

then we can determine the state feedback matrix by the structural algorithm for the coefficient matrix sequence of $R_r(s)$. And in the transfer function of the closed loop system, $R_r(s)$ is eleminated[10].

Conclusion

This paper proposes the structural algorithm of the matrix sequence consisted of the coefficient matrix of polynomial matrix. The algorithm always stops by finite steps. At the end of carrying out the algorithm, we can obtain a row proper polynomial matrix and the power of row, and can record the transfer matrix and/or its inverse according to the request.

Using the structural algorithm, we can solve some problems in polynomial matrix theory and linear systems theory. For example, we can find the supremal (A, B)-invariant subspace of a system and solve its DDP; get the row proper of a polynomial matrix; find the r.c.r.d. of two polynomical matrices, solve the unilateral Diophantine equation of a polynomial matrix; carry out the left decomposition and left irreducible decomposition of a rational matrix; find the minimal realization of a transfer function matrix; find the transfer matrix and sub-matrices which bring a complete controllable system to Yokoyama controllable canonical form and find the state feedback matrix which eliminates the zeros of a transfer function, etc. Using the principle of duality, we can solve the dual problems as well.

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矩阵序列的结构算法和线性系统

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摘 要

本文给出了矩阵序列的结构算法．当结构算法结束时，可得正则阵列及相应的结构指数 $\{m_1:\alpha_1,\alpha_2,\cdots,\alpha_{m_1}\}$.

利用结构算法于线性系统，可以构造属于 KerC 的极大 (A,B) 不变子空间，从而可以解决系统的干扰解耦问题．

该结构算法还能用于多项式矩阵的行正则化；求两个多项式矩阵的最大右公因及右互质部份；求解多项式矩阵的 Diophantine 方程；有理分式阵的分解及（作为传递函数阵的）最小实现；化系统为 Yokoyama 标准形；用状态反馈消去系统零点．